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THERMAL STRATIFICATION AND CIRCULATION OF WATER BODIES SUBJECTED TO THERMAL DISCHARGE

BY

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ABSTRACT

A three-dimensional analytical model for large water bodies is presented. Time histories and spatial distribution of pressure, velocity and temperature in water bodies, subjected to thermal discharge, are determined employing a digital computer. The dynamic response is obtained for a rectangular water body by applying a finite difference method to the mass, momentum and energy balance equations. These partial differential equations are algebraically manipulated to obtain; 1) three parabolic differential equations integrated temporally to find the horizontal velocity components and temperature; 2) one algebraic integral equation to get the vertical velocity component; 3) one elliptic differential equation integrated spatially to find the pressure; and 4) one differential equation to get the water level. Numerical stability criteria are developed which facilitate the selection of space and time increments for stable numerical integration.

The distinctive feature of this analysis, as compared to previous studies, is the calculation of pressure and water level from equations of motion without simplifying assumptions such as hydrostatic pressure approximation and rigid-lid concept.

The mathematical formulation is verified by applying this analysis to cases where the final steady state flow patterns have been determined analytically or experimentally by others. In particular, the final steady state solution obtained from this

dynamic analysis is verified with existing flow measurements of laminar flow development in a square duct. Furthermore, the natural circulation flows developed by this analysis are verified with known flow patterns in partially heated ponds.

The problem of thermal discharge entering a river with known initial velocity and temperature distribution is then analyzed. The time histories of the velocity and temperature distribution as well as the velocity and temperature profiles are obtained. These results provide the values of temperature rise and the rate of temperature rise needed for the assessment of the extent of thermal pollution in water bodies.

APPROVAL OF DISSERTATION

THERMAL STRATIFICATION AND CIRCULATION OF
WATER BODIES SUBJECTED TO THERMAL DISCHARGE

BY

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FOR

DEPARTMENT OF MECHANICAL ENGINEERING
NEW JERSEY INSTITUTE OF TECHNOLOGY

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1. INTRODUCTION

Thermal power and industrial processing plants use large quantities of cooling water from natural or artificial water bodies and return the water to the source at higher temperatures. Changes that occur, either in the temperature of the water or in temperature-related water quality constituents, may adversely affect the aquatic environment of the water source. When the thermal discharge from a plant, into a water body, raises the water temperature by such an extent as to damage aquatic life or other legitimate uses of the water source, some degree of thermal pollution exists.

To assess the extent of a thermal pollution, utilities, ecologists, federal and state agencies as well as the public are interested in determining the space and time distribution of temperature within a water body, (such as lakes, reservoirs and rivers) when it is subjected to a prescribed thermal load.

The temperature distribution within a water impoundment is intimately associated with the hydrodynamic behavior of the water mass. Fluid flow in the impoundment is, in turn, affected by the hydrodynamic boundary conditions, three-dimensionality of the water body and the degree of stratification imposed on the water mass. The problem is further complicated by the heat transfer process through the water surface, internal mixing, inflows and outflows. For these reasons, a quantitative prediction of the temperature field for a particular water source must consider the mass-transport, momentum-transport, and heat-transport processes which produce the temperature changes.

The theoretical analysis of the dynamic behavior of a large water body, subjected to thermal discharge, is based on the solution of space- and time-dependent conservation and state equations. Conservation of mass, momentum, and energy for the water flow in the impoundment together with the equation of state, provide a sufficient number of partial differential equations and algebraic relations in space and time for the solution of the problem. Although, these equations have been known for over a century, due to their highly nonlinear nature, a direct closed form analytical solution is considered practically impossible for most general cases involving realistic geometries and boundary conditions. The available analytical solutions of problems involving thermal discharge is severely limited by the many simplifying assumptions made in order to achieve a solution. In fact, analytical solutions to the problems of thermal discharge are obtained only for problems involving simple geometry and boundary conditions. Solutions to more realistic geometries and boundary conditions can be sought only by the development of computer modeling in three-dimensions under transient conditions.

Current three-dimensional computer simulation of thermal discharge into water bodies are based on a number of simplifying assumptions discussed hereunder:

1. Compliance with the Conservation of Mass and Momentum

Generally, the three components of water velocity are found from the longitudinal, lateral, and vertical momentum equations. These three components of velocity should satisfy the conservation of mass equation. A major difficulty arises when simultaneous compliance with the principles of conservation of mass and momentum cannot be ensured. Under the Boussinesq approximation, the conservation of mass reduces to zero divergence for the water velocity (with no mass storage term) i.e., the sum of all inflows to and outflows from any fluid control volume in the flow field must vanish. Since the new velocity components, calculated by the integrations of the momentum equations do not necessarily satisfy the continuity equation, the mass balance would be disturbed, thereby yielding a small surplus or deficit inflow at certain control volumes. To resolve this dilemma, Brady and Geyer (2)[1] hypothesize that the surplus or deficit inflows redistribute in all directions by equal magnitude. Obviously, the disadvantage of this velocity adjustment technique is that the adjusted velocities no longer satisfy the momentum equations. Considering a water body, such as a river, flowing longitudinally with a laterally uniform velocity distribution, and applying the approximate boundary conditions on the lateral velocities at the thermal discharge location, they found that the out-of-balance surplus inflow to each adjacent cell is exceptionally large. When their model attempts to distribute this surplus inflow among the surrounding

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[1]Underlined numerals in parenthesis designate references listed in Section 8.

fluid cells, the resulting vertical velocity adjustment becomes excessively large which aborts the run. Brady and Geyer referred to this problem as the "vertical over-responsiveness of the model" and devised schemes to suppress the associated undesirable vertical fluctuations by either: 1) temporarily suppressing vertical redistribution such that each fluid layer achieves its own mass balance independently; or 2) temporarily enforcing a rigid lid at the fluid free surface such that a zero vertical velocity is maintained at the most upper layer of the model at every fluid cell. The rigid lid assumption is used by many authors to simplify the solution to the stratification and circulation in water bodies (18, 20, 32). Obviously, this situation calls for an improved approach for a simultaneous compliance with the conservation equations of mass and momentum.

2. Surface Flow Distribution

The treatment of the slope variation at the water free surface constitutes another major difficulty. The upwelling (or downwelling) flow at the surface creates positive (or negative) surge waves (36, 10). The mathematical modeling of these phenomena under unsteady flow conditions is complicated. To resolve this difficulty, Waldrop and Farmer (39) first assumed that the vertical component of velocity near the surface will be redirected in the horizontal direction (in x and y directions) and the surface will move. Later, Waldrop and Farmer (40), in a major effort to resolve the surface flow distribution problem, introduced a mass balance at the free surface and extended their horizontal momentum equations to elements near the water surface. This situation calls for further studies of the behavior of the free surface under unsteady flow conditions.

3. Pressure Distribution

The velocity components in the flow field are extremely sensitive to values of nodal pressures. Slight changes in the pressure distribution would affect the circulation in the water body considerably. The calculation of the correct pressure distribution is, therefore, of paramount importance. Most authors (41, 42, 19, 4, 16, 23, 8) assume that the total dynamic pressure at each point in the flow is equal to hydrostatic pressure obtained under static conditions. This assumption, referred to as hydrostatic approximation, leads to inaccuracies in regions of severe upwelling and downwelling which will adversely affect the correctness of circulation velocities. Thus, a better technique for the computation of the pressure field is called for.

4. Numerical Stability

Calculations of space and time increments for obtaining a stable numerical solution of partial differential equations of mass, momentum and energy conservation is a difficult task. Most authors resort to overly simplified criteria for the establishment of the upper bound of the integration time step for an assumed set of space increments. Brady and Geyer (2) state that the maximum integration step appears to be limited by relationships between the velocities and the smallest cell dimensions in each direction. Waldrop and Farmer (39) and Harlow and Welch (9) resort to a stability limit calculated on the basis of one-dimensional, unsteady, incompressible flow equations with a free surface. This limit imposes an upper bound on their integration time step equal to the ratio of the smallest cell size and the surface wave speed. More accurate means for the prediction of the space and time increments are needed.

Most authors believe that much work is still required to refine various features of the present unsteady three-dimensional computer models before the present models can be applied to situations involving significant stratification effects, or flow fluctuations, in water bodies.

1.1 History and Background

Wind-induced circulation in shallow waters has been studied by Liu and Perez (22), Liggett and Hadjitheodorou (18, 19, 20), among others. These studies neglect thermal stratification and employ unsteady momentum and continuity equations to determine the pressure and velocity distribution as functions of space and time in large water bodies. The problem of circulation in stratified lakes idealized as a two layer lake, in which epilimnion and hypolimnion are considered homogeneous has been analyzed by Liggett and Lee (15, 21, 10). These studies neglect the existence of the thermocline and are unrealistic.

The aforementioned studies do not take into account the coupled hydrometerological phenomena controlling the disposition of energy at the surface of water bodies and neglect the mechanisms that are responsible for distributing energy internally throughout the fluid mass.

Orlob and Selna (27) developed a digital computer program for the calculation of temperature variations in deep reservoirs. Their analysis is limited by the assumption that all transfer of heat energy is accomplished along the vertical axis. Stefan (35) presented a two-dimensional mathematical formulation consisting of continuity, momentum and energy equations for the modeling of heated water over lakes. However, he made no attempt to solve the governing equations.

Brady and Geyer (2) provided a comprehensive review of analytical modeling techniques for the solution of problems involving thermal discharges and presented a general computer model for simulating thermal discharge in three dimensions. They established the feasibility of their technique by a sample application to a river site where density effects are relatively insignificant. The compliance with the conservation of mass and momentum is obtained by redistributing the surplus or deficit inflows in a control volume in all directions by equal magnitudes. This feature was responsible for their observed "vertical over-responsiveness of the model" leading to excessive vertical velocities, as discussed earlier. They recommended further development work for situations involving significant stratification effects and fluctuating flows.

Waldrop and Farmer (39, 40), in a three dimensional study of buoyant plumes, examined the flow field resulting from the interaction of a stream with the much larger body of flowing water into which it debouches. They expressed the local density of the water as function of salinity (39) and temperature (40). Three dimensional unsteady conservation equations, used in their study to describe the interaction, included the effects of buoyancy, inertia, and the difference in the hydrostatic heads of the two currents. They solved the conservation equations with a time-dependent finite difference technique. The treatment of the slope variation at the water free surface is accomplished as discussed earlier in surface flow distribution. The pressure distribution is calculated from the vertical momentum equations after neglecting the fluid vertical inertia.

Leendertse et al (16), in a study similar to that of Waldrop and Farmer (40), developed a three-dimensional model for estuaries, bays, and coastal seas in which non-isotropic density conditions exist. They performed a numerical integration of the finite difference equations of motion, transport, and continuity and applied their computational method to the analysis of a number of basins with boundaries of increasing complexity. They employ the hydrostatic approximation discussed earlier and neglect the vertical acceleration altogether.

Mercier (24, 25) developed a predictor-corrector method for the solution of incompressible viscous fluid and applied this method to the transient flow in a density stratified reservoir. His solution is restricted to two-dimensional and adiabatic flow. Rather than considering the density as a function of temperature, Mercier obtains the water density by setting the material derivative of the density equal to zero.

Spradley and Churchill (34) examined the problem of pressure- and buoyancy-driven thermal convection in a rectangular enclosure for unsteady laminar compressible flow at various reduced levels of gravity. The enclosure was heated on one side and cooled on the opposite side. The conservation equations of mass, momentum, and energy were solved numerically for a compressible, heat-conducting ideal gas employing an explicit finite-difference technique. Their solution show that the thermally induced motion is acoustic in nature owing to the sonic character of the induced pressure waves.

Further reviews of the state of art for the solution of the conservation equations of mass, momentum, and energy as well as the study of the behavior of water bodies subjected to thermal dischage are available (28, 33, 30, 29) and interested readers are referred to these references for additional information.

A careful examination of the above references demonstrates that a completely adequate mathematical technique for the combined analysis of thermal stratification and circulation of water bodies is still lacking. A rigorous mathematical simulation for the prediction of the thermal and hydraulic behavior of impoundments is needed.

1.2 Scope and Objectives of the Present Study

The main objectives of the present study are as follows:

1) To develop a three-dimensional model for the mathematical description of a large rectangular water body free from the major shortcomings described earlier.

2) To develop a numerical integration technique for solving the above mathematical model on a digital computer.

3) To develop a digital computer program for predicting the thermal stratification and circulation phenomenon in a given rectangular water body subjected to thermal discharge and to determine the time histories and spatial distribution of pressure, velocity and temperature fields within the water body.

The major contributions of the present study, as compared with the previous works in this general area, can be summarized as follows:

1) Compliance with the conservation of mass and momentum is obtained by the introduction of two flow regions in the entire flow field: a) the water-level region containing a portion of water near the free surface in which the water-level rises or falls, as the case may be, during the dynamic solution; and b) the sub-water-level region located under the water-level region which remains totally filled with fluid at all times during the transient. As will be seen later, the superimposition of a three-dimensional grid on these two regions facilitates the simultaneous compliance of the principles of conservation of mass and momentum for the entire flow field without any mass imbalance.

2) A different set of differential equations for the conservation of mass, momentum and energy is applied to the cells located in the water-level and sub-water-level flow regions. This is necessary because the cells in the water-level region have a variable height while the cells in the sub-water level region have a constant height. This feature facilitates the calculation of surface flow distribution.

3) The pressure distribution is obtained by combining the momentum and continuity equations. This feature eliminates the need for the hydrostatic pressure approximation. It also eliminates the need for neglecting the acceleration terms in the momentum equations in the vertical direction.

4) A detailed numerical stability analysis is performed which provides accurate criteria for the selection of the space and time increments.

2. MATHEMATICAL FORMULATION

2.1 Simplifying Assumptions

The mathematical formulation in this study is based on the following assumptions:

1) The equations of motion, energy and continuity are applied in their time-smoothed form to turbulent incompressible three-dimensional flow with Cartesian coordinates, as shown in Figure 1, with z positive upward.

2) It is assumed that Boussinesq's approximation applies. In other words, densities are treated as constants except in terms involving gravity. This allows natural circulation to take place. Furthermore, density in the body force term is considered to be a function of temperature only.

3) The effects of turbulence are modeled by using eddy transport coefficients. Horizontal and vertical momentum eddy viscosities and thermal eddy diffusivities are considered as constants throughout the body of water though different magnitudes for horizontal and vertical directions.

4) It is assumed that there are no internal heat sources and the heat exchange between the water body and the atmosphere takes place near the water free surface.

5) Loss of mass due to evaporation at the surface and conductive heat transfer through the impoundment solid boundaries are generally small and are neglected.

