THE SETTLING OF PARTICLES IN A SQUARE CONTAINED MEDIUM

BY

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ABSTRACT

Stokes' Law, has been standardly utilized to calculate the terminal velocity of falling particles. However, the limitation of Stokes' Law is that it does not take into account the container walls and their resulting drag forces. Extensive work with additional drag due to cylindrical container walls has been examined by many investigators. The classical and earliest is the well know Ladenburg Correction which cannot be utilized with non-cylindrical containers. This experimental thesis examines the analog of the Ladenburg relationship for a square container. This experimental thesis was undertaken to experimentally determine the value of the constant K_1 for a square contained medium. The theoretical relationships that were previously done utilized a calculated theoretical value of the constant K_1 in the formula $y \simeq K (1 - K_1 x)$.

In this series of experiments, measurements were taken on the weight, diameter and density of the spheres utilized. Temperature dependent properties of viscosity and density of the fluid medium were measured and plotted. Actual settling velocities of the spheres were measured along with fluid medium temperatures. Because of the differences in the sphere densities and temperature differences of the fluid medium each data point was considered independently. Each data point had its unique Stokes' settling velocity and this was taken into account during the calculations. The data points were plotted and

computer analyzed for the constant value K_1.

This series of experiments has experimentally determined the value of the constant K_1 to be 1.8932. This differs from the theoretically calculated K_1 value of 1.903 by 0.51%.

The plotted data points indicate increased scattering as the spheres become smaller. This appears to be directly related to the convection currents in the fluid medium.

APPROVAL OF THESIS

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PREFACE

The settling of particles is an integral part of many common processes. These processes vary from standard fluidized beds to the more recent liquid membrane modes of separation. These real life conditions can be better understood by scale modeling. An excellent model used for this purpose may be the settling of spheres in a square contained medium.

In the past there has been extensive work done on models of a falling sphere in a cylindrical or spherically contained medium. Unfortunately, in these cases, scaling up to a real life situation is difficult.

Dr. E. Bart of the New Jersey Institute of Technology Chemical Engineering Department who provided the theoretical background and purpose was also the advisor for this experimental thesis. Dr. E. Bart (2) did the theoretical calculations for the settling of spheres in a square contained medium. This theoretical work should lead to more accurate scaling up from models to a real life situation.

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BACKGROUND/THEORY

A spherical particle falling under the influence of gravity in a viscous fluid ultimately comes to a uniform terminal settling velocity. This terminal velocity is when the gravitational forces experienced by the sphere are counter balanced by the hydrodynamic forces.

Taking into account the density difference between the sphere and the surrounding fluid the gravitational force acting on the particle is:

$$Fg = (\rho s - \rho)\ g\ 4/3\ \pi\ a^3 \tag{1}$$

Where Fg = gravitational force
ρs = sphere density
ρ = fluid density
g = local acceleration of gravity
a = sphere radius

Stokes' Law for the frictional resistance or drag on a sphere is:

$$Ff = 6\ \pi \mu\ a\ Us \tag{2}$$

Where Ff = frictional resistance
μ = fluid viscosity
Us = Stokes' terminal velocity (in an unbounded medium)

Ladenburg's (4) investigation of the sphere falling in a cylinder led to his correction to the drag force:

$$Fz = 6\ \pi \mu\ Us\ a\ (1 + K\ a/Ro) \tag{3}$$

Where Ro = cylinder radius
K = constant

The Ladenburg value of K was later corrected to 2.10444. This value works only when a/Ro is small. This correction is a poor representation of a model for an array. The Ladenburg model has a cylindrical envelope that surrounds the particle. The problem with the Ladenburg model is that the spaces between the packed cylindrical envelopes is not accounted for.

Bärt (2) investigated a sphere falling in a square container. The sphere at the center of a cube should be an excellent model for a three-dimensional array.

Bärt's correction to the drag force is:

$$F_z \simeq 6 \pi \mu U a (1 + 1.903266\, a/l) \tag{4}$$

Where l = square container half width

Equating the gravitational force acting on a sphere Eqn. (1) with Stokes' Law for the frictional resistance Eqn. (2), results in Stokes' Law for the calculation of the terminal velocity of spheres falling in an unbounded medium. The standard from is:

$$U_s = \frac{g d^2 (\rho_s - \rho)}{18 \mu} \tag{5}$$

Where d = sphere diameter

This equation is only valid for laminar flow with Reynold's numbers less than one. In order to take in account for any wall effects the standard equation must be modified. The normal modification is a polynomial expansion that is generally known in the field of low Reynold's number flows. Investigators like Wakiya, Faxen and Dahl and others summarized in Happel and Brenner (3) utilize this general form. The only differences in the actual polynomial expansions utilized are in the dimensionless parameters and the coefficients determined by the particular geometry.

The following equation takes into account the drag force resulting from the container walls:

$$U = \frac{gd^2 (\rho s - \rho)}{18\mu} \quad (1 - K_1 \, d/D + K_2 \, (d/D)^2 - \ldots.) \qquad (6)$$

Which is also $U = Us \, (1 - K_1 \, d/D + K_2 \, (d/D)^2 - \ldots.)$

Where K_1, K_2 = coefficients (constants)
U = actual particle settling velocity
D = square duct width

U is a measurable quantity while Us is a theoretical abstraction, since an unbounded medium does not really exist. If the spheres become very small mathematically, it is the same as if the bounded medium becomes infinitely large. In this way, the measured settling velocity becomes the Stokes' Law for an unbounded medium:

Divide equation (6) by d^2, resulting in:

$$U/d^2 = \frac{g(\rho s - \rho)}{18\mu} \quad (1 - K_1 \, (d/D) + K_2 \, (d/D)^2 - \ldots.) \qquad (7)$$

Let $K = \frac{g(\rho s - \rho)}{18\mu}$ which is constant if temperature is constant

And let $y = U/d^2$, $x = d/D$

These substitutions into equation (7) results in:

$$y = K \, (1 - K_1 \, x + K_2 \, x^2 - \ldots.) \qquad (8)$$

For very small x, x^2 becomes negligible and equation (8) becomes:

$$y \simeq K\ (1 - K_1\ x) \tag{9}$$

Since K is not constant equation (9) must be divided by K to give a simple straight line relationship. The resulting final equation is:

$$y/K \simeq 1 - K_1\ x \tag{10}$$

Therefore, a straight line relationship for y/K versus x should be observed for small values of x. The slope should be K_1 with an intercept of 1.00.

Bart (2) determined the theoretical value of K_1 to be 1.903. The present work determined a value for this same K_1 by direct experimentation.