6) The Coriolis forces acting on the water body is considered to

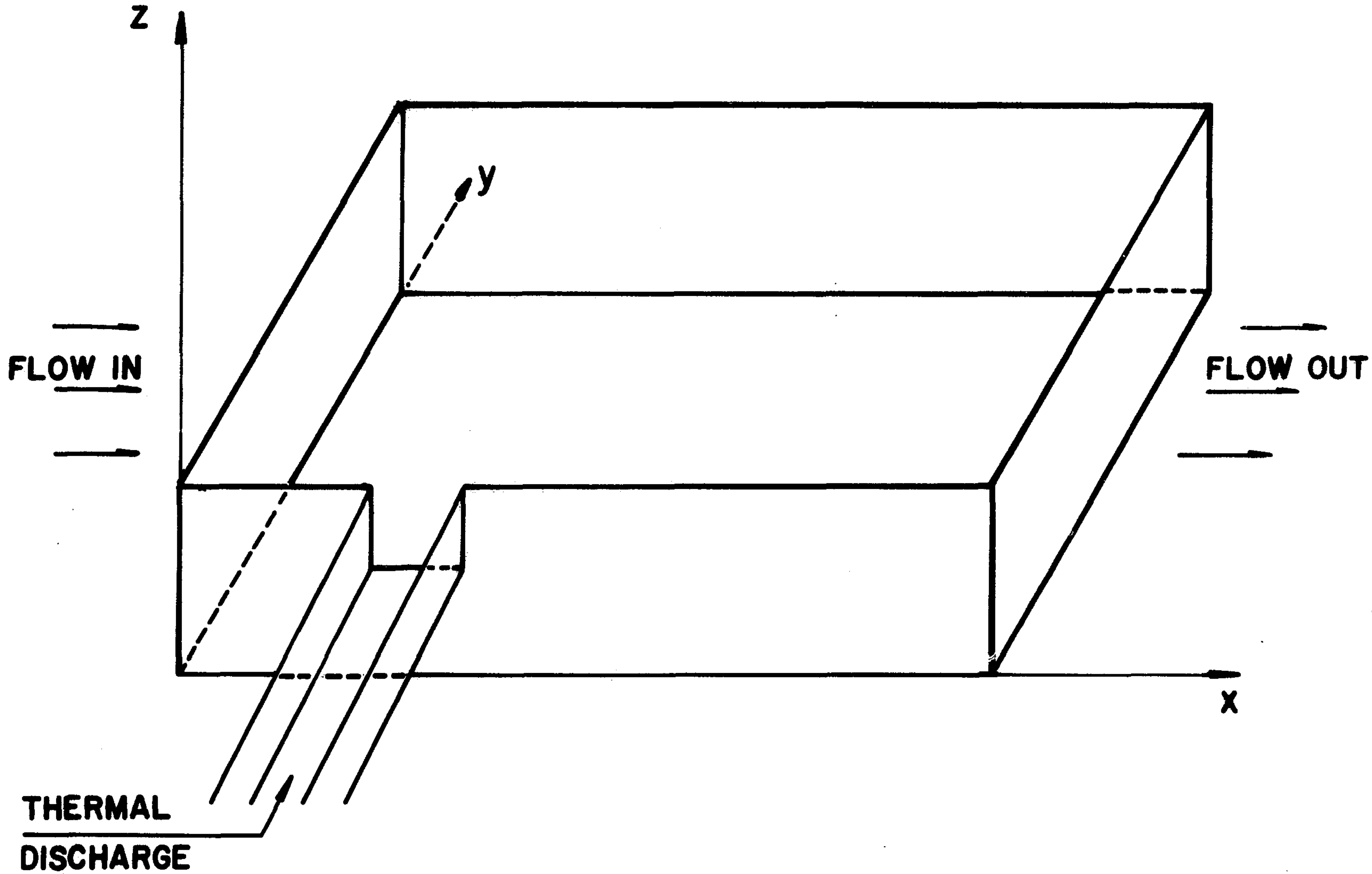


FIG 1. SCHEMATIC DIAGRAM OF THE WATER BODY

be negligible.

The boundary conditions of the problem involve: 1) the geometry of the impoundment including the thermal discharge inlet, as well as the river inflow and outflow configurations; 2) the mass flow rates of the inflows across the boundaries; and 3) the level, pressure and temperature along the inlet boundaries. It is assumed that all of the above parameters are known.

The initial conditions of the problem includes the pressure, velocity, level and temperature distributions of the impoundment at time t = 0. These variables are computed by the present program by first performing a dynamic analysis under no thermal discharge conditions and then using the calculated pressure, velocity, level and temperature distributions, as initial conditions, in a dynamic analysis involving a thermal discharge.

2.2 Governing Equations

The mathematical formulation of the sub-water-level flow region consists of the following fundamental equations derived in Appendix 1.

Momentum equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu_h\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \nu_v\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \nu_h\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \nu_v\frac{\partial^2 v}{\partial z^2} \qquad (2)$$

$$\frac{\partial w}{\partial t} + u\,\frac{\partial w}{\partial x} + v\,\frac{\partial w}{\partial y} + w\,\frac{\partial w}{\partial z} = -\,\frac{1}{\rho_0}\,\frac{\partial p}{\partial z} + \nu_h\,\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \nu_v\,\frac{\partial^2 w}{\partial z^2} - g\,\frac{\rho}{\rho_0} \tag{3}$$

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Energy equation:

$$\frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} + w\,\frac{\partial T}{\partial z} = D_h\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + D_v\,\frac{\partial^2 T}{\partial z^2} \tag{5}$$

Equation of state:

$$\rho = \rho\,(T) \tag{6}$$

In the above formulation, u, v and w are the local velocity components in the x, y and z directions respectively; p is the pressure; T is the local water temperature; ρ is the local density of water; ρ_0 is a fixed reference density to be defined later; and ν_h, ν_v, D_h and D_v are the horizontal and vertical components of eddy viscosity and eddy diffusivity of heat, respectively. The above differential equations are solved simultaneously, subject to the boundary conditions described in the following section. To solve the above mathematical formulation, the following steps will be taken:

1) Non-dimensionalization of the governing equations and derivation of time-derivative equations for velocity components u, v, w and temperature T.

2) Derivation of the final equations including the pressure equation.

In this study, the thermal discharge parameters are used as reference values for the non-dimensionalization of the governing equations. Let U_0, ρ_0, T_0 and d_0 be the velocity, density, temperature and half width of thermal discharge flow respectively. The dimensionless quantities are defined by

$$x^* = \frac{x}{d_0} \qquad y^* = \frac{y}{d_0} \qquad z^* = \frac{z}{d_0} \tag{7}$$

$$u^* = \frac{u}{U_0} \qquad v^* = \frac{v}{U_0} \qquad w^* = \frac{w}{U_0} \tag{8}$$

$$t^* = \frac{tU_0}{d_0} \qquad T^* = \frac{T}{T_0} \qquad p^* = \frac{p}{\rho_0 g d_0} \tag{9}$$

$$\rho^* = \frac{\rho}{\rho_0} \tag{10}$$

Employing the above dimensionless variables in the continuity, momentum and energy equations, one obtains the time derivatives for velocity components u, v, w and temperature T in parabolic differential form as follows:

$$\frac{\partial u^*}{\partial t^*} = - u^* \frac{\partial u^*}{\partial x^*} - v^* \frac{\partial u^*}{\partial y^*} - w^* \frac{\partial u^*}{\partial z^*} - \frac{1}{F_0^2} \frac{\partial p^*}{\partial x^*}$$
$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 u^*}{\partial z^2} \tag{11}$$

$$\frac{\partial v^*}{\partial t^*} = - u^* \frac{\partial v^*}{\partial x^*} - v^* \frac{\partial v^*}{\partial y^*} - w^* \frac{\partial v^*}{\partial z^*} - \frac{1}{F_0^2} \frac{\partial p^*}{\partial y^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 v^*}{\partial z^{*2}} \tag{12}$$

$$\frac{\partial w^*}{\partial t^*} = - u^* \frac{\partial w^*}{\partial x^*} - v^* \frac{\partial w^*}{\partial y^*} - w \frac{\partial w^*}{\partial z^*} - \frac{1}{F_0^2} \frac{\partial p^*}{\partial z^*} - \frac{\rho^*}{F_0^2}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 w^*}{\partial z^{*2}} \tag{13}$$

$$\frac{\partial T^*}{\partial t^*} = - u^* \frac{\partial T^*}{\partial x^*} - v^* \frac{\partial T^*}{\partial y^*} - w^* \frac{\partial T^*}{\partial z^*}$$

$$+ \frac{D_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^*}\right) + \frac{D_v}{\nu_0} \frac{\partial^2 T^*}{\partial z^{*2}} \tag{14}$$

with continuity and state equations given by

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{15}$$

$$\rho^* = \rho^* (T^*) \tag{16}$$

In the above equations, the following dimensionless numbers are used

1) Froude number $F_0 = U_0/(gd_0)^{1/2}$ (17)

2) Reynolds number $R_0 = U_0 d_0/\nu_0$ (18)

Equations (11) to (16) are a set of six equations that will be used for the determination of six unknowns, u^*, v^*, w^*, p^*, ρ^*, and T^*, as functions of time and space.

The final equations, including the pressure equation, are obtained from the above equations as follows. First, the continuity equation is differentiated with respect to time to give

$$\frac{\partial}{\partial t^*}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right) = 0 \tag{19}$$

The order of differentiation in equation (19) is then interchanged

$$\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial t^*}\right) + \frac{\partial}{\partial y^*}\left(\frac{\partial v^*}{\partial t^*}\right) + \frac{\partial}{\partial z^*}\left(\frac{\partial w^*}{\partial t^*}\right) = 0 \tag{20}$$

To facilitate spatial differentiations indicated by equation (20), equations (11) through (13) are first rewritten in the following form

$$\frac{\partial u^*}{\partial t^*} = Q_x^* - \frac{1}{F_0^2}\frac{\partial p^*}{\partial x^*} \tag{21}$$

$$\frac{\partial v^*}{\partial t^*} = Q_y^* - \frac{1}{F_0^2}\frac{\partial p^*}{\partial y^*} \tag{22}$$

$$\frac{\partial w^*}{\partial t^*} = Q_z^* - \frac{1}{F_0^2}\frac{\partial p^*}{\partial z^*} \tag{23}$$

where

$$Q_x^* = - u^* \frac{\partial u^*}{\partial x^*} - v^* \frac{\partial u^*}{\partial y^*} - w^* \frac{\partial u^*}{\partial z^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 u^*}{\partial z^{*2}} \tag{24}$$

$$Q_y^* = - u^* \frac{\partial v^*}{\partial x^*} - v^* \frac{\partial v^*}{\partial y^*} - w^* \frac{\partial v^*}{\partial z^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 v^*}{\partial z^{*2}} \tag{25}$$

$$Q_z^* = - u^* \frac{\partial w^*}{\partial x^*} - v^* \frac{\partial w^*}{\partial y^*} - w^* \frac{\partial w^*}{\partial z^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 w^*}{\partial z^{*2}} - \frac{\rho^*}{F_0^2} \tag{26}$$

with

$$\frac{\partial w^*}{\partial z^*} = - \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial z^*}\right) \tag{27}$$

substituting equations (21) through (23) into (20), yields

$$\nabla^2 p^* = F_0^2 \left(\frac{\partial Q_x^*}{\partial x^*} + \frac{\partial Q_y^*}{\partial y^*} + \frac{\partial Q_z^*}{\partial z^*}\right) \tag{28}$$

This elliptic differential equation will provide a means for the calculation of the pressure distribution in the water body. Equations (27), (24), (25), (26), (28), (21), (22), (14), and (16) constitute our final equations which will be solved for updated values of w^*, Q_x^*,

Q_y^*, Q_z^*, p^*, u^*, v^*, T^*, and ρ^*, respectively. It should be noted that the vertical momentum equation (23) is used indirectly in this computation through its usage in the derivation of the pressure equation (28). Since the water-level variation provides a vertical freedom for the water body, it is imperative to calculate the vertical velocity component w^* from the continuity equation and not by the application of the momentum equation in the vertical direction equation (23). However, the momentum balance in the vertical direction is fully satisfied since equation (23) is expliticly used in the derivation of the pressure equation (28). In fact, equation (23) may be easily derived from the final equations stated above.

The mathematical formulation for the water-level flow region is derived by the application of the fundamental equations of mass, momentum, and energy as follows. The water-level in cells located at the air-water interface varies with time and space. For this reason, the continuity equation, as given by equation (27), is not valid for these cells. Application of a mass balance to such a cell with all sides fixed in space except the top which moves with the water-level, yields

$$\begin{aligned}\frac{\partial}{\partial t}(\rho \Delta x \Delta y\, H) &= (\rho u \Delta y\, H)_x - (\rho u \Delta y\, H)_{x+\Delta x} \\ &+ (\rho v \Delta x\, H)_y - (\rho v \Delta x\, H)_{y+\Delta y} \\ &+ (\rho w \Delta x \Delta y)_z - (\rho w \Delta x \Delta y)_{z+H} \qquad (29)\end{aligned}$$

The last lerm represents the mass flux leaving the water-level surface. Since the evaporation losses are considered insignificant, this term is equal to zero. Dividing both sides of the above equation by $\rho\Delta x\Delta y$, considering ρ to be constant and equal to ρ_0 consistent with Boussinesq approximation used earlier, and letting Δx, Δy approach zero, yields

$$\frac{\partial H}{\partial t} = - \frac{\partial}{\partial x} (uH) - \frac{\partial}{\partial y} (vH) + w \tag{30}$$

The above equation will be integrated in time to obtain the water-level. In nondimensional form, the above equation becomes

$$\frac{\partial H^*}{\partial t^*} = - \frac{\partial}{\partial x^*} (u^* H^*) - \frac{\partial}{\partial y^*} (v^* H^*) + w^* \tag{31}$$

where

$$H^* = \frac{H}{d_0} \tag{32}$$

Similarly, the energy equation, as given by equation (14), is not valid for the water-level elements. Application of the energy balance to these cells yields

$$\begin{aligned} \frac{\partial}{\partial t} (\rho C_p \Delta x \Delta y H T) = {} & (\rho C_p u \Delta y H T)_x - (\rho C_p u \Delta y H T)_{x + \Delta x} \\ & + (\rho C_p v \Delta x H T)_y - (\rho C_p v \Delta x H T)_{y + \Delta y} \\ & + (\rho C_p w \Delta x \Delta y T)_z - (\rho C_p w \Delta x \Delta y T)_{z + H} \end{aligned}$$

$$+ (-\rho C_p D_x \Delta y H \frac{\partial T}{\partial x})_x - (-\rho C_p D_x \Delta y H \frac{\partial T}{\partial x})_{x + \Delta x}$$

$$+ (-\rho C_p D_y \Delta x H \frac{\partial T}{\partial y})_y - (-\rho C_p D_y \Delta x H \frac{\partial T}{\partial y})_{y + \Delta y}$$

$$+ (-\rho C_p D_z \Delta x \Delta y \frac{\partial T}{\partial z})_z - (-\rho C_p D_z \Delta x \Delta y \frac{\partial T}{\partial z})_{z + H} \tag{33}$$

It should be noted again that the enthalpy flux leaving the water-level surface is equal to zero. Furthermore, the turbulent thermal diffusion at the water-level surface, is (3)

$$(\rho C_p D_z \frac{\partial T}{\partial z})_{z + H} = - K (T - E) \tag{34}$$

Considering the above statements, dividing energy equation (33) by $\rho C_p \Delta x \Delta y$ with $\rho = \rho_0$, and letting Δx, Δy approach zero, yields

$$\frac{\partial}{\partial t} (HT) = \frac{\partial}{\partial x} (uHT) - \frac{\partial}{\partial y} (vHT) + wT$$

$$+ D_x \frac{\partial}{\partial x} (H \frac{\partial T}{\partial x}) + D_y \frac{\partial}{\partial y} (H \frac{\partial T}{\partial y})$$

$$- D_z \frac{\partial T}{\partial z} - \frac{K (T - E)}{\rho_0 C_p} \tag{35}$$

Performing the differentiations indicated by the first three terms of the above, and employing equation (30) gives

$$\frac{\partial T}{\partial t} = - u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + \frac{D_x}{H} \frac{\partial}{\partial x} (H \frac{\partial T}{\partial x})$$

$$+ \frac{D_y}{H} \frac{\partial}{\partial y} (H \frac{\partial T}{\partial y}) - \frac{D_z}{H} \frac{\partial T}{\partial z} - \frac{K (T - E)}{\rho_0 C_p H} \tag{36}$$

By a similar reasoning, after employing the surface boundary conditions, to be derived later, the velocity components in the water-level element is found to be

$$\frac{\partial u}{\partial t} = - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \frac{1}{\rho_0 H} \frac{\partial Hp}{\partial x} + \frac{\nu_h}{H} \frac{\partial}{\partial x} (H \frac{\partial u}{\partial x})$$

$$+ \frac{\nu_h}{H} \frac{\partial}{\partial y} (H \frac{\partial u}{\partial y}) - \frac{\nu_v}{H} \frac{\partial u}{\partial z} + \frac{1}{2} \frac{C_{fx} \rho_a W_x^2}{\rho_0 H} \tag{37}$$

$$\frac{\partial v}{\partial t} = - u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - \frac{1}{\rho_0 H} \frac{\partial Hp}{\partial y} + \frac{\nu_h}{H} \frac{\partial}{\partial x} (H \frac{\partial v}{\partial x})$$

$$+ \frac{\nu_h}{H} \frac{\partial}{\partial y} (H \frac{\partial v}{\partial y}) - \frac{\nu_v}{H} \frac{\partial v}{\partial z} + \frac{1}{2} \frac{C_{fy} \rho_a W_y^2}{\rho_0 H} \tag{38}$$

After setting $D_x = D_y = D_h$, and $D_z = D_v$, equations (36), (37), and (38) in non-dimensional form become

$$\frac{\partial T^*}{\partial t^*} = - u^* \frac{\partial T^*}{\partial x^*} - v^* \frac{\partial T^*}{\partial y^*} + \frac{D_h}{\nu_0} \frac{1}{R_0 H^*} (\frac{\partial}{\partial x^*} (H^* \frac{\partial T^*}{\partial x^*})$$

$$+ \frac{\partial}{\partial y^*} (H^* \frac{\partial T^*}{\partial y^*})) - \frac{\nu_v}{\nu_0} \frac{1}{R_0 H^*} \frac{\partial T^*}{\partial z^*} - \frac{S_0}{H^*} (T^* - E^*) \tag{39}$$

$$\frac{\partial u^*}{\partial t^*} = - u^* \frac{\partial u^*}{\partial x^*} - v^* \frac{\partial u^*}{\partial y^*} - \frac{1}{F_0^2 H^*} \frac{\partial H^* p^*}{\partial x^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0 H^*} \left(\frac{\partial}{\partial x^*} \left(H^* \frac{\partial u^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*} \left(H^* \frac{\partial u^*}{\partial y^*}\right)\right)$$

$$- \frac{\nu_v}{\nu_0} \frac{1}{R_0 H^*} \frac{\partial u^*}{\partial z^*} + \frac{1}{2} \frac{C_{fx} \rho_a^* W_x^{*2}}{H^*} \qquad (40)$$

$$\frac{\partial v^*}{\partial t^*} = - u^* \frac{\partial v^*}{\partial x^*} - v^* \frac{\partial v^*}{\partial y^*} - \frac{1}{F_0^2 H^*} \frac{\partial H^* p^*}{\partial y^*}$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0 H^*} \left(\frac{\partial}{\partial x^*} \left(H^* \frac{\partial v^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*} \left(H^* \frac{\partial v^*}{\partial y^*}\right)\right)$$

$$- \frac{\nu_v}{\nu_0} \frac{1}{R_0 H^*} \frac{\partial v^*}{\partial z^*} + \frac{1}{2} \frac{C_{fy} \rho_a^* W_y^{*2}}{H} \qquad (41)$$

where S_0, the Stanton number, is defined by

$$S_0 = \frac{K}{\rho_0 C_p U_0} \qquad (42)$$

and

$$\rho_a^* = \frac{\rho_a}{\rho_0} \qquad (43)$$

$$W_x^* = \frac{W_x}{U_0} \qquad W_y^* = \frac{W_y}{U_0} \qquad (44)$$

The pressure at the air-water interface is always atmospheric and can be considered equal to zero no matter where the water-level may be at any instant of time. For this reason, the pressure at the center of the water-level element can be considered to be hydrostatic, i.e., for the water-level element

$$P^* = \rho^* H^* \tag{45}$$

This equation may also be derived by applying a momentum balance, in the vertical direction, on the water-level flow region and noting that the water vertical velocity in this region is relatively small and, therefore, negligible. If the pressure distribution given by equation (45) is substituted into the horizontal momentum equations, the terms involving pressure gradient would simplify as follows

$$\frac{1}{F_0^2 H^*} \frac{\partial H^* P^*}{\partial x^*} = \frac{1}{F_0^2} \left(H^* \frac{\partial \rho^*}{\partial x^*} + 2\rho^* \frac{\partial H^*}{\partial x^*}\right) \tag{46}$$

$$\frac{1}{F_0^2 H^*} \frac{\partial H^* p^*}{\partial y^*} = \frac{1}{F_0^2} \left(H^* \frac{\partial \rho^*}{\partial y^*} + 2\rho^* \frac{\partial H^*}{\partial y^*}\right) \tag{47}$$

It is important to realize the difference between the flow patterns in the sub-water-level and the water-level elements. In the sub-water-level elements, an increase in the fluid boundary velocity will induce horizontal velocity components. However, the time constant associated with the momentum equations will not permit these equations to respond instantaneously to the boundary velocity disturbance at the river or thermal discharge inlets. Because of the flow incompressibility and since the fluid vertical inertia is considerably smaller than the horizontal inertias, the mass imbalance will result in a positive vertical fluid motion designated as "upwelling". This vertical motion will continue upward until it reaches the water-level element where by virtue of equation (30) will cause the water-level to rise. This rise is large for elements close to the boundary disturbance and small for elements farther away. This spatial change affects the horizontal momentum equations (40) and (41) and induce a horizontal flow in the water-level elements which in turn tends to reduce the water-level through equation (30). A reverse situation takes place when the boundary velocity is decreased. The mass imbalance results in a negative vertical fluid motion, designated as "downwelling". This will cause the lowering of the water-level with an attendant horizontal flow into the element to increase the water-level.

2.3 Boundary Conditions

The water body boundary surfaces may be classified as follows:

Type (1): The surface boundary located at the water-level free surface.

Type (2): The lateral solid-fluid boundary located at the interface between the impoundment wall and the water body.

Type (3): The bottom boundary located at the bottom of the impoundment.

Type (4): The fluid-fluid boundary located at the interface between the water body in the region of interest and surrounding waters.

Implementation of the boundary conditions for the above boundary surfaces involves calculation of the first spatial derivative of the system variables at the boundary. Two methods have been in usage for the calculation of the derivative:

1) Application of the reflection principle and usage of fictitious fluid nodes behind the solid boundary;

2) Calculation of the derivative at the boundary on the basis of three internal nodes adjacent to the boundary node.

The latter technique requires less computer storage space and is employed in the present study as detailed in the Appendix.

Pressure boundary conditions. For the surface boundary, the pressure is considered atmospheric at the air-water interface and is set equal to zero.

For the lateral solid-fluid and bottom boundaries, application of the momentum equation in the direction normal to the boundaries gives the following equations respectively:

$$\frac{\partial p^*}{\partial y^*} = F_0^2 \frac{\nu_h}{\nu_0} \frac{1}{R_0} \frac{\partial^2 v^*}{\partial y^{*2}} \qquad (48)$$

$$\frac{\partial p^*}{\partial z^*} = - \rho^* + F_0^2 \frac{\nu_v}{\nu_0} \frac{1}{R_0} \frac{\partial^2 w^*}{\partial z^{*2}} \qquad (49)$$

For the fluid-fluid boundaries, the pressure distribution at the inflow interfaces is considered to be static. The pressure distribution at any outflow interface node is set equal to that of the adjacent upstream node in the flow field. The surrounding waters entering a water body is characterized by a uniform channel flow. The pressure variation at any cross section of a uniform channel flow may be proved to be hydrostatic by applying the Bernoulli's equation. This equation is applied to two streamlines, one at the channel top and the other at an arbitrary depth. Since, in steady, frictionless, one-dimensional, uniform channel flow, the constants in the Bernoulli's equation are the same for all streamlines, it can be easily shown that the pressure distribution across the channel, as well as the interface between the channel and the impoundment is hydrostatic.

Velocity boundary conditions. For the surface boundary, the wind effect produces shear at the water surface which is equal to the shear stress in the fluid near the surface.

$$\rho_0 \nu_v \left(\frac{\partial u}{\partial z}\right)_{z = z_s} = \frac{1}{2} \rho_a C_{fx} W_x \left| W_x \right| \tag{50}$$

$$\rho_0 \nu_v \left(\frac{\partial v}{\partial z}\right)_{z = z_s} = \frac{1}{2} \rho_a C_{fy} W_y \left| W_y \right| \tag{51}$$

where C_{fx} and C_{fy} are skin coefficients along x and y axis respectively; ρ_a and ρ_0 are the air and water densities; W_x and W_y are x and y components of the wind velocity; u and v are the fluid velocity components along x and y; and ν_v is the vertical eddy viscosity. Equations (50) and (51) are based on the assumption that the wind velocity components are steady and moderate with zero vertical component. Under these conditions, the water surface will remain smooth and the skin coefficients may be taken from experimental measurements (22, 16). In dimensionless form, the equations (50) and (51) become

$$\frac{\nu_v}{\nu_0}\frac{1}{R_0}\left(\frac{\partial u^*}{\partial z^*}\right)_{z^* = z_s^*} = \frac{1}{2} C_{fx} \rho_a^* W_x^* \left| W_x^* \right| \tag{52}$$

$$\frac{\nu_v}{\nu_0}\frac{1}{R_0}\left(\frac{\partial v^*}{\partial z^*}\right)_{z^* = z_s^*} = \frac{1}{2} C_{fy} \rho_a^* W_y^* \left| W_y^* \right| \tag{53}$$

For the lateral solid-fluid and bottom boundaries, the velocity components normal to the boundaries is always zero. The tangential velocity components is calculated by the application of no-slip condition. With this condition, both tangential velocity components at the solid boundary are set equal to zero. The no-slip boundary condition is ideal for very fine grid spacing. However, computational economy requires a coarse grid spacing for most dynamic three-dimensional hydrothermal analysis. Since,a coarse grid does not afford sufficient resolution at the boundary, a limited amount of numerical error will be induced in the solution.

For the fluid-fluid boundaries, the velocity distribution at the inflow interfaces is set equal to the channel inflow distribution which is considered to be a known function of time. The velocity distribution at any outflow interface node is set equal to that of the adjacent upstream node in the flow field.

Temperature boundary condition. For the surface boundary, the rate of heat dispersion through the water (involving both conduction and turbulent thermal diffusion) is equal to the heat transfer from water to air.

$$\rho_0 \, C_p D_v \, \left(\frac{\partial T}{\partial z}\right)_{z = z_s} = - K \, (T_s - E) \tag{54}$$

where ρ_0, C_p, and D_v are the density, specific heat, and eddy diffusivity of heat respectively; T_s is the water surface temperature; K is the heat exchange coefficient; and E is the equilibrium water temperature. Value of K and E may be estimated in terms of meteorological conditions (3). In dimensionless form, equation (54) becomes

$$\frac{D_v}{\nu_0} \frac{1}{R_0} \left(\frac{\partial T^*}{\partial z^*}\right)_{z^* = Z_s^*} = - S_0 \, (T_s^* - E^*) \tag{55}$$

For the lateral solid-fluid and bottom boundaries, the heat transfer is considered to be negligible leading to zero temperature gradient at the boundary, i.e.,

$$\frac{\partial T^*}{\partial n^*} = 0 \tag{56}$$

The temperature distribution at the fluid-fluid boundaries is treated similar to the velocity distribution. The temperature at the inflow interface is set equal to the channel inflow temperature which is considered to be a known function of time. The temperature at any outflow interface node is set equal to that of the adjacent upstream node in the flow field.

Water-level boundary condition. The water-level at the fluid-fluid boundaries is treated similar to the temperature distribution. The water-level at the inflow interface is set equal to the channel inflow which is considered to be known. The water-level at outflow interface nodes is set equal to that of the adjacent upstream nodes in the flow field. For the solid-fluid, the water level is calculated by simplifying the continuity equation for the water-level-region considering no-slip boundary conditions.

Boundary conditions on Q_x^*, Q_y^* and Q_z^*. In the development of pressure equation, values of $\partial Q_x^*/\partial x^*$, $\partial Q_y^*/\partial y^*$, and $\partial Q_z^*/\partial z^*$ should be evaluated in the flow field. It was shown earlier that variables Q_x^*, Q_y^*, and Q_z^*, are related to the velocity field by equations (24), (25), and (26). It should be realized that the only spatial derivatives of Q_x^* needed in the present analysis is $\partial Q_x^*/\partial x^*$. Spatial derivatives $\partial Q_x^*/\partial y^*$ and $\partial Q_x^*/\partial z^*$ are not required. This situation simplifies the analysis. Calculation of $\partial Q_x^*/\partial x^*$ in the fluid cell next to the boundary requires the calculation of Q_x^* at the boundaries normal to the x axis. Similarly, calculation of $\partial Q_y^*/\partial y^*$ and $\partial Q_z^*/\partial z^*$ require the calculation of Q_y^* and Q_z^* at the boundaries normal to the the y and z axes respectively.