MEASUREMENT OF PHYSICAL PROPERTIES

The viscous fluid medium utilized in this experimental work was Ucon Lubricant, type 50 HB - 5100 by Union Carbide. The viscosity of this Ucon Lubricant is very high, similar to molasses. Its viscosity and transparency made it a good choice for reading slow settling velocities. This material is temperature sensitive. Any temperature changes or gradients would result in viscosity variations. With this in mind, the density and viscosity of the Ucon Lubricant were measured over the probable operating temperatures.

The density was measured by a calibrated 25 ml volumetric flask suspended in a constant temperature bath. A waiting period of 10-15 minutes was utilized for the Ucon Lubricant to reach equilibrium. The Ucon Lubricant was added or removed by a disposable pipet. The volumetric flask was weighed on a Mettler H-8 analytical balance. The results were plotted on a graph of density versus temperature °Celsius. (Appendix, Figure Two, pg. 22). The data points were fed into a Hewlett Packard HP-9820 Computer for least squares analysis, yielding:

$$D = 1.3733 - 1.2813\ t(^\circ c)$$ with a goodness of fit = -0.99918.

The viscosity was measured with a Cannon-Fenske Viscometer, Standard Test ASTM-D-445, with a 2.572 centistokes per second constant. Centistokes can be multiplied by the density at the given temperature to give centipoise. The viscometer was suspended in a constant temperature bath for the density readings. The results were plotted on a graph of viscosity versus temperature (Appendix, Figure Three, pg. 23). The data points were fed into a Hewlett Packard HP-9820 Computer for least squares analysis, yielding:

$$V = 4.0833 - .0080\ t(^{\circ}c) \text{ with a goodness of fit } = -0.99726.$$

The spheres were white Delrin precision made ball bearings from Industrial Tectonics. The theoretical specific gravity of Delrin is 1.425. The sphere sizes were nominally 1/8, 3/16, 1/4, 5/16, 3/8, 1/2, 3/4 inch. The individual spheres were measured for their particular density and diameter.

The individual sphere densities were determined by measuring the density of an organic mixture with the same density. The individual spheres were placed into an organic solvent mixture and the appropriate organic solvent was added with agitation until the sphere just started to rise from the bottom of the container or started to drop from the surface. The solvent mixture was pipeted into a 25 ml calibrated volumetric flask and weighed. The solvents utilized were Carbon Tetrachloride (Tetrachloromethane) Sp.G = 1.595 and 1, 2 - Di-Chloroethane Sp.G = 1.256. Each Delrin sphere was separated from each other in small labeled containers after their density measurement. Each Delrin sphere had its diameter measured by a micrometer. The average of ten readings were recorded. Sets were made up in small labeled boxes consisting of one sphere from each size. There were not enough spheres of various sizes to have one of each size in the six groups.

Each sphere was weighed on the H-8 Mettler balance. From the weights and densities, the diameters were also calculated.

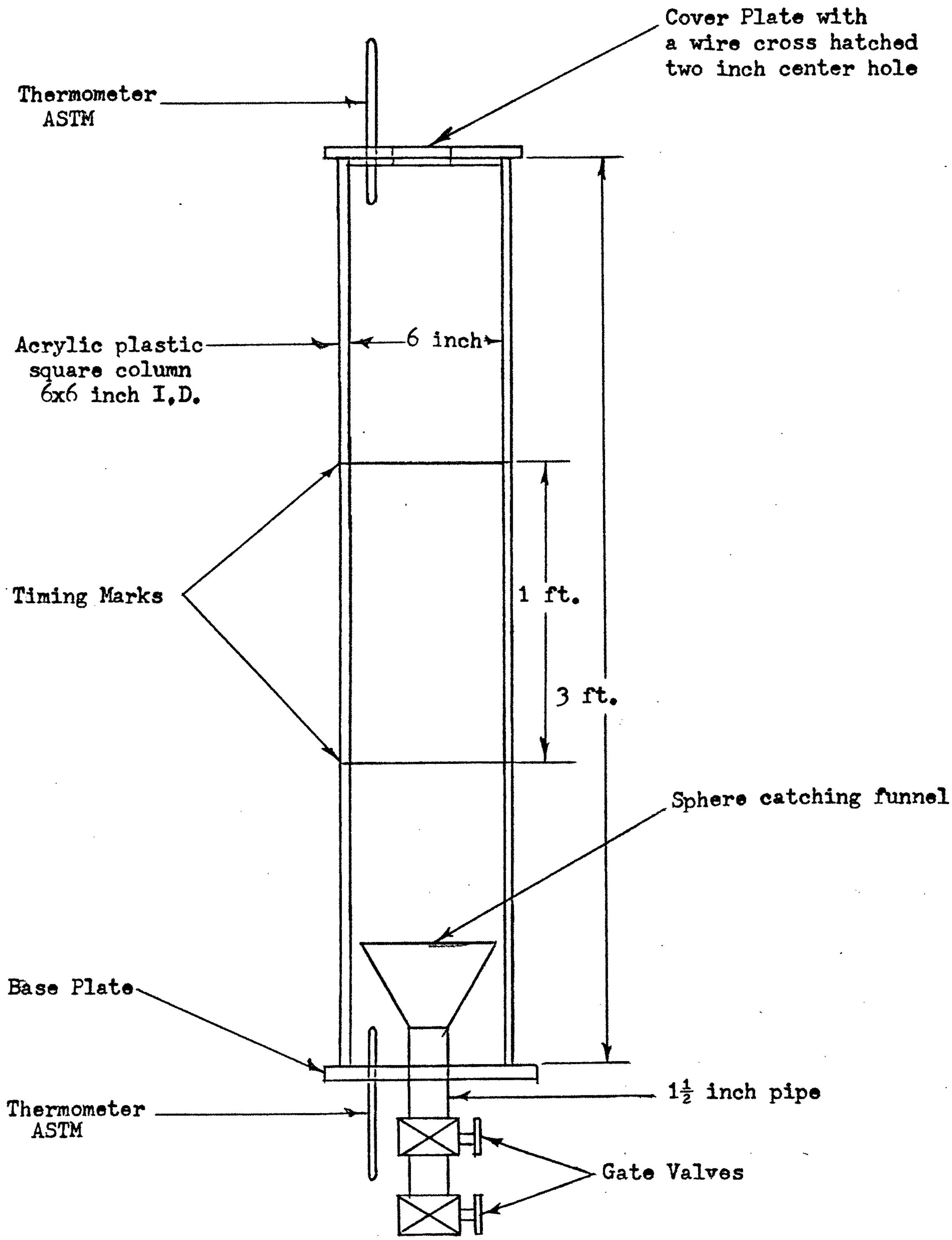
Each set of spheres was kept isolated from the other sets and labeled. This was necessary in order not to lose sphere identity since there were differences in dimensions and densities.