For the surface boundary, only Q_z^* is needed. As stated earlier, since the vertical velocity component in the water-level element is small, equation (26) gives

$$(Q_z^*)_{z = z_s^*} = - \frac{\rho^*}{F_0^2} \tag{57}$$

For the lateral solid-fluid boundaries, only the component of Q^* normal to the solid boundary is required. Since the velocity component normal to the solid boundary is zero, the component Q^* normal to the solid boundary can be obtained after simplifying equation (25) as follows:

$$(Q_y^*)_{y^* = 0} = (Q_y^*)_{y = y_{max}} = \frac{\nu_h}{\nu_0} \frac{1}{R_0} \frac{\partial^2 v^*}{\partial y^{*2}} \tag{58}$$

For the bottom boundary only Q_z^* is needed. Simplification of equation (26) at the bottom boundary gives

$$(Q_z^*)_{z^* = 0} = - \frac{\rho^*}{F_0^2} + \frac{\nu_v}{\nu_0 R_0} \left(\frac{\partial^2 w^*}{\partial z^{*2}}\right) \tag{59}$$

For the fluid-fluid boundary, the surrounding waters entering the water body are characterized by a uniform channel flow, with a uniform velocity at any cross section and zero axial pressure gradient. As an example, application of the momentum equation along the channel flowing in the x-direction gives

$$(Q_x^*)_{x^* = 0} = 0 \tag{60}$$

Furthermore, at the outflow interface nodes, the values of Q_x^* are set equal to that of the adjacent upstream nodes in the flow field.

3. NUMERICAL SOLUTION

The numerical scheme, used in this study, is mainly based on the application of spatial and temporal finite difference technique to the governing equation described in the previous section. This scheme is divided into the following steps:

1) Initialization and setting of boundary conditions;

2) Calculation of vertical velocity components;

3) Calculation of pressure distribution;

4) Calculation of the time derivatives for the horizontal velocity components and temperature in the elements of the sub-water-level region;

5) Calculation of the time derivative for the water-level, horizontal velocity components and temperature in the elements of the water-level region;

6) Time integration and updating of the horizontal velocity components and temperature in the sub-water-level region;

7) Calculation of the water-level, velocity components, and temperature in the water-level region;

8) Calculation of density distribution.

After the completion of the last step, the problem time is incremented, the computational procedure is repeated starting at the second step, and the integration cycle is continued until the final problem time is reached.

The initial conditions of the problem consist of the initial values of temperature, velocity components, and pressure at time t = 0. These quantities are most easily obtained by the application of the present dynamic analysis. To demonstrate this application, let us consider that it is desired to determine the dynamic response of a water body when it is subjected to a prescribed thermal discharge. The solution to the problem can be obtained in two steps. In the first step, the dynamic response is obtained by setting the thermal discharge temperature equal to the river inflow temperature. Maintaining the atmospheric and the inflow conditions unchanged, the water body will reach equilibrium conditions and the pressure, velocity and temperature distribution will be obtained. In the second step, these equilibrium conditions are entered as the initial conditions and the thermal discharge temperature is set equal to the prescribed value. Again, if the atmospheric and inflow conditions are held unchanged, the water body will reach new equilibrium conditions. An examination of this dynamic response will enable the program user to predict the effects of the thermal discharge on the state of the water body. The results of this study could then be used to determine if the state's allowable temperature standards have been violated. These standards generally vary from state to state. Many states impose a maximum allowable water temperature, a temperature rise, and a rate of temperature rise on the use of water bodies.

From the temporal viewpoint, the boundary conditions stated earlier are of two types -- time-independent and time-dependent. The setting of the time-independent boundary conditions are undertaken prior to the beginning of the dynamic solution. The time-dependent boundary conditions are satisfied during the execution of the integration cycle. For example, the time-dependent boundary conditions on vertical velocity and pressure are satisfied during the execution of steps 2 and 3 stated above respectively.

3.1 Calculation of the Vertical Velocity Components

As discussed earlier, the vertical component of velocity w^* is calculated from the continuity equation (27). This equation can be used to determine the vertical velocity component w^*, once the horizontal velocity components u^* and v^* are known throughout the flow field. For the first step of numerical integration, the latter quantities as initialized will be used to calculate the velocity component w^*. For the numerical integration steps higher than the first, the updated values of u^* and v^* will be available by the time integration of the momentum equations in the x and y directions to be discussed later.

To facilitate the writing of finite difference equations, to be presented in this section, it is convenient to introduce the following conventions:

1) Variables defining any quantity within an element i,j,k will be written without subscripts, for example, $u^*_{i,j,k}$ will be written as u^*.

2) Variables defining any quantity within an element adjacent

to element i,j,k will be written with the subscript that is different from i,j,k, for example $u^*_{i+1,\, j,\, k}$ or $u^*_{i,j-1,k}$ will be written as u^*_{i+1} or u^*_{j-1} respectively.

The finite difference form of $\partial u^*/\partial x^*$ and $\partial v^*/\partial y^*$, using a parabolic representation (three points with unequal spacing), are given in the Appendix 3.

Hence, expressing the right hand side equation (27) in finite difference form, yields:

$$\frac{\partial w^*}{\partial z^*} = a_x \frac{u^*_{i+1} - u^*}{x^*_{i+1} - x^*} + b_x \frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}}$$

$$+ a_y \frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} + b_y \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}} \qquad (61)$$

where

$$a_x = \frac{x^* - x^*_{i-1}}{x^*_{i+1} - x^*_{i-1}} \qquad (62)$$

$$b_x = \frac{x^*_{i+1} - x^*}{x^*_{i+1} - x^*_{i-1}} \qquad (63)$$

$$a_y = \frac{y^* - y^*_{j-1}}{y^*_{j+1} - y^*_{j-1}} \qquad (64)$$

and

$$b_y = \frac{y^*_{j+1} - y^*}{y^*_{j+1} - y^*_{j-1}} \tag{65}$$

Starting from the bottom boundary, equation (59) is then integrated along z to obtain the vertical velocity component in each element.

3.2 Calculation of Pressure Distribution

The updated values of pressure in the flow field are found from equations (24), (25), (26), and (28) by applying a centered three-point finite difference for the first spatial derivatives of u^*, v^*, w^*, Q^*_x, Q^*_y, and Q^*_z as well as for the second spatial derivatives of p^*, u^*, v^*, and w^*. A variable mesh size finite difference scheme is used as detailed in Appendix 3. Equation (28) becomes

$$c_x \left(\frac{p^*_{i+1} - p^*}{x^*_{i+1} - x^*} - \frac{p^* - p^*_{i-1}}{x^* - x^*_{i-1}}\right) + c_y \left(\frac{p^*_{j+1} - p^*}{y^*_{j+1} - y^*} - \frac{p^* - p^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$+ c_z \left(\frac{p^*_{k+1} - p^*}{z^*_{k+1} - z^*} - \frac{p^* - p^*_{k-1}}{z^* - z^*_{k-1}}\right) = \xi \tag{66a}$$

where

$$\xi = F_0^2 \left(a_x \frac{(Q^*_x)_{i+1} - Q^*_x}{x^*_{i+1} - x^*} + b_x \frac{Q^*_x - (Q^*_x)_{i-1}}{x^* - x^*_{i-1}}\right. \tag{66b}$$

$$+ a_y \frac{(Q^*_y)_{j+1} - Q^*_y}{y^*_{j+1} - y^*} + b_y \frac{Q^*_y - (Q^*_y)_{j-1}}{y^* - y^*_{j-1}}$$

$$+ a_z \frac{(Q_z^*)_{k+1} - Q_z^*}{z_{k+1}^* - z^*} + b_z \frac{Q_z^* - (Q_z^*)_{k-1}}{x^* - z_{k-1}^*})$$

$$a_x = (x^* - x_{i-1}^*)/(x_{i+1}^* - x_{i-1}^*)$$

$$b_x = (x_{i+1}^* - x^*)/(x_{i+1}^* - x_{i-1}^*)$$

$$c_x = 2/(x_{i+1}^* - x_{i-1}^*)$$

$$a_y = (y^* - y_{j-1}^*)/(y_{j+1}^* - y_{j-1}^*)$$

$$b_y = (y_{j+1}^* - y^*)/(y_{j+1}^* - y_{j-1}^*)$$

$$c_y = 2/(y_{j+1}^* - y_{j-1}^*)$$

$$a_z = (z^* - z_{k-1}^*)/(z_{k+1}^* - z_{k-1}^*)$$

$$b_z = (z_{k+1}^* - z^*)/(z_{k+1}^* - z_{k-1}^*)$$

$$c_z = 2/(z_{k+1}^* - z_{k-1}^*) \qquad (67)$$

$$Q_x^* = - u^* \left(a_x \frac{u_{i+1}^* - u^*}{x_{i+1}^* - x^*} + b_x \frac{u^* - u_{i-1}^*}{x^* - x_{i-1}^*}\right)$$

$$- v^* \left(a_y \frac{u^*_{j+1} - u^*}{y^*_{j+1} - y^*} + b_y \frac{u^* - u^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$- w^* \left(a_z \frac{u^*_{k+1} - u^*}{z^*_{k+1} - z^*} + b_z \frac{u^* - u^*_{k-1}}{z^* - z^*_{k-1}}\right)$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(c_x \left(\frac{u^*_{i+1} - u^*}{x^*_{i+1} - x^*} - \frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}}\right)\right.$$

$$\left. + c_y \left(\frac{u^*_{j+1} - u^*}{y^*_{j+1} - y^*} - \frac{u^* - u^*_{j-1}}{y^* - y^*_{j-1}}\right)\right)$$

$$+ \frac{\nu_v}{\nu_0 R_0} C_z \left(\frac{u^*_{k+1} - u^*}{z^*_{k+1} - z^*} - \frac{u^* - u^*_{k-1}}{z^* - z^*_{k-1}}\right) \tag{68}$$

$$Q^*_y = - u^* \left(a_x \frac{v^*_{i+1} - v^*}{x^*_{i+1} - x^*} + b_x \frac{v^* - v^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$- v^* \left(a_y \frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} + b_y \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$- w^* \left(a_z \frac{v^*_{k+1} - v^*}{z^*_{k+1} - z^*} + b_z \frac{v^* - v^*_{k-1}}{z^* - z^*_{k-1}}\right)$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} \left(c_x \left(\frac{v^*_{i+1} - v^*}{x^*_{i+1} - x^*} - \frac{v^* - v^*_{i-1}}{x^* - x^*_{i-1}}\right)\right.$$

$$+ c_y \left(\frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} - \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}}\right))$$

$$+ \frac{\nu_v}{\nu_0 R_0} c_z \left(\frac{v^*_{k+1} - v^*}{z^*_{k+1} - z^*} - \frac{v^* - v^*_{k-1}}{z^* - z^*_{k-1}}\right) \tag{69}$$

and

$$Q^*_z = - u^* \left(a_x \frac{w^*_{i+1} - w^*}{x^*_{i+1} - x^*} + b_x \frac{w^* - w^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$- v^* \left(a_y \frac{w^*_{j+1} - w^*}{y^*_{j+1} - y^*} + b_y \frac{w^* - w^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$- w^* \left(a_z \frac{w^*_{k+1} - w^*}{z^*_{k+1} - z^*} + b_z \frac{w^* - w^*_{k-1}}{z^* - z^*_{k-1}}\right)$$

$$+ \frac{\nu_h}{\nu_0 R_0} (c_x \left(\frac{w^*_{i+1} - w^*}{x^*_{i+1} - w^*} - \frac{w^* - w^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$+ c_y \left(\frac{w^*_{j+1} - w^*}{y^*_{j+1} - y^*} - \frac{w^* - w^*_{j-1}}{y^* - y^*_{j-1}}\right))$$

$$+ \frac{\nu_v}{\nu_0 R_0} c_z \left(\frac{w^*_{k+1} - w^*}{z^*_{k+1} - z^*} - \frac{w^* - w^*_{k-1}}{z^* - z^*_{k-1}}\right) - \frac{1}{F_0^2} \rho^* \tag{70}$$

Numerical values for Q_x^*, Q_y^*, and Q_z^* at any element i, j, k needed for the right hand side of equation (66b) are provided by equation (68), (69), and (70) respectively. Numerical values for $(Q_x^*)_{i+1}$, $(Q_x^*)_{i-1}$, $(Q_y^*)_{j+1}$, $(Q_y^*)_{j-1}$, $(Q_z^*)_{k+1}$, and $(Q_z^*)_{k-1}$, also needed for the right hand side of equation (66b) are provided by the same equations, after applying these equations to the entire flow field from the first cell (i=1, j=1, k=1) to the last ($i=i_{max}$, $j=j_{max}$, $k=k_{max}$).

Defining

$$e_x = \frac{c_x}{x_{i+1}^* - x^*} \qquad f_x = \frac{c_x}{x^* - x_{i-1}^*}$$

$$e_y = \frac{c_y}{y_{j+1}^* - y^*} \qquad f_y = \frac{c_y}{y^* - y_{j-1}^*}$$

$$e_z = \frac{c_z}{z_{k+1}^* - z^*} \qquad f_z = \frac{c_z}{z^* - z_{k-1}^*} \tag{71}$$

Equation (66a) becomes

$$e_x\,(p_{i+1}^* - p^*) - f_x\,(p^* - p_{i-1}^*) +$$

$$e_y\,(p_{j+1}^* - p^*) - f_y\,(p^* - p_{j-1}^*) +$$

$$e_z\,(p_{k+1}^* - p^*) - f_z\,(p^* - p_{k-1}^*) = \xi \tag{72}$$

Within one small time increment, values e_x, e_y, e_z, f_x, f_y, f_z, and ξ are constants and equation (72) constitutes a system of linear algebraic equations of p^* which can be solved simultaneously. An iterative method, based on a modified Gauss-Seidel iteration technique, called successive over-relaxation is employed to solve the above system of equations. This method basically consists of solving equation (72) for p^* and designating that by p_t^*.

$$p_t^* = \frac{e_x\, p_{i+1}^* + f_x\, p_{i-1}^* + e_y p_{j+1}^* + f_y\, p_{j-1}^* + e_z\, p_{k+1}^* + f_z\, p_{k-1}^* - \xi}{e_x + f_x + e_y + f_y + e_z + f_z} \tag{73}$$

and then calculating the actual pressure distribution employing a weighted average of this value and the value of pressure found from the preceding iteration such that

$$p_{\ell+1}^* = \omega(p_t^*)_{\ell+1} + (1 - \omega)\, p_\ell^* \tag{74}$$

where

p_ℓ^* = pressure in cell i, j, k at the preceding iteration ℓ

$(p_t^*)_{\ell+1}$ = pressure in cell i, j, k calculated from equation (73) at the present iteration $\ell+1$

$p_{\ell+1}^*$ = actual pressure in cell i, j, k calculated from equation (74) at the present iteration $\ell+1$

ω = relaxation factor to be discussed later in this section

The pressure calculation, employing equation (74) is performed for every cell in the flow field and old values of p_{ℓ}^{*} are replaced by $p_{\ell+1}^{*}$ as soon as the latter value is calculated for any cell. The iterative procedure, involving the calculation of the pressure for the entire flow field, is repeated until values of pressure at every cell converge within a prescribed error. It should be noted that for $\omega = 1$, this approach becomes identical with the Gauss-Seidel iterative method. The quantity ω, designated as relaxation factor varies between 0 and 2. For values of $\omega < 1$, the iterative method is referred to as underrelaxation and is usually employed to make a nonconvergent iterative process converge. For values of $1 < \omega < 2$, the iterative method is referred to as overrelaxation and is ordinarily used to accelerate an already convergent iterative process. In this study, successive overrelaxation procedure is used and value of ω is chosen between one and two.

The optimum value of ω for finite difference method applied to the elliptic differential equation, involved in this study, is found (11, 6, 37) from

$$\omega_{opt} = \frac{2}{1 + (1 - \lambda^2)^{1/2}} \tag{75}$$

with

$$\lambda^2 = \lim \frac{||Y_{\ell+1}||}{||Y_{\ell}||} \tag{76}$$

and

$$\| Y_{\ell + 1} \| = \Sigma_{i = 1}^{i_{max}} \Sigma_{j = 1}^{j_{max}} \Sigma_{k = 1}^{k_{max}} \left| (p_t^*)_{\ell + 1} - (p_t^*)_{\ell} \right| \tag{77}$$

where

ω_{opt} = optimum value of overrelaxation factor

λ^2 = norms ratio

$\| Y_{\ell + 1} \|$ = norm of the pressure at the present iteration $\ell + 1$

$\| Y_{\ell} \|$ = norm of the pressure at the preceding iteration ℓ

The above procedure indicates that the calculation of the optimum value of overrelaxation factor is based on the ratio of norms at the present iteration $\ell + 1$ and the preceding iteration as given by equations (76) and (77) using Gauss-Seidel iterative scheme ($\omega = 1$). After having found the optimum value of ω, the numerical scheme employs the modified Gauss-Seidel technique with ω set equal to the optimum value. A question now arises as to how many Gauss-Seidel iterations should be performed before switching to the modified Gauss-Seidel scheme. If the number of iterations ℓ is too few, the value of λ^2 may become larger than unity which will not result in an optimum value for ω. On the other hand, if the number of iterations ℓ are too many, the value of λ^2 will approach unity ($\omega_{opt} = 2$). In this case, the converged values of pressure would be obtained by using the uneconomical Gauss-Seidel iterations. One effective procedure to solve this problem is to take enough Gauss-Seidel iterations to ensure that the estimate of λ^2 has settled down to a reasonably constant level less than one, and then employ equation (75) to estimate ω_{opt}. This estimate of ω_{opt} is then used in equation (74) and the numerical solution iterated to convergence using this overrelaxation factor.

From the foregoing analysis, it is evident that the optimum value of ω depends on the pressure distribution which is a time-dependent function. For this reason, the optimum value of ω also becomes time-dependent and should be calculated once during each integration step. However, since values of ω will only affect the speed of convergence and not the accuracy of the solution, it is not mandatory to compute the optimum value of ω at each integration step. A considerable amount of computer time will be saved if one calculates the optimum value of ω for the first integration step and use that value for the following integration steps. Experience with this numerical approach has indicated that since the time variation of pressure in the flow field is not excessively large, the optimum value of ω found in the first integration step provides a near optimum value for the following steps and the calculation of ω_{opt} for all integration steps may be eliminated at a considerable cost saving.

3.3 Calculation of the Time Derivative for the Horizontal Velocity Components and Temperature

Time derivatives of horizontal velocity components and temperature in the flow field are found from equations (21), (22), and (14) by applying a centered three point finite difference for the first spatial derivative of p^* and T^* as well as the second spatial derivative of T^*. A variable mesh size finite difference scheme is used as detailed in Appendix 3.

$$\frac{\partial u^*}{\partial t^*} = Q_x^* - \frac{1}{F_0^2} \left(a_x \frac{p_{i+1}^* - p^*}{x_{i+1}^* - x^*} + b_x \frac{p^* - p_{i-1}^*}{x^* - x_{i-1}^*}\right) \tag{78}$$

$$\frac{\partial v^*}{\partial t^*} = Q_y^* - \frac{1}{F_0^2} \left(a_y \frac{p_{j+1}^* - p^*}{y_{j+1}^* - y^*} + b_y \frac{p^* - p_{j-1}^*}{y^* - y_{j-1}^*}\right) \tag{79}$$

$$\begin{aligned}
\frac{\partial T^*}{\partial t^*} = & - u^* \left(a_x \frac{T_{i+1}^* - T^*}{x_{i+1}^* - x^*} + b_x \frac{T^* - T_{i-1}^*}{x^* - x_{i-1}^*}\right) \\
& - v^* \left(a_y \frac{T_{j+1}^* - T^*}{y_{j+1}^* - y^*} + b_y \frac{T^* - T_{j-1}^*}{y^* - y_{j-1}^*}\right) \\
& - w^* \left(a_z \frac{T_{k+1}^* - T^*}{z_{k+1}^* - z^*} + b_z \frac{T^* - T_{k-1}^*}{z^* - z_{k-1}^*}\right) \\
& + \frac{D_h}{\nu_0} \frac{1}{R_0} \left(c_x \left(\frac{T_{i+1}^* - T^*}{x_{i+1}^* - x^*} - \frac{T^* - T_{i-1}^*}{x^* - x_{i-1}^*}\right)\right. \\
& \left. + c_y \left(\frac{T_{j+1}^* - T^*}{y_{j+1}^* - y^*} - \frac{T^* - T_{j-1}^*}{y^* - y_{j-1}^*}\right)\right) \\
& + \frac{D_v}{\nu_0} \frac{1}{R_0} c_z \left(\frac{T_{k+1}^* - T^*}{z_{k+1}^* - z^*} - \frac{T^* - T_{k-1}^*}{z^* - z_{k-1}^*}\right)
\end{aligned} \tag{80}$$

where Q_x^*, Q_y^*, a_x, b_x, a_y, b_y, a_z, b_z, c_x, c_y, and c_z, are previously defined by equations (68), (69), and (67).

3.4 Calculation of the Time Derivative for the Water -Level, Horizontal Velocity Components and Temperature in the Water-Level Elements

Time-derivatives of the water-level horizontal velocity components and temperature in the water-level region are found from equations (31), (41), (40), and (39), respectively, by applying a centered three-point finite difference for the first and second spatial derivatives of H^*, u^*, v^*, and T^*. A variable mesh size finite difference scheme is used as detailed in Appendix 3.

$$\frac{\partial u^*}{\partial t^*} = - u^* \left(a_x \frac{u^*_{i+1} - u^*}{x^*_{i+1} - x^*} + b_x \frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$- v^* \left(a_y \frac{u^*_{j+1} - u^*}{y^*_{j+1} - y^*} + b_y \frac{u^* - u^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$- \frac{H^*}{F_0^2} \left(a_x \frac{\rho^*_{i+1} - \rho^*}{x^*_{i+1} - x^*} + b_x \frac{\rho^* - \rho^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$+ \left(a_x \frac{H^*_{i+1} - H^*}{x^*_{i+1} - x^*} + b_x \frac{H^* - H^*_{i-1}}{x^* - x^*_{i-1}}\right) ,$$

$$\left(\frac{\nu_h}{\nu_0} \frac{1}{R_0 H^*} \left(a_x \frac{u^*_{i+1} - u^*}{x^*_{i+1} - x^*} + b_x \frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}}\right) - \frac{2\rho^*}{F_0^2}\right)$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} c_x \left(\frac{u^*_{i+1} - u^*}{x^*_{i+1} - x^*} - \frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}} \right)$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0 H^*} \left(a_y \frac{H^*_{j+1} - H^*}{y^*_{j+1} - y^*} + b_y \frac{H^* - H^*_{j-1}}{y^* - y^*_{i-1}} \right) \cdot$$

$$\left(a_y \frac{u^*_{j+1} - u^*}{y^*_{j+1} - y^*} + b_y \frac{u^* - u^*_{j-1}}{y^* - y^*_{j-1}} \right)$$

$$+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} c_y \left(\frac{u^*_{j+1} - u^*}{y^*_{j+1} - y^*} - \frac{u^* - u^*_{j-1}}{y^* - y^*_{j-1}} \right)$$

$$- \frac{\nu_v}{\nu_0} \frac{1}{R_0 H^*} \left(\frac{u^* - u^*_{k-1}}{z^* - z^*_{k-1}} \right) + \frac{1}{2H^*} c_{fx} \rho^*_a W^{*2}_x \qquad (81)$$

$$\frac{\partial v^*}{\partial t^*} = - u^* \left(a_x \frac{v^*_{i+1} - v^*}{x^*_{i+1} - x^*} + b_x \frac{v^* - v^*_{i-1}}{x^* - x^*_{i-1}} \right)$$

$$- v^* \left(a_y \frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} + b_y \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}} \right)$$

$$- \frac{H^*}{F_0^2} \left(a_y \frac{\rho^*_{j+1} - \rho^*}{y^*_{j+1} - y^*} + b_y \frac{\rho^* - \rho^*_{j-1}}{y^* - y^*_{j-1}} \right)$$

$$+ \left(a_y \frac{H^*_{j+1} - H^*}{y^*_{j+1} - y^*} + b_y \frac{H^* - H^*_{j-1}}{y^* - y^*_{j-1}} \right) \cdot$$

$$\left(\frac{\nu_h}{\nu_0}\frac{1}{R_0 H^*}\left(a_y \frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} + b_y \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}}\right) - \frac{2\rho^*}{F_0^2}\right)$$

$$+ \frac{\nu_h}{\nu_0}\frac{1}{R_0} c_x \left(\frac{v^*_{i+1} - v^*}{x^*_{i+1} - x^*} - \frac{v^* - v^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$+ \frac{\nu_h}{\nu_0}\frac{1}{R_0 H^*}\left(a_x \frac{H^*_{i+1} - H^*}{x^*_{i+1} - x^*} + b_x \frac{H^* - H^*_{i-1}}{x^* - x^*_{i-1}}\right) \cdot$$

$$\left(a_x \frac{v^*_{i+1} - v^*}{x^*_{i+1} - x^*} + b_x \frac{v^* - v^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$+ \frac{\nu_h}{\nu_0}\frac{1}{R_0} c_y \left(\frac{v^*_{j+1} - v^*}{y^*_{j+1} - y^*} - \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$- \frac{\nu_v}{\nu_0}\frac{1}{R_0 H^*}\left(\frac{v^* - v^*_{k-1}}{z^* - z^*_{k-1}}\right) + \frac{1}{2H^*} c_{fy}\, \rho^*_a\, W^{*2}_y \qquad (82)$$

$$\frac{\partial T^*}{\partial t^*} = - u^* \left(a_x \frac{T^*_{i+1} - T^*}{x^*_{i+1} - x^*} + b_x \frac{T^* - T^*_{i-1}}{x^* - x^*_{i-1}}\right)$$

$$- v^* \left(a_y \frac{T^*_{j+1} - T^*}{y^*_{j+1} - y^*} + b_y \frac{T^* - T^*_{j-1}}{y^* - y^*_{j-1}}\right)$$

$$+ \frac{D_h}{\nu_0} \frac{1}{R_0 H^*} (a_x \frac{H^*_{i+1} - H^*}{x^*_{i+1} - x^*} + b_y \frac{H^* - H^*_{i-1}}{x^* - x^*_{i-1}}) \cdot$$

$$(a_x \frac{T^*_{i+1} - T^*}{x^*_{i+1} - x^*} + b_x \frac{T^* - T^*_{i-1}}{x^* - x^*_{i-1}})$$

$$+ \frac{D_h}{\nu_0} \frac{1}{R_0} c_x (\frac{T^*_{i+1} - T^*}{x^*_{i+1} - x^*} - \frac{T^* - T^*_{i-1}}{x^* - x^*_{i-1}})$$

$$+ \frac{D_h}{\nu_0} \frac{1}{R_0 H^*} (a_y \frac{H^*_{j+1} - H^*}{y^*_{j+1} - y^*} + b_y \frac{H^* - H^*_{j-1}}{y^* - y^*_{j-1}}) \cdot$$

$$(a_y \frac{T^*_{j+1} - T^*}{y^*_{j+1} - y^*} + b_y \frac{T^* - T^*_{j-1}}{y^* - y^*_{j-1}})$$

$$+ \frac{D_h}{\nu_0} \frac{1}{R_0} c_y (\frac{T^*_{j+1} - T^*}{y^*_{j+1} - y^*} - \frac{T^* - T^*_{j-1}}{y^* - y^*_{j-1}})$$

$$- \frac{D_v}{\nu_0} \frac{1}{R_0 H^*} (\frac{T^* - T^*_{k-1}}{z^* - z^*_{k-1}}) - \frac{S_0}{H^*} (T^* - E^*) \qquad (83)$$

$$\frac{\partial H^*}{\partial t^*} = - u^* (\frac{H^* - H^*_{i-1}}{x^* - x^*_{i-1}}) - v^* (\frac{H^* - H^*_{j-1}}{y^* - y^*_{j-1}})$$

$$- H^* (\frac{u^* - u^*_{i-1}}{x^* - x^*_{i-1}} + \frac{v^* - v^*_{j-1}}{y^* - y^*_{j-1}}) + w^*_{k-1} \qquad (84)$$

It should be noted that a backward spatial differencing method is employed in equation (84) to ensure a conditional stability of this equation.

3.5 Calculations of the Horizontal Velocity Components and Temperature in the Sub-Water-Level-Region

The updated values of horizontal velocity components and temperature in the sub-water-level region are found by applying a forward two-point finite difference for the first temporal derivatives.

$$u^{*n} = u^* + \Delta t^* \left(\frac{\partial u^*}{\partial t^*}\right) \tag{85}$$

$$v^{*n} = v^* + \Delta t^* \left(\frac{\partial v^*}{\partial t^*}\right) \tag{86}$$

$$T^{*n} = T^* + \Delta t^* \left(\frac{\partial T^*}{\partial t^*}\right) \tag{87}$$

where superscript n represents the updated values, Δt^* is time integration step, and values $(\partial u^*/\partial t^*)$, $(\partial v^*/\partial t^*)$, and $(\partial T^*/\partial t^*)$ are calculated from equations (78), (79), and (80).

3.6 Calculation of the Water-Level, Velocity Components and Temperature in the Water-Level Element

The updated values of the water-level, velocity components and temperature in the water-level region are found by applying a forward two-point finite difference for the first temporal derivatives.

$$u^{*n} = u^* + \Delta t^* \left(\frac{\partial u^*}{\partial t^*}\right) \tag{88}$$

$$v^{*n} = v^* + \Delta t^* \left(\frac{\partial v^*}{\partial t^*}\right) \tag{89}$$

$$T^{*n} = T^* + \Delta t^* \left(\frac{\partial T^*}{\partial t^*}\right) \tag{90}$$

$$H^{*n} = H^* + \Delta t^* \left(\frac{\partial H^*}{\partial t^*}\right) \tag{91}$$

The values of $(\partial u^*/\partial t^*)$, $(\partial v^*/\partial t^*)$, $(\partial T^*/\partial t^*)$, and $(\partial H^*/\partial t^*)$ are calculated from equations (81), (82), (83), and (84).