THE EXPERIMENTAL COLUMN

The column (Figure One, The Experimental Column) was previously made by Mr. L. B. Dight (1) for a senior project. All dimensions and construction were verified and/or repaired. The column was constructed of acrylic plastic. The sides were 3/8" thick and the base 3/4" deep. The height of the column was three feet tall and measured six inches square. The 3/8" thick acrylic top was modified for this experiment. A two inch hole was drilled in the top and a cross hatch of wires was affixed to the top. Where the wires crossed maintained the constant starting point for the dropping of the spheres.

Directly in the center of the base was a funnel leading to a two gate valve system to recover the dropped spheres. Two ASTM thermometers, calibrated to 0.01 °c were mounted on the column, one on the top and one in the base. The front and back of the column had timing marks placed on the outside at the one and two foot distances. The marks on the back were placed to ensure consistent readings. The column was adjusted by use of a bubble level to ensure proper alignment.

FIGURE ONE - THE EXPERIMENTAL COLUMN



EXPERIMENTAL PROCEDURE

All spheres were dipped into Ucon Lubricant and dropped into the column at the crossed wires by use of a pair of tweezers. The stop watch was started as the sphere passed the first timing mark and stopped when the sphere passed the second timing mark. The time was then recorded. The smallest sphere was dropped first. This procedure was repeated until all the spheres of that particular set were dropped according to increasing size. Only after this particular set of spheres was removed were any other spheres dropped.

EXPERIMENTAL RESULTS

The raw data from the experiments are shown in the Appendix, Table Two, pp. 25-29. This data was worked up to apply to equation (9). The results of this work up are summarized in the Appendix, Tables Two and Three. Sample Calculations are shown in Appendix (p. 20) for a typical run.

The resulting data were used in a least squares analysis (using a Hewlett Packard HP-9820 Computer and standard program). The results of this least squares evaluation are summarized below:

A. Calculated Sphere Diameters - All data points

Goodness of Fit: -0.7434
Intercept: 0.9972
Slope: -1.7121

B. Measured Sphere Diameters - All data points

Goodness of Fit: -0.7052
Intercept: 0.9923
Slope: -1.6326

Because of the wide range of data points with the 1/8 inch spheres a computer analysis was performed without the 1/8 inch sphere data points.

C. Calculated Sphere Diameters - Less 1/8 inch sphere data points

Goodness of Fit: -0.8839
Intercept: 1.0125
Slope: -1.8932

D. <u>Measured</u> <u>Sphere</u> <u>Diameters</u> - Less 1/8 inch sphere data points

Goodness of Fit: -0.8807
Intercept: 1.0095
Slope: -1.8366

Percent Difference from Theoretical Calculated K_1 value of 1.903.

A. $\frac{1.903 - 1.7121}{1.903} \times 100 = 10.0\%$

B. $\frac{1.903 - 1.6326}{1.903} \times 100 = 14.2\%$

C. $\frac{1.903 - 1.8932}{1.903} \times 100 = 0.51\%$

D. $\frac{1.903 - 1.8366}{1.903} \times 100 = 3.5\%$

DISCUSSION OF RESULTS

The best experimental K_1 value is 1.8932 which is approximately 0.5% different from the theoretical value of 1.903.

The data resulting from the calculated sphere diameters, omitting the 1/8 inch sphere data points, gives the least percent difference to the theoretical calculated K_1 value. The calculated diameters should give the hydraulic diameter which would take in account surface imperfections. The hydraulic diameter should have resulted in more accurate results and did so by comparing calculations C and D.

The differences between the top and bottom thermometers differed up to 2.2 °Celsius. This means a difference from the top and bottom of the column of 0.0018 gm/cc in density and 2.60 Stokes in viscosity. The temperatures in the Ucon Lubricant varied as much as 1.1 °Celsius during experimentation utilizing any single box of spheres. This resulted in a change of 0.0008 gm/cc in density and 1.25 Stokes in viscosity while the experiment was being conducted. With these different and changing temperatures, convective heat currents probably were formed. These convective currents would easily account for the increased scatter in data as the spheres got smaller.

Because of the temperature differences of the Ucon Lubricant and the density differences of the Delrin Spheres, it was absolutely necessary to calculate a K value for each data point.

Careful measurements were extremely important in this experiment due to sources of possible error. These experiments essentially involved measurement of the small deviations from Stokes' Law. When the deviations are small compared to the measured parameters, error propagation will be most unfavorable, as is the situation in these experiments.

CONCLUSIONS/RECOMMENDATIONS

The theoretical value of K_1 calculated by Bart (2) to be 1.903 has been experimentally verified well within experimental error.

All future experimentation in this area should include two things:

1.) A constant temperature room.

2.) Careful preliminary work determining the actual physical properties of the spheres and fluid to be utilized.

APPENDIX

CALCULATIONS

The average temperature (Degrees Celsius), assuming a straight line relationship, was used to calculate the density and viscosity of the Ucon Lubricant during each sphere's fall. The viscosity was changed to poise for this calculation.

Viscosity: $\mu = \frac{t^{\circ}\text{Celsius} - 40.8327}{0.00805} \times \frac{\rho}{100}$ =(poise)

Density: $\rho = \frac{t^{\circ}\text{Celsius} - 1,373.2625}{-1,281.2827}$ = gm/cc

$y = U/d^2$

$K = \frac{g(\rho s - \rho)}{18\mu}$

981.456 cm/sec^2 = g
2.54 cm/inch = 30.48 cm/ft

$K = \frac{981.456\ (\rho s - \rho)}{18\mu}$

SAMPLE CALCULATIONS

Run #1 A - Box 1/8" sphere

d(in)	ρs(gm/cc)	t(°c)	Time
0.1250	1.34364	19.0	473.6 sec.

$$\rho = \frac{19.0 - 1,373.2626}{-1,281.2827} = 1.05696 \text{ gm/cc}$$

$$\mu = \frac{19.0 - 40.8326}{0.00805} \times \frac{1.05696}{100} = 28.6342 \text{ poise}$$

$$473.6 \text{ sec/ft} \rightarrow 0.002111 \text{ ft/sec} = U$$

$$x = d/D = \frac{0.1250}{6} = 0.0208$$

$$y = U/d^2 = \frac{0.002111 \text{ ft/sec}}{(0.1250 \text{ inch})^2} \frac{(144 \text{ inch}^2)}{(1 \text{ ft}^2)} = 19.4549 \text{ 1/ft-sec}$$

$$K = \frac{g(\rho s - \rho)}{18\mu} = \frac{981.456 \text{ cm/sec}^2 (1.34364 - 1.05696 \text{ gm/cc})}{18 (28.6342 \text{ gm/cm sec})}$$

$$= 0.5458 \text{ 1/cm-sec}$$

$$K = 0.5458 \text{ 1/cm-sec} \times 30.48 \text{ cm/ft} = 16.6389 \text{ 1/ft-sec}$$

$$y/K = \frac{19.4549 \text{ 1/ft-sec}}{16.6389 \text{ 1/ft-sec}} = 1.1692$$

TABLE ONE - EXPERIMENTAL DATA

Viscosity Measurements of Ucon Lubricant.