3.7 Calculation of Density Distribution

The fluid density is considered to be a function of temperature only. The dependence of density on pressure, within the range of the variation of pressure in this study, is extremely weak and is, therefore, neglected. The functional relationship between the density and temperature is defined in tabular form as shown in columns 1 and 2 of Table 1 and is designated by

$$\rho = \rho\ (T) \tag{92}$$

in which T is the fluid temperature in degrees Fahrenheit, and ρ is the density in lbm/ft^3. In non-dimensional form, equation (92) becomes

$$\rho^* = \rho^*\ (T^*) \tag{93}$$

as exemplified in columns 3 and 4 of Table 1 with temperatures normalized to 90°F.

TABLE 1. DENSITY VERSUS TEMPERATURE

No.	T °F	$\rho(\frac{lbm}{ft^3})$	Dimensionless T	Dimensionless ρ
1	70	62.30296	.7777777	1.0030153
2	72	62.29876	.8000000	1.0028028
3	74	62.27426	.8222222	1.0025533
4	76	62.25487	.8444444	1.0022411
5	78	62.23938	.8666666	1.0019917
6	80	62.22002	.8888888	1.0016800
7	82	62.20065	.9111111	1.0013682
8	84	62.18132	.9333333	1.0010570
9	86	62.16199	.9555555	1.0007458
10	88	62.13882	.9777777	1.0003728
11	90	62.11566	1.0000000	1.0000000
12	92	62.09251	1.0222222	.99962730
13	94	62.06940	1.0444444	.99925526
14	96	62.04628	1.0666666	.99888305
15	98	62.02319	1.0888888	.998511325
16	100	61.99628	1.1111111	.99807810
17	102	61.96939	1.1333333	.997645199
18	104	61.94252	1.1555555	.99721262
19	106	61.91566	1.1777777	.9967802
20	108	61.88885	1.2000000	.9963485
21	110	61.86205	1.2222222	.9959171
22	112	61.83145	1.2444444	.9954245
23	114	61.80470	1.2666666	.99499385
24	116	61.77415	1.2888888	.9945020
25	118	61.74364	1.3111111	.9940108
26	120	61.71315	1.3333333	.9935199

4. NUMERICAL STABILITY AND CONVERGENCE

The numerical procedure, presented in the previous section, involves spatial integrations for the calculation of the vertical velocity component and pressure, as well as time integrations for the calculation of the horizontal velocity components, temperature and water-level. These spatial and time integration procedures are plagued by numerical stability problems caused by round-off errors in the numerical solution and convergence problems caused by the truncation error due to the finite magnitude of the space and time increments. The numerical solution is said to be stable if round-off errors remain either constant or decrease as the solution to the difference equations is carried out. The numerical solution is termed convergent if, as the space or time increment approaches zero, the numerical solution approaches the exact solution to the differential equations. Using these definitions, stability is a necessary condition for convergence and in this sense the problems of stability and convergence are interdependent.

To achieve numerical stability in nonlinear numerical solutions, the governing system of differential equations must first be linearized and the absolute value of the largest eigenvalue of the amplification matrix for the system of differential equations should be made less or equal to unity (1, 26). This condition will provide an upper bound on the integration step size that ensures the numerical stability of the solution. The computation of the eigenvalues of a large system of differential equations is very laborious and time consuming. It constitutes a problem of the same order of difficulty as the problem to be solved. Fortunately, an exact calculation of the upper bound of integration step size is not necessary. A conservative estimate of the upper bound of integration step size is all that is required. The calculation of a conservative estimate for spatial and time steps will be based on the following arguments:

1) In general, numerical instabilities develop locally in one variable, grow with increasing time and adversely affect other variables through coupling terms in the governing differential equations. If the differential equation for every variable is numerically stable in any small region of the domain, without feedback from the others, instability will not have its origin within the domain and, therefore, will not occur . Employing this hypothesis, each of the differential equations involved in the analysis will be examined independently and their stability criterion will be determined.

2) Exprience with the computational properties of various numerical schemes for the solution of the heat and mass transport phenomena has led to a simple rule (3) based on the fact that water temperature is generally a non-negative quantity. This rule, which applies to an explicit computational scheme, is that the coefficients multiplying each of the temperature values should be positive quatities. This rule, used herein for the numerical stability analysis of the differential equations for the horizontal velocity components and temperature, is substantiated in Appendix 2.

4.1 Stability Analysis of Horizontal Momentum and Energy Equations

As discussed earlier, the horizontal momentum and energy equations are solved explicitly.

Using the above-mentioned rule on these explicit solutions, one obtains a set of stability conditions which must be satisfied, otherwise the solutions will not converge. The stability analysis is demonstrated for energy equation only. In view of similarity between the momentum and energy equations, the same results are applicable to the horizontal momentum equations. After linearizing and expressing the energy equation in an explicit finite difference form in terms of a small variation in temperature, as exemplified in Appendix 2, one obtains:

$$\dot{T}^{n} = \dot{T}_{i+1}\left(-a_x \frac{u\Delta t}{x_{i+1}-x} + c_x \frac{D_h\Delta t}{x_{i+1}-x}\right)$$

$$+ \dot{T}\left(1 + a_x \frac{u\Delta t}{x_{i+1}-x} - b_x \frac{u\Delta t}{x-x_{i-1}} + a_y \frac{v\Delta t}{y_{j+1}-y} - b_y \frac{v\Delta t}{y-y_{j-1}}\right.$$

$$+ a_z \frac{w\Delta t}{z_{k+1}-z} - b_z \frac{w\Delta t}{z-z_{k-1}} - c_x \frac{D_h\Delta t}{x_{i+1}-x} - c_x \frac{D_h\Delta t}{x-x_{i-1}} - c_y \frac{D_h\Delta t}{y_{j+1}-y}$$

$$\left. - c_y \frac{D_h\Delta t}{y-y_{j-1}} - c_z \frac{D_v\Delta t}{z_{k+1}-z} - c_z \frac{D_v\Delta t}{z-z_{k-1}}\right)$$

$$+ \dot{T}_{i-1}\left(b_x \frac{u\Delta t}{x-x_{i-1}} + c_x \frac{D_h\Delta t}{x-x_{i-1}}\right)$$

$$+ \dot{T}_{j+1} \left(- a_y \frac{v\Delta t}{y_{j+1}-y} - c_y \frac{D_h\Delta t}{y_{j+1}-y}\right)$$

$$+ \dot{T}_{j-1} \left(b_y \frac{v\Delta t}{y-y_{j-1}} + c_y \frac{D_h\Delta t}{y-y_{j-1}}\right)$$

$$+ \dot{T}_{k+1} \left(-a_z \frac{w\Delta t}{z_{k+1}-z} + c_z \frac{D_v\Delta t}{z_{k+1}-z}\right)$$

$$+ \dot{T}_{k-1} \left(b_z \frac{w\Delta t}{z-z_{k-1}} + c_z \frac{D_v\Delta t}{z-z_{k-1}}\right) \qquad (94)$$

where superscript $(\cdot)$ indicates a small variation in temperature. Setting the temperature coefficients equal to positive value and substituting for a_x, b_x, c_x, a_y, b_y, c_y, a_z, b_z, and c_z from equations (67) yields

$$x - x_{i-1} < \frac{2D_h}{|u|}$$

$$x_{i+1}-x < \frac{2D_h}{|u|}$$

$$y - y_{j-1} < \frac{2D_h}{|v|}$$

$$y_{j+1}-y < \frac{2D_h}{|v|}$$

$$z-z_{k-1} \leqslant \frac{2D_v}{|w|}$$

$$z_{k+1}-z \leqslant \frac{2D_v}{|w|} \tag{95}$$

and

$$\Delta t \leqslant 1/(\frac{2D_h}{(x-x_{i-1})^2} + \frac{2D_h}{(y-y_{j-1})^2} + \frac{2D_v}{(z-z_{i-1})^2}) \tag{96}$$

The increments of the nodal coordinates may be expressed in terms of the elements mesh size Δx, Δy, and Δz by

$$x-x_{i-1} = \frac{1}{2}(\Delta x + \Delta x_{i-1})$$

$$y-y_{j-1} = \frac{1}{2}(\Delta y + \Delta y_{j-1})$$

$$z-z_{k-1} = \frac{1}{2}(\Delta z + \Delta z_{k-1}) \tag{97}$$

Substituting equations (97) into equations (95) and (96) results in

$$\Delta x + \Delta x_{i-1} \leqslant \frac{4D_h}{|u|}$$

$$\Delta y + \Delta y_{j-1} \leqslant \frac{4D_h}{|v|}$$

$$\Delta z + \Delta z_{k-1} \leqslant \frac{4D_v}{|w|}$$

$$\Delta t \leqslant 1/\left(\frac{8D_h}{(\Delta x+\Delta x_{i-1})^2} + \frac{8D_h}{(\Delta y+\Delta y_{j-1})^2} + \frac{8D_v}{(\Delta z+\Delta z_{k-1})^2}\right) \tag{98}$$

Since the energy equation has the same form as the momentum equations, equations (97) and (98) constitute the numerical stability criteria for the energy and horizontal momentum equations.

4.2 Convergence of Pressure Equation

As discussed in Section 3.2, the pressure equation provides a system of linear equations, which is solved by the modified Gauss-Seidel iterative method designated as successive over-relaxation. A sufficient condition for the convergence of this iterative process is given by Isaacson and Keller (12) and Williams (43) as follows:

$$\Delta = \frac{\sum\limits_{m=1}^{m'} |a_{rm}|}{|a_{rr}|} \leqslant 1 \tag{99}$$

where a_{rr} is the value of the diagonal terms of matrix and a_{rm} is the value of the off-diagonal terms, with m' notation signifying that the value of a_{rr} is omitted from the summation. The convergence of the numerical solution is ensured if $\Delta < 1$ for at least one equation in the system of linear pressure equations, given by equations (72). An examination of the later equations shows that for nodal points within the fluid field $\Delta = 1$ except for nodal points just below the water-level region or near other boundaries for which pressure is known. The known value of pressure causes a transfer of the corresponding term to the right hand side of equation (72) leading to $\Delta < 1$ which is a sufficient condition for convergence.

4.3 Stability Analysis of Water-Level Equation

The stability criterion for the water-level equation is obtained by linearizing equation (31), as exemplified in Appendix 2 for the energy equation, as follows:

$$\frac{\partial \dot{H}}{\partial t} = - u \frac{\partial \dot{H}}{\partial x} - v \frac{\partial \dot{H}}{\partial y} \tag{100}$$

where superscript (·) indicates small variation about steady-state. In this study, equation (100) is expressed in a finite difference form, by applying a forward two point finite difference approximation to the temporal derivative and backward finite difference approximation to first spatial derivatives respectively. Applying Von-Neumann stability analysis, shown in Appendix 2, to the discretized equation accounting for variable mesh size, one obtains:

$$e^{\delta \Delta t} = 1 - \frac{u \Delta t}{x - x_{i-1}} (1 - e^{-i\alpha \Delta x_{i-1}})$$

$$- \frac{v \Delta t}{y - y_{j-1}} (1 - e^{-i\beta \Delta y_{j-1}}) \tag{101}$$

The necessary and sufficient condition for equation (101) to be stable is that the absolute value of $e^{\delta \Delta t}$ must be less than or equal to one.

After expressing the exponential terms in trigonometric forms and calculating the absolute value of $e^{\delta\Delta t}$, equation (101) reduces to

$$\left|e^{\delta\Delta t}\right| = (1 - 4\,(1 - \frac{u\Delta t}{x-x_{i-1}} - \frac{v\Delta t}{y-y_{j-1}})\,\frac{u\Delta t}{x-x_{i-1}}\sin^2\frac{\alpha\Delta x_{i-1}}{2}$$

$$- 4\,(1 - \frac{u\Delta t}{x-x_{i-1}} - \frac{v\Delta t}{y-y_{j-1}})\,\frac{v\Delta t}{y-y_{j-1}}\sin^2\frac{\beta\Delta y_{j-1}}{2}$$

$$- 4\,(\frac{u\Delta t}{x-x_{i-1}})\;(\frac{v\Delta t}{y-y_{j-1}})\,\sin^2\frac{\alpha\Delta x_{i-1} - \beta\Delta y_{j-1}}{2}) \leqslant 1 \qquad (102)$$

In the above inequality, the maximum values of the trigonometric terms are equal to one. Therefore, further examination shows that this inequality will be satisfied if

$$1 - \frac{u\Delta t}{x-x_{i-1}} - \frac{v\Delta t}{y-y_{j-1}} \geqslant 0 \qquad (103)$$

To satisfy the above stability conditions for negative values of the velocity components, one must have

$$\Delta t \leqslant 1/(|u|/(x-x_{i-1}) + |v|/(y-y_{j-1})) \qquad (104)$$

Expressing equation (104) in terms of Δx and Δy as shown in Section (4.1), one obtains

$$\Delta t \leqslant 1/(2|u|/(\Delta x + \Delta x_{i-1}) + 2|v|/(\Delta y + \Delta y_{j-1})) \qquad (105)$$

It should be noted that estimates of the largest values of the velocity components would suffice for the calculation of conservative values of space and time increments.

5. PRESENTATION OF RESULTS

The mathematical formulation and the computer program, developed in this study, is verified by applying the dynamic analysis to a number of cases where the final steady-state solution is either known experimentally or the final steady-state flow pattern has been determined by other investigators. Employing the present program, the fluid flow problem considered is brought to steady-state condition through a dynamic analysis. The initial values of thermal and hydraulic variables in the flow field, needed for this purpose, were selected in such a way to facilitate the entry of input data. Starting from these rough initial values and maintaining the appropriate boundary conditions, the present program is employed to bring the flow problem to steady-state. These steady-state solutions are then compared either with the experimental work or the known flow patterns. These verification studies are considered in the next section.

5.1 Verification Studies

The following problems were selected for verification purposes:

1) Experimental studies on the laminar flow development in a square duct (7)

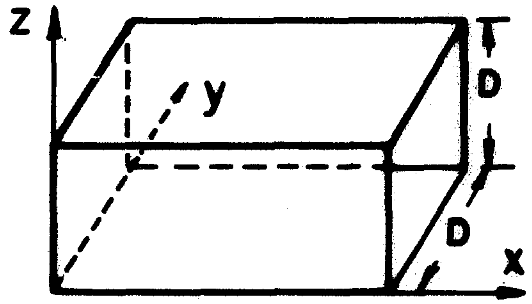
2) Natural circulation in a pond partially heated from the bottom or the side.

The first study consists of a duct with a square cross section with a spatially and temporally uniform input velocity at the duct entrance. Since the geometric dimensions of the laboratory model used in the experimental studies were small and our interest here is in water bodies of appreciable size, the similarity between the present and the experimental study is based on two dimensionless parameters: 1) the quotient of aspection ration (x/D) to Reynolds number; and 2) the ratio of the fluid velocity to the average inlet velocity. The geometric and hydraulic input data for this case is given in Table 2 and Fig. 2a. The computational grid used in this dynamic analysis consists of 12 x 5 x 5 mesh points with non-uniform spacing in the longitudinal direction and uniform spacing in the square cross section as shown in Table 3. The initial values of the velocity components in the flow field were selected uniform in the longitudinal direction equal to the inlet velocity with zero transversal and vertical velocities. The dynamic solution was continued until the flow pattern in the duct reached steady-state. Typical time histories for a number of upstream and a downstream points along the duct axial central plane are obtained and used to plot the final steady-state centerline and vertical velocity profiles in the square duct. These velocity profiles, in non-dimensional form, are compared in Figs. 2b and 2c with the experimental results of Goldstein and Kreid (37) who measured the laminar flow development in a square duct using laser-doppler flowmeter. This comparison shows that the results of this study are in good agreement with the experimental data. This confirms that the present formulation can predict the three dimensional flow behavior of water bodies.

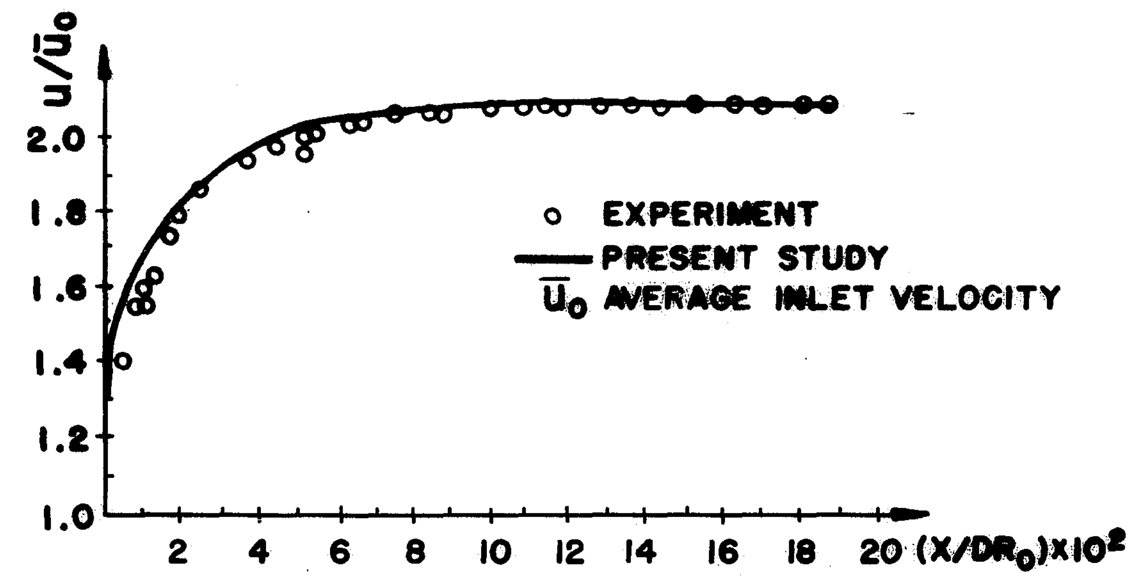
The second study consists of natural circulation in a pond par-

TABLE 2. THE GEOMETRIC AND HYDRAULIC INPUT DATA
FOR LAMINAR FLOW IN A SQUARE DUCT

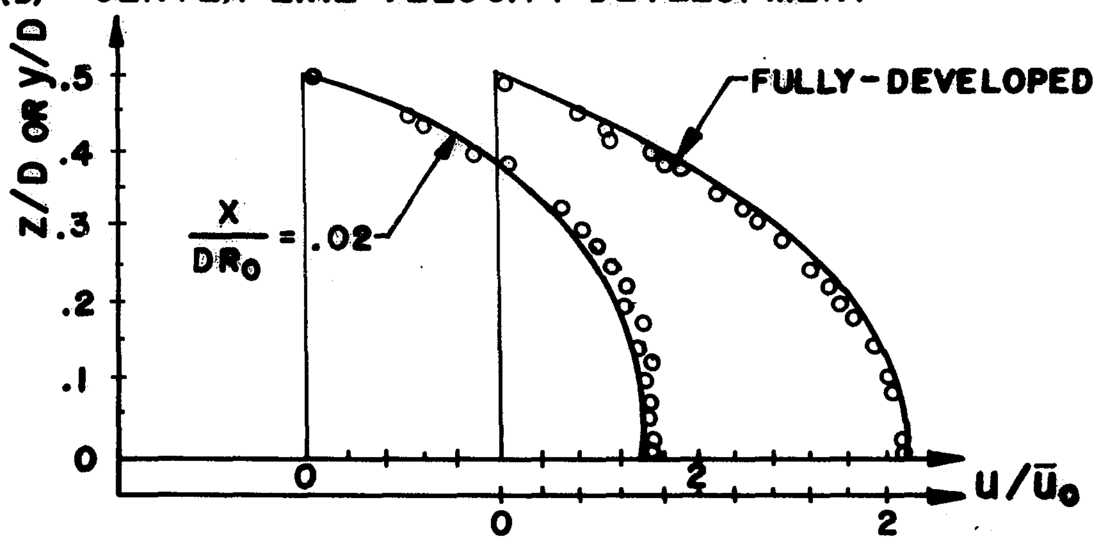
Specifications	Dimensions	
	British Unit	SI Unit
Water Body Length	280 ft.	85.34 m
Water Body Width	72 ft.	21.946 m
Water Body Depth	72 ft.	21.946 m
Inlet Water Velocity	1 ft/sec	0.3048 m/sec
Water Temperature	75^oF	$23.89\ ^oC$
Reynolds Number, R_0	20.83	20.83
Hydraulic Diameter of Square Duct, D	72 ft.	21.946 m



(a) THE SQUARE DUCT CONFIGURATION



(b) CENTER-LINE VELOCITY DEVELOPMENT



(c) DEVELOPMENT OF VELOCITY PROFILE
CENTRAL PLANE

FIG 2. VELOCITY DEVELOPMENT IN A SQUARE DUCT

TABLE 3. GRID DIMENSIONS FOR THE LAMINAR FLOW IN A SQUARE DUCT

Element	x		y		z	
No.	ft	m	ft	m	ft	m
1	10	3.048	18	5.486	18	5.486
2	10	3.048	18	5.486	18	5.486
3	10	3.048	18	5.486	18	5.486
4	10	3.048	18	5.486	18	5.486
5	20	6.096	18	5.486	18	5.486
6	30	9.144				
7	30	9.144				
8	30	9.144				
9	30	9.144				
10	30	9.144				
11	30	9.144				
12	30	9.144				
13	30	9.144				

tially heated from the bottom or the side with uniform initial temperature, zero initial velocities and zero inlet and outlet mass flux. The geometric and hydraulic input data for this case is given in Table 4. The computational grid used in the dynamic analysis consists of 10 x 6 x 6 mesh points with uniform spacing in the longitudinal and transversal directions and non-uniform spacing in the vertical direction as shown in Table 5. Two cases were examined. In these cases, a step temperature is applied to a portion of side wall or the bottom surface of the pond. The dynamic response of the pond is obtained for each case and the natural circulation flows developed are compared with expected flow patterns. Typical natural circulation flow patterns for partially heated side wall and bottom surface are shown in Figs. 3, 4, 5, and 6 at times t = 40 and t = 25 seconds, respectively, into the transient. An examination of these flow patterns demonstrates that for the case of partially heated side wall one natural circulation vortex in transversal direction and two symmetric natural circulation vortices in the longitudinal direction are developed which lose their strength as the distance from the heated wall increases (see Figs. 3 and 4). For the case of partially heated bottom surface two symmetric natural vortices in vertical longitudinal and vertical diagonal planes are developed which lose their strength with increasing distance from the heated surface (see Figs. 5 and 6). In both cases, described above, the natural circulation vortices formed and the resultant mixing are caused by density differences in the water body. These results establish that the present formulation can predict the three dimensional flow and thermal

TABLE 4. GEOMETRIC AND HYDRAULIC INPUT DATA FOR PARTIALLY HEATED POND

Specifications	Dimensions: British Unit	Dimensions: SI Unit
Water Body Length	540 ft	164.592 m
Water Body Width	250 ft	76.2 m
Water Body Depth	20.5 ft	6.248 m
Water Body Temperature	75 ^oF	23.89 ^oC
Water Body Equilibrium Temperature	74 ^oF	23.33 ^oC
Temperature of Heated Area	100 ^oF	37.77 ^oC
Reference Velocity, U_0	1 ft/sec	0.3048 m/sec
Reference Length, d_0	50 ft	15.24 m
Reference Temperature, T_0	100 ^oF	37.77 ^oC
Reference Time, t_0	50 sec	50 sec
Reference Density, ρ_0	61.9963 lbm/ft^{-3}	960.590 $kg\ m^{-3}$
Reference Pressure, p_0	21.52 $lbf\ in^{-2}$	14639.4 $kg\ m^{-2}$
Reynolds Number, R_0	0.6061×10^7	0.6061×10^7
Heat Exchange Coefficient, K	100 $Btu/ft^2\ day\ ^oF$	23.64 $W/m^2\ ^oC$

TABLE 5. GRID DIMENSIONS FOR PARTIALLY HEATED POND

Element	x		y		z	
No.	ft	m	ft	m	ft	m
1	60	18.29	50	15.24	3.5	1.07
2	60	18.29	50	15.24	4.	1.22
3	60	18.29	50	15.24	5	1.52
4	60	18.29	50	15.24	4	1.22
5	60	18.29	50	15.24	4	1.22
6	60	18.29	50	15.24	3.5	1.07
7	60	18.29				
8	60	18.29				
9	60	18.29				
10	60	18.29				

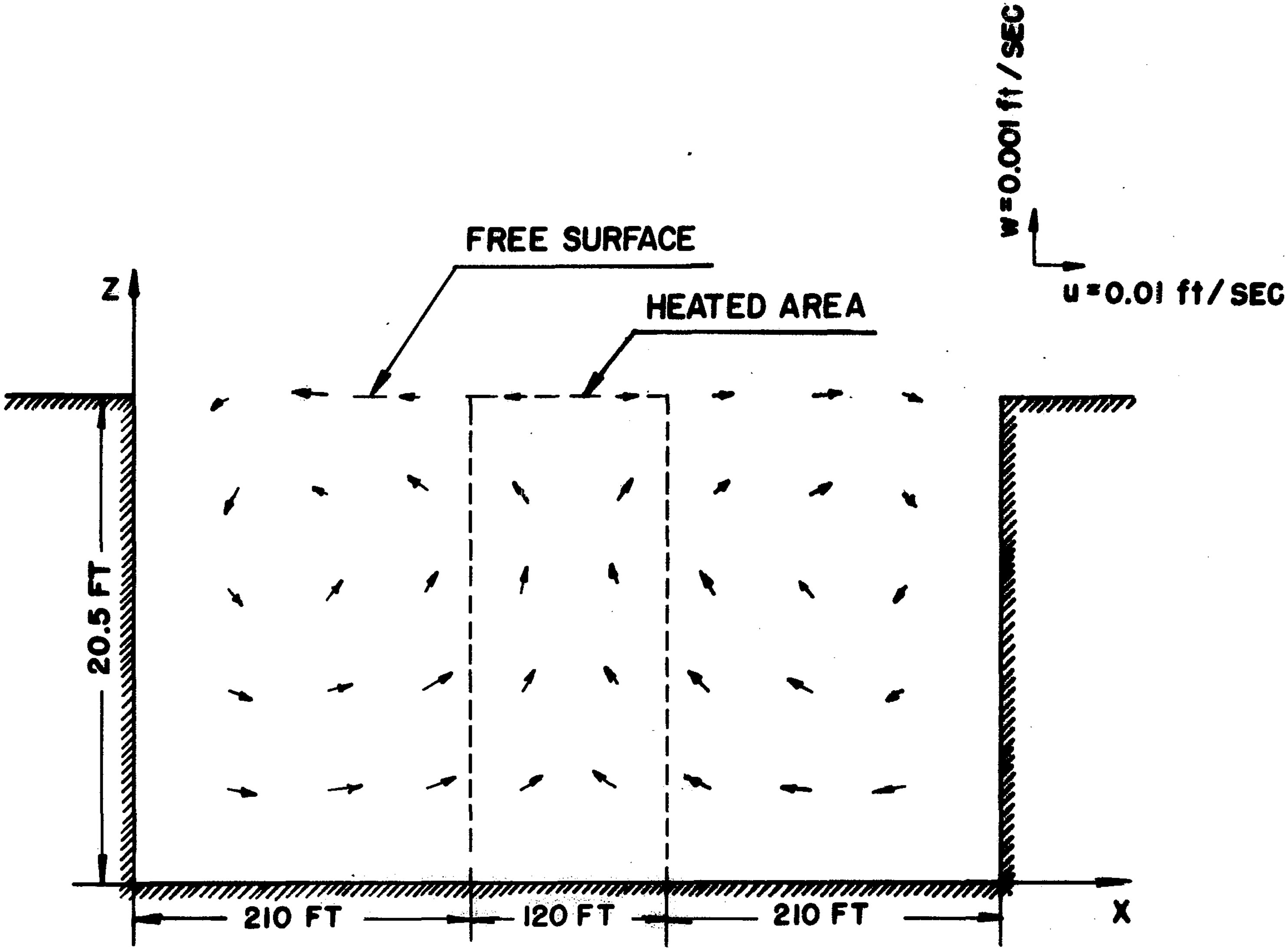


FIG 3. NATURAL CIRCULATION AT $y=100$ FT. AND $t=40$ SEC IN A POND PARTIALLY HEATED FROM SIDE.