Run	t(°c)	T(min-sec)	T(sec)	(Centistokes) Time x 2.572
1	20	16m - 45s	1005	2,584.86
2	20	16m - 58s	1018	2,618.35
3	22	14m - 59s	899	2,312.18
4	22	15m - 03s	903	2,322.52
5	24	13m - 25s	805	2,071.44
6	24	13m - 25s	805	2,070.97
7	26	12m - 04s	723	1,861.41
8	26	12m - 03s	723	1,860.48

Density Measurements of Ucon Lubricant.

Run	t(°c)	(GMS) Gross Wt.	(GMS) Net Wt.	(gm/cc) Density
1	19.4	53.571	26.414	1.05656
2	18.7	53.584	26.427	1.05708
3	21.2	53.542	26.385	1.05540
4	23.2	53.502	26.345	1.05380
5	24.8	53.465	26.308	1.05232
6	26.2	53.440	26.283	1.05132
7	27.7	53.413	26.256	1.05024

Sphere Weights: (GMS)

Sphere	A-Box	B-Box	C-Box	D-Box
1/8	0.023	0.023	0.023	0.023
3/16	0.077	0.078	0.078	0.078
1/4	0.181	0.184	0.181	0.184
5/16	0.372	0.372	0.371	0.372
3/8	0.630	0.629	0.626	0.627
1/2	1.464	1.487	1.484	1.485
3/4	4.995	4.976	5.032	5.032

Sphere Weights: (GMS)

Sphere	E-Box	F-Box
3/16	0.078	0.077
1/4	0.184	0.182
5/16	0.371	0.371
3/8	0.628	0.629
1/2	1.480	1.482

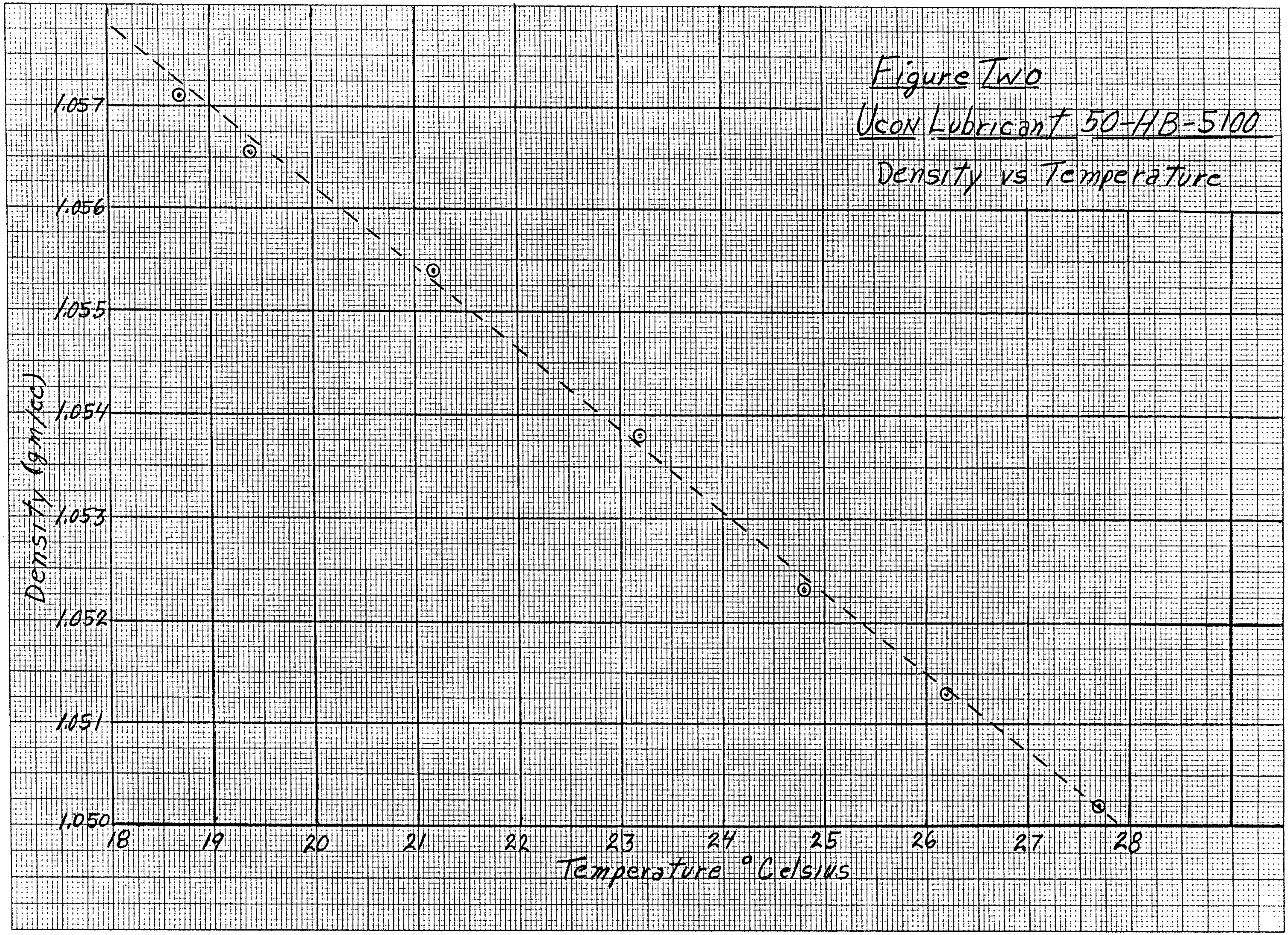


Figure Two
Ucon Lubricant 50-HB-5100
Density vs Temperature
Density (gm/cc)
1.057
1.056
1.055
1.054
1.053
1.052
1.051
1.050
18
19
20
21
22
23
24
25
26
27
28
Temperature °Celsius

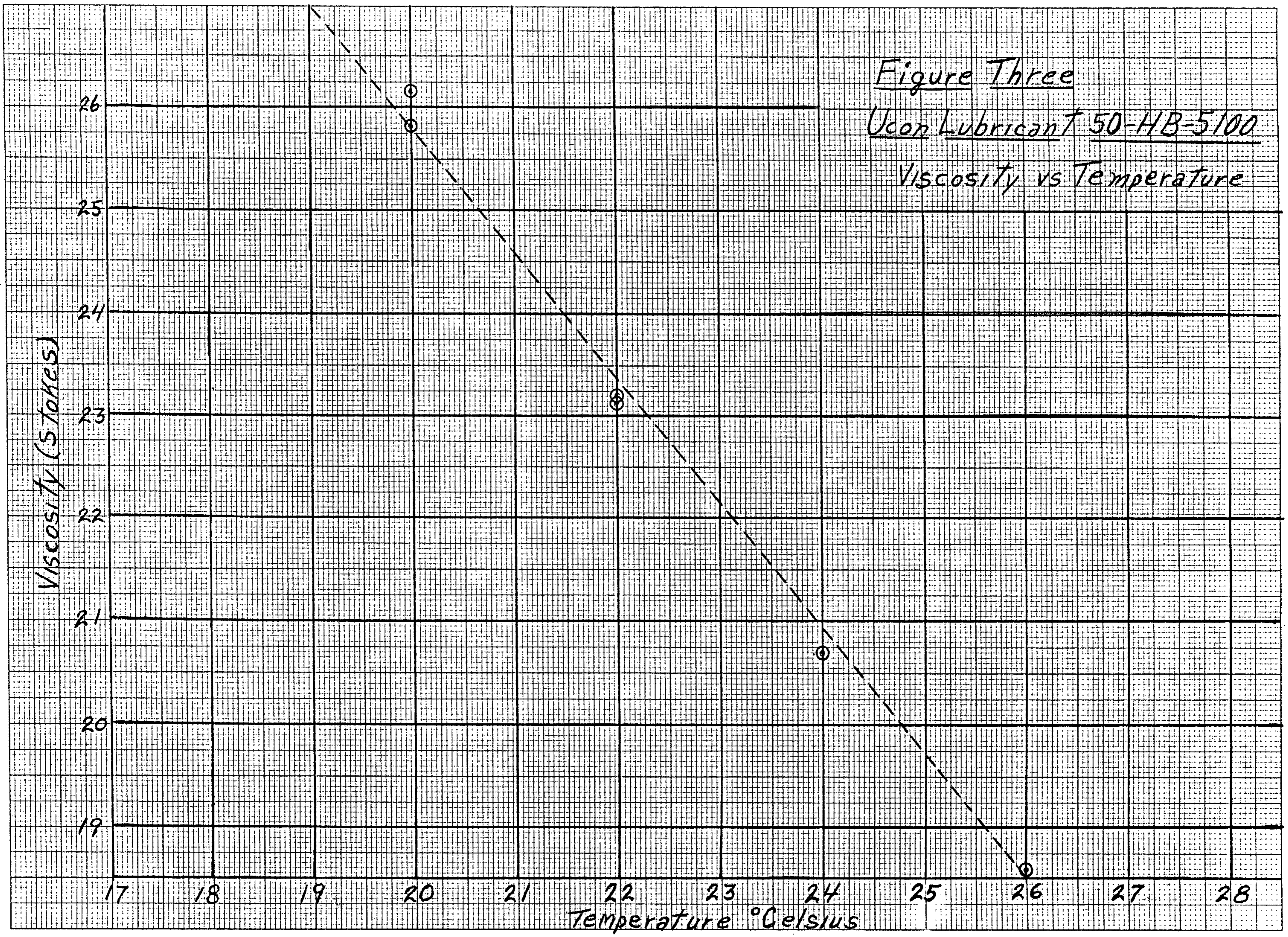


Figure Three
Ucon Lubricant 50-HB-5100
Viscosity vs Temperature
Viscosity (Stokes)
26
25
24
23
22
21
20
19
17
18
19
20
21
22
23
24
25
26
27
28
Temperature °Celsius

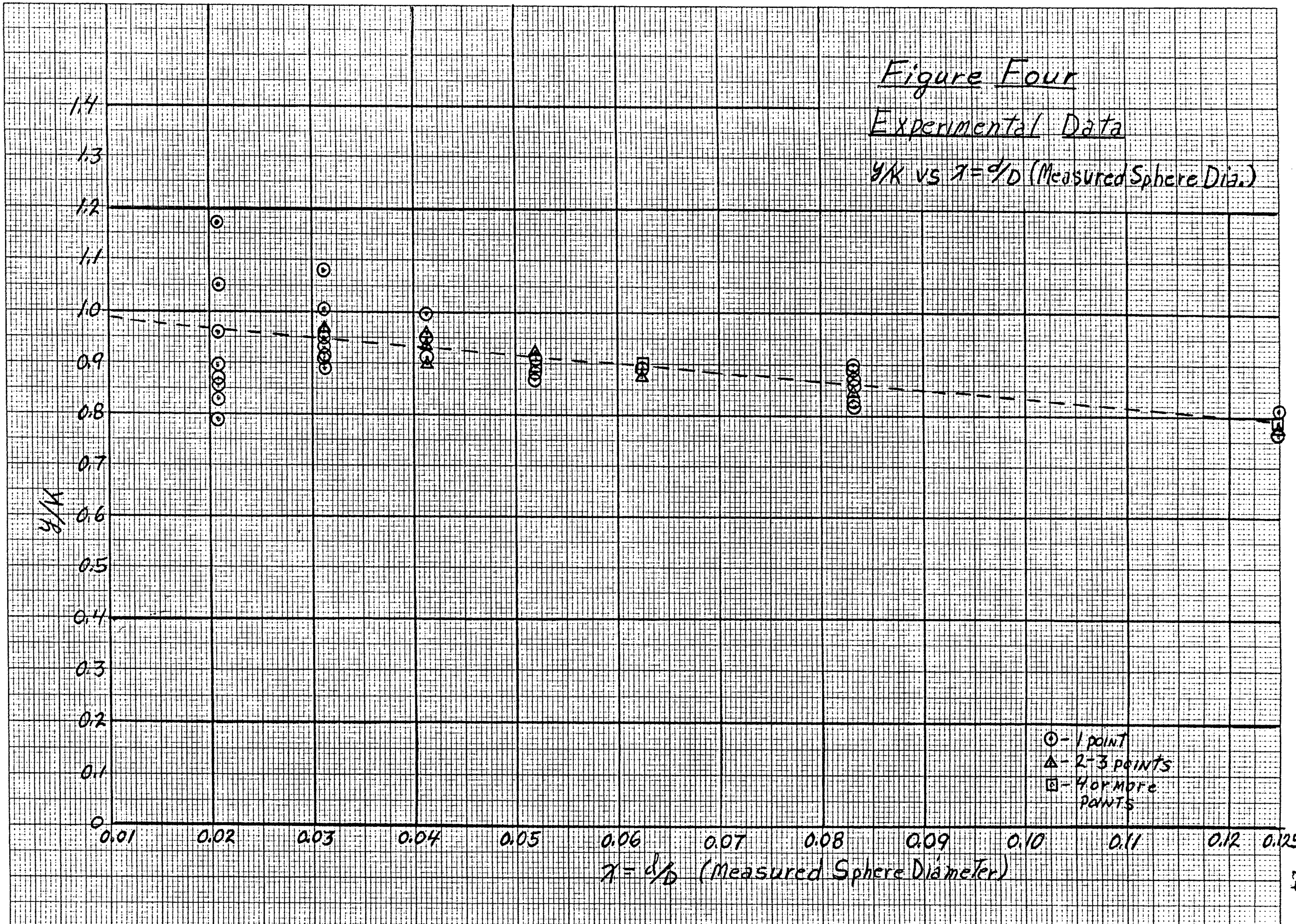


Figure Four
Experimental Data
y/K vs λ = d/D (Measured Sphere Dia.)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y/K
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.125
λ = d/D (Measured Sphere Diameter)
⊙ - 1 point
△ - 2-3 points
⊡ - 4 or more points

TABLE TWO - EXPERIMENTAL RESULTS

Run #1 A - Box

d(in)	ρ_s(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1250	1.34364	473.6	19.0	1.05696	28.6342
0.1872	1.37336	204.6	19.2	1.05680	28.3677
0.2488	1.37336	125.3	19.3	1.05672	28.2344
0.3122	1.42776	74.5	19.45	1.05660	28.0346
0.3752	1.38720	58.3	19.5	1.05656	27.9679
0.4995	1.36820	38.1	19.6	1.05649	27.8350
0.7498	1.37276	17.8	19.6	1.05649	27.8350

U(ft/sec)	x	y	K	y/K
0.002111	0.0208	19.4549	16.6389	1.1692
0.004888	0.0312	20.0854	18.5458	1.0830
0.007981	0.0414	18.5660	18.6380	0.9961
0.013423	0.0520	19.8311	22.0029	0.9013
0.017153	0.0625	17.5459	19.6475	0.8930
0.026247	0.0832	15.1485	18.6111	0.8139
0.056179	0.1249	14.3894	18.8834	0.7620

Run #2 A - Box

d(in)	ρ_s(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1250	1.34364	593.2	21.3	1.05516	25.5741
0.1872	1.37336	214.2	↓	↓	↓
0.2488	1.37336	123.5	↓	↓	↓
0.3122	1.42776	68.9	↓	↓	↓
0.3752	1.38720	53.0	↓	↓	↓
0.4995	1.36820	33.5	↓	↓	↓
0.7498	1.37276	15.8	↓	↓	↓

U(ft/sec)	x	y	K	y/K
0.001686	0.0208	15.5382	18.7468	0.8288
0.004669	0.0312	19.1856	20.6782	0.9278
0.008097	0.0414	18.8359	20.6782	0.9109
0.014514	0.0520	21.4429	24.2134	0.8856
0.018868	0.0625	19.3002	21.5776	0.8945
0.029851	0.0832	17.2286	20.3429	0.8469
0.063291	0.1249	16.2111	20.6392	0.7854

TABLE TWO - EXPERIMENTAL RESULTS

Run #3 B - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1242	1.40392	391.7	21.3	1.05516	25.5741
0.1872	1.37896	201.2			
0.2494	1.37696	117.7			
0.3124	1.42776	67.9			
0.3750	1.38880	53.0			
0.5000	1.38652	31.6			
0.7495	1.37072	16.0			

U(ft/sec)	x	y	K	y/K
0.002553	0.0207	23.8325	22.6642	1.0515
0.004970	0.0312	20.4224	21.0421	0.9705
0.008496	0.0416	19.6691	20.9122	0.9405
0.014728	0.0521	21.7312	24.2134	0.8975
0.018868	0.0625	19.3208	21.6816	0.8911
0.031646	0.0833	18.2281	21.5334	0.8465
0.062500	0.1249	16.0214	20.5066	0.7813

Run #4 C - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1255	1.31556	565.2	21.2	1.05524	25.7070
0.1877	1.37276	204.5			
0.2483	1.37336	120.2			
0.3122	1.42776	68.7			
0.3742	1.38720	52.9			
0.4993	1.38652	31.6			
0.7495	1.38424	15.3			

U(ft/sec)	x	y	K	y/K
0.001769	0.0209	16.1735	16.8294	0.9610
0.004889	0.0313	19.9827	20.5273	0.9735
0.008319	0.0414	19.4303	20.5661	0.9511
0.014556	0.0520	21.5049	24.0831	0.8929
0.018904	0.0624	19.4406	21.4609	0.9059
0.031646	0.0832	18.2792	21.4169	0.8535
0.065359	0.1249	16.7542	21.2695	0.7877

TABLE TWO - EXPERIMENTAL RESULTS

Run #5 D - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1255	1.33812	692.7	21.4	1.05509	25.4415
0.1872	1.37276	222.6	21.4	1.05509	25.4415
0.2493	1.37696	122.5	21.3	1.05516	25.5741
0.3125	1.42776	60.5	↓	↓	↓
0.3745	1.38880	53.6			
0.5000	1.38152	31.9			
0.7492	1.38656	15.5			

U(ft/sec)	x	y	K	y/K
0.001444	0.0209	13.2021	15.4323	0.8555
0.004492	0.0312	18.4583	20.7514	0.8895
0.008163	0.0416	18.9133	20.9122	0.9044
0.014388	0.0521	21.2159	24.2135	0.8762
0.018657	0.0624	19.1558	21.6816	0.8835
0.031348	0.0833	18.0564	21.2085	0.8514
0.064516	0.1249	16.5514	21.5360	0.7854

Run #6 B - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1242	1.40392	449.9	21.2	1.05524	25.7070
0.1872	1.37896	214.3	↓	↓	↓
0.2494	1.37696	122.4			
0.3124	1.42776	69.4			
0.3750	1.38880	53.8			
0.5000	1.38652	32.1			
0.7495	1.37072	16.3			

U(ft/sec)	x	y	K	y/K
0.002223	0.0207	17.8848	22.5418	0.7934
0.004666	0.0312	19.1732	20.9282	0.9161
0.008169	0.0416	18.9120	20.7988	0.9093
0.014409	0.0521	21.2605	24.0831	0.8828
0.018587	0.0625	19.0331	21.5643	0.8826
0.031153	0.0833	17.9441	21.4169	0.8378
0.061349	0.1249	15.7263	20.3955	0.7711

TABLE TWO - EXPERIMENTAL RESULTS

Run #7 A - Box

d(in)	ρs(gm/cc)	t(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1250	1.34364	572.2	21.0	1.05539	25.9726
0.1872	1.37336	220.1			
0.2488	1.37336	126.2			
0.3122	1.42776	70.6			
0.3752	1.38720	54.8	20.9	1.05548	26.1058
0.4995	1.36820	34.3			
0.7498	1.37276	16.2			

U(ft/sec)	x	y	K	y/K
0.001747	0.0208	16.1003	18.4445	0.8729
0.004543	0.0312	18.6678	20.3462	0.9175
0.007924	0.0414	18.4334	20.3367	0.9064
0.014409	0.0520	21.2878	23.8272	0.8934
0.018587	0.0625	19.0128	21.1178	0.9003
0.031153	0.0832	17.9801	19.9082	0.9031
0.061349	0.1249	15.7137	20.1985	0.7779

Run #8 C - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1255	1.31556	589.1	21.75	1.05481	24.9767
0.1877	1.37276	201.8	21.65	1.05489	25.1095
0.2483	1.37336	117.8			
0.3122	1.42776	66.7			
0.3742	1.38720	52.0			
0.4993	1.38652	30.6			
0.7495	1.38424	14.5			

U(ft/sec)	x	y	K	y/K
0.001698	0.0209	15.5243	17.3501	0.8947
0.004955	0.0313	20.2525	21.0389	0.9626
0.008489	0.0414	19.8274	21.0787	0.9406
0.014993	0.0520	22.1506	24.6793	0.8975
0.019231	0.0624	19.7768	21.9947	0.8992
0.032679	0.0832	18.8759	21.9497	0.8599
0.068966	0.1249	17.6789	21.7988	0.8110

TABLE TWO - EXPERIMENTAL RESULTS

Run #9 E - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1880	1.37896	199.3	21.65	1.05489	25.1095
0.2500	1.37696	112.9	↓	↓	↓
0.3122	1.42776	65.0			
0.3750	1.38880	51.2			
0.4998	1.37896	30.6	21.60	1.05493	25.1759

U(ft/sec)	x	y	K	y/K
0.005018	0.0313	20.4445	21.4493	0.9532
0.008857	0.0417	20.4065	21.3169	0.9573
0.015385	0.0520	22.7297	24.6793	0.9210
0.019531	0.0625	19.9997	22.1006	0.9049
0.032679	0.0833	18.8382	21.3901	0.8807

Run #10 F - Box

d(in)	ρs(gm/cc)	T(sec)	t(°c)	ρ(gm/cc)	μ(poise)
0.1875	1.37896	190.3	21.6	1.05493	25.1759
0.2488	1.37896	113.2	↓	↓	↓
0.3125	1.42776	65.3			
0.3750	1.38880	51.1			
0.5000	1.37896	30.4			

U(ft/sec)	x	y	K	y/K
0.005255	0.0312	21.5245	21.3901	1.0063
0.008834	0.0414	20.5503	21.3901	0.9607
0.015314	0.0521	22.5814	24.6115	0.9175
0.019569	0.0625	20.0386	22.0397	0.9092
0.032895	0.0833	18.9475	21.3901	0.8858

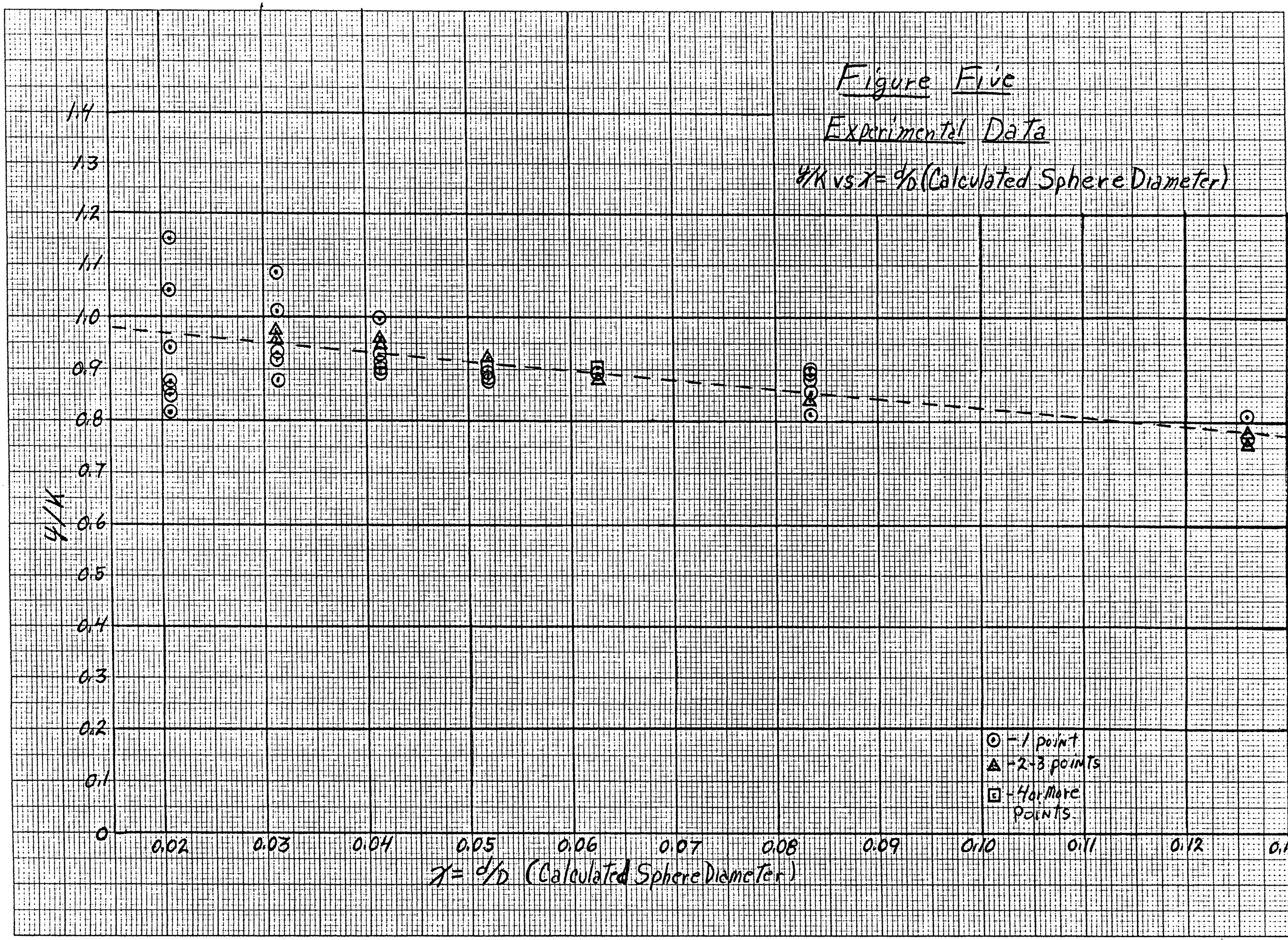


Figure Five
Experimental Data
y/K vs λ = d/D (Calculated Sphere Diameter)
y/K
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
λ = d/D (Calculated Sphere Diameter)
- 1 point
- 2-3 points
- 4 or more points

TABLE THREE - EXPERIMENTAL RESULTS

Calculated Sphere Diameters Based on Density and Weight.

Run #1 A - Box

d(in)	U(ft/sec)	x=d/D	$y=U/d^2$	K	y/K
0.1258	0.002111	0.0210	19.2083	16.6389	1.1544
0.1870	0.004888	0.0312	20.1285	18.5458	1.0853
0.2486	0.007981	0.0414	18.5959	18.6380	0.9977
0.3120	0.013423	0.0520	19.8565	22.0029	0.9024
0.3755	0.017153	0.0626	17.5179	19.6475	0.8916
0.4996	0.026247	0.0833	15.1425	18.6111	0.8136
0.7513	0.056179	0.1252	14.3321	18.8834	0.7590

Run #2 A - Box

d(in)	U(ft/sec)	x=d/D	$y=U/d^2$	K	y/K
0.1258	0.001686	0.0210	15.3411	18.7468	0.8183
0.1870	0.004669	0.0312	19.2266	20.6782	0.9298
0.2486	0.008097	0.0414	18.8662	20.6782	0.9124
0.3120	0.014514	0.0520	21.4704	24.2134	0.8867
0.3755	0.018868	0.0626	19.2694	21.5776	0.8930
0.4996	0.029851	0.0833	17.2217	20.3429	0.8466
0.7513	0.063291	0.1252	16.0646	20.6392	0.7784

Run #3 B - Box

d(in)	U(ft/sec)	x=d/D	$y=U/d^2$	K	y/K
0.1241	0.002553	0.0207	23.8709	22.6642	1.0532
0.1875	0.004970	0.0313	20.3571	21.0421	0.9674
0.2497	0.008496	0.0416	19.6218	20.9122	0.9383
0.3120	0.014728	0.0520	21.7870	24.2134	0.8998
0.3751	0.018868	0.0625	19.3105	21.6816	0.8906
0.5000	0.031646	0.0833	18.2281	21.5334	0.8465
0.7507	0.062500	0.1251	15.9702	20.5066	0.7788

Run #4 C - Box

d(in)	U(ft/sec)	x=d/D	$y=U/d^2$	K	y/K
0.1268	0.001769	0.0211	15.8435	16.8294	0.9414
0.1877	0.004889	0.0313	19.9827	20.5273	0.9734
0.2486	0.008319	0.0414	19.3835	20.5661	0.9425
0.3117	0.014556	0.0520	21.5740	24.0831	0.8958
0.3747	0.018904	0.0625	19.3887	21.4609	0.9034
0.5004	0.031646	0.0834	18.1990	21.4169	0.8497
0.7506	0.065359	0.1251	16.7052	21.2695	0.7854

TABLE THREE - EXPERIMENTAL RESULTS

Calculated Sphere Diameters Based on Density and Weight.

d(in)	U(ft/sec)	x=d/D	$y=U/d^2$	K	y/K
Run #5	**D - Box**				
0.1261	0.001444	0.0210	13.0767	15.4323	0.8474
0.1878	0.004492	0.0313	18.3405	20.7514	0.8838
0.2497	0.008163	0.0416	18.8528	20.9122	0.9015
0.3120	0.014388	0.0520	21.2840	24.2135	0.8790
0.3747	0.018657	0.0625	19.1354	21.6816	0.8826
0.5004	0.031348	0.8340	18.0276	21.2085	0.8500
0.7506	0.064516	0.1251	16.4897	21.5360	0.7657
Run #6	**B - Box**				
0.1241	0.002223	0.0207	20.7854	22.5418	0.9221
0.1875	0.004666	0.0313	19.1119	20.9282	0.9132
0.2497	0.008169	0.0416	18.8666	20.7988	0.9071
0.3120	0.014409	0.0520	21.3151	24.0831	0.8851
0.3751	0.018587	0.0625	19.0229	21.5643	0.8821
0.5000	0.031153	0.0833	17.9441	21.4169	0.8378
0.7507	0.061349	0.1251	15.6761	20.3955	0.7686
Run #7	**A - Box**				
0.1258	0.001747	0.0210	15.8962	18.4445	0.8618
0.1870	0.004543	0.0312	18.7078	20.3462	0.9195
0.2486	0.007924	0.0414	18.4631	20.3367	0.9079
0.3120	0.014409	0.0520	21.3151	23.8272	0.8946
0.3755	0.018587	0.0626	18.9824	21.1178	0.8989
0.4996	0.031153	0.0833	17.9729	19.9082	0.9028
0.7513	0.061349	0.1252	15.6510	20.1985	0.7749
Run #8	**C - Box**				
0.1268	0.001698	0.0211	15.2076	17.3501	0.8765
0.1877	0.004955	0.0313	20.2525	21.0389	0.9626
0.2486	0.008489	0.0414	19.7796	21.0787	0.9384
0.3117	0.014993	0.0520	22.2217	24.6793	0.9004
0.3747	0.019231	0.0625	19.7241	21.9947	0.8968
0.5004	0.032679	0.0834	18.7930	21.9497	0.8562
0.7506	0.068966	0.1251	17.6271	21.7988	0.8086

TABLE THREE - EXPERIMENTAL RESULTS

Calculated Sphere Diameters Based on Density and Weight.

Run #9 E - Box

d(in)	U(ft/sec)	x=d/D	y=U/d^2	K	y/K
0.1875	0.005018	0.0313	20.5537	21.4493	0.9582
0.2497	0.008857	0.0416	20.4556	21.3169	0.9596
0.3117	0.015385	0.0520	22.8027	24.6793	0.9240
0.3749	0.019531	0.0625	20.0104	22.1006	0.9054
0.5001	0.032679	0.0834	18.8156	21.3901	0.8796

Run #10 F- Box

d(in)	U(ft/sec)	x=d/D	y=U/d^2	K	y/K
0.1867	0.005255	0.0311	21.7093	21.3901	1.0149
0.2487	0.008834	0.0415	20.5669	21.3901	0.9615
0.3117	0.015314	0.0520	22.6975	24.6115	0.9222
0.3751	0.019569	0.0625	20.0280	22.0397	0.9087
0.5003	0.032895	0.0834	18.9248	21.3901	0.8847

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