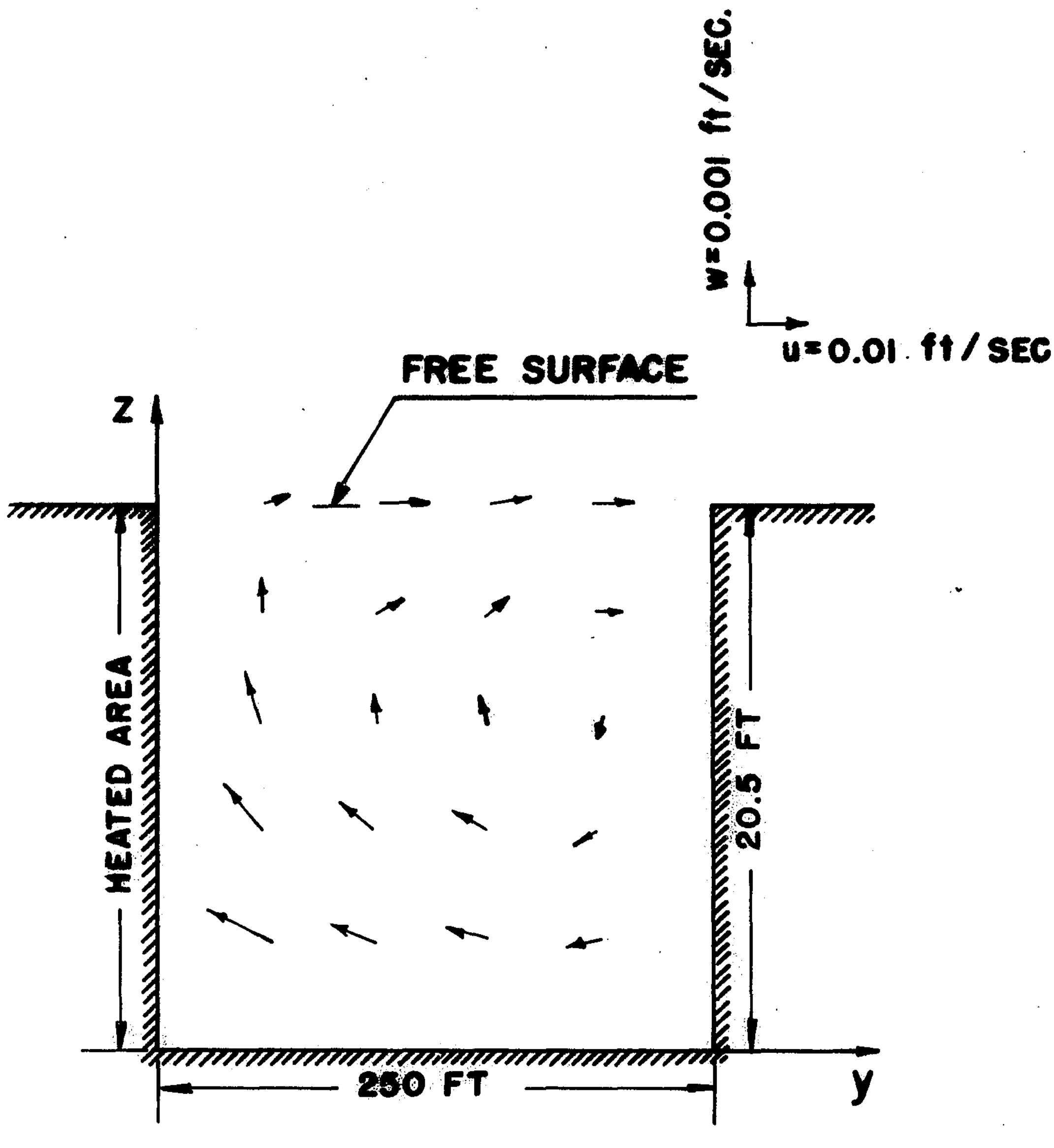


FIG 4. NATURAL CIRCULATION AT X=240 FT. AND t=40 SEC. IN A POND PARTIALLY HEATED FROM SIDE.

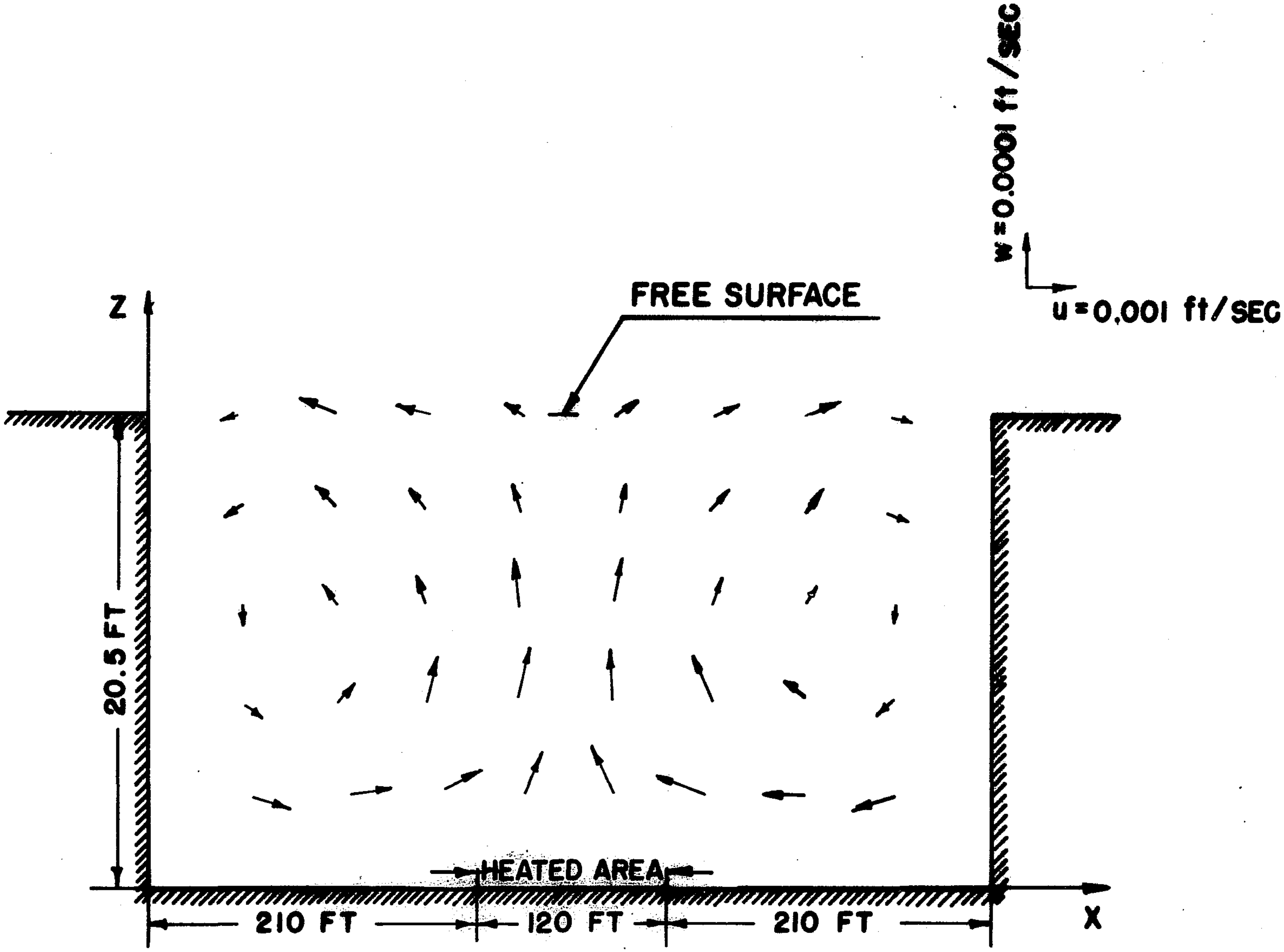


FIG 5. NATURAL CIRCULATION AT y=100 FT. AND t= 25 SEC.
IN A POND PARTIALLY HEATED FROM BOTTOM.

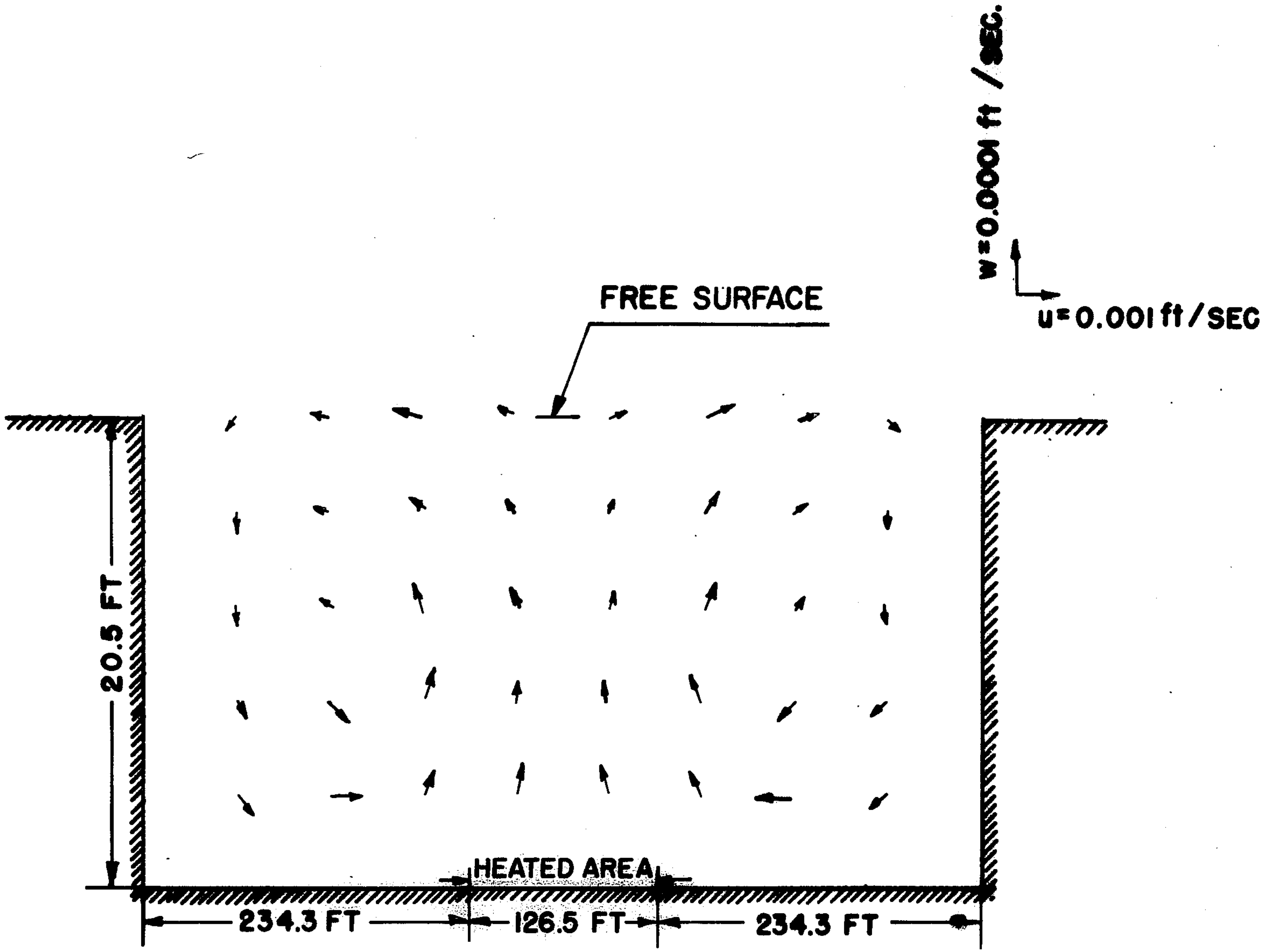


FIG 6. NATURAL CIRCULATION AT t= 25 SEC IN VERTICAL DIAGONAL PLANE IN A POND PARTIALLY HEATED FROM BOTTOM

aspects of water bodies.

5.2 Circulation and Stratification in Water Bodies

The mathematical formulation, presented in this study, is applied to the study of circulation and stratification of large water bodies. The water body is considered to be flowing initially at a uniform velocity and temperature and suddenly exposed to a 90° angle jet with higher velocity but the same temperature. This problem is referred to as three-dimensional non-buoyant jet in a cross current. When the dynamic problem reaches steady-state, the water jet temperature is suddenly raised to simulate a thermal discharge. This problem is referred to as three-dimensional buoyant jet in a cross current. To facilitate the understanding of the results, the two problems indicated above (i.e., non-buoyant and buoyant jets) are discussed separately in the following sections but the time histories of the results are shown on the same plots for easy reference.

5.3 Three Dimensional Non-Buoyant Jet in a Cross Current

The problem of a three-dimensional non-buoyant jet in a cross current, as modeled in Fig. 1, was first analyzed. The geometric and hydraulic input data for the case studied are given in Table 6.

The water body is assumed to be flowing initially at a uniform velocity of 0.40 ft/sec when suddenly exposed to a jet velocity of 2.00 ft/sec at a 90 degree angle. The surface wind velocity are considered to be zero. Furthermore, the temperature of the thermal

TABLE 6. GEOMETRIC AND HYDRAULIC INPUT DATA FOR THREE-DIMENSIONAL, NON-BUOYANT AND BUOYANT JETS IN A CROSS CURRENT

Specifications	Dimensions: British Unit		Dimensions: SI Unit	
Water Body Length	600	ft	182.88	m
Water Body Width	360	ft	109.73	m
Water Body Depth	16.5	ft	5.03	m
Jet Width, 2 d_0	100	ft	30.48	m
Jet Depth	5.75	ft	1.75	m
Jet Velocity, U_0	2	ft/sec	.61	m/sec
River Velocity	.4	ft/sec	.12	m/sec
Water Body Temperature	75	^oF	23.89	^oC
Water Body Equilibrium Temperature	74	^oF	23.33	^oC
Thermal Discharge Temperature, T_0	90	^oF	32.22	^oC
Reference Time, t_0	25	sec	25	sec
Reference Density, ρ_0	62.1156	lbm ft^{-3}	962.730	kg m^{-3}
Reference Pressure, P_0	21.56	lbm in^{-2}	14672.0	kg m^{-2}
Reynolds Number, R_0	0.1212×10^8		0.1212×10^8	
Heat Exchange, Coefficient, k	100	Btu/ft^2 day ^oF	23.64	W/m^2 ^oC

discharge is made equal to the water body temperature to simulate a non-buoyant jet. A three-dimensional grid was superimposed on the flow field as detailed in Table 7. The boundaries of this grid coincide with the physical river boundaries. The nodal points, where hydrothermal variables are defined, are located at the point i,j,k of each grid and are shown in Fig. 7.

The time histories of the dynamic variables are shown, for points A. B, C, D, E, F, G, and H marked on Fig. 7, as follows:

1) Velocity components and water-level for two nodes A and B in front of the incoming jet at the free surface shown in Figs. 8 and 9.

2) Velocity components for two nodes C and D in front of the incoming jet at 12.75 feet below the free surface shown in Figs. 10 and 11.

3) Velocity components and water-level for two nodes E and F located upstream and downstream respectively at the free surface shown in Figs. 12 and 13.

4) Velocity components of two nodes G and H located upstream and downstream respectively at 12.75 feet below the free surface shown in Figs. 14 and 15.

Examination of the above plots shows the following features:

Considering the flow at the water surface, the transversal velocities at points A and B, in front of the incoming jet, rise initially with a subsequent rise in the vertical velocity leading to a rise in water-level in the neighborhood of the incoming jet which

TABLE 7. GRID DIMENSIONS FOR THREE-DIMENSIONAL NON-BUOYANT AND BUOYANT JETS IN A CROSS CURRENT

Element	x		y		z	
No.	ft	m	ft	m	ft	m
1	65	19.81	50	15.24	3.5	1.07
2	60	18.29	50	15.24	4.0	1.22
3	55	16.76	55	16.76	5.0	1.52
4	50	15.24	60	18.29	4.0	1.22
5	50	15.24	65	19.81	3.5	1.07
6	55	16.76	70	21.34		
7	60	16.76	70	21.34		
8	65	19.81				
9	70	21.34				
10	70	21.34				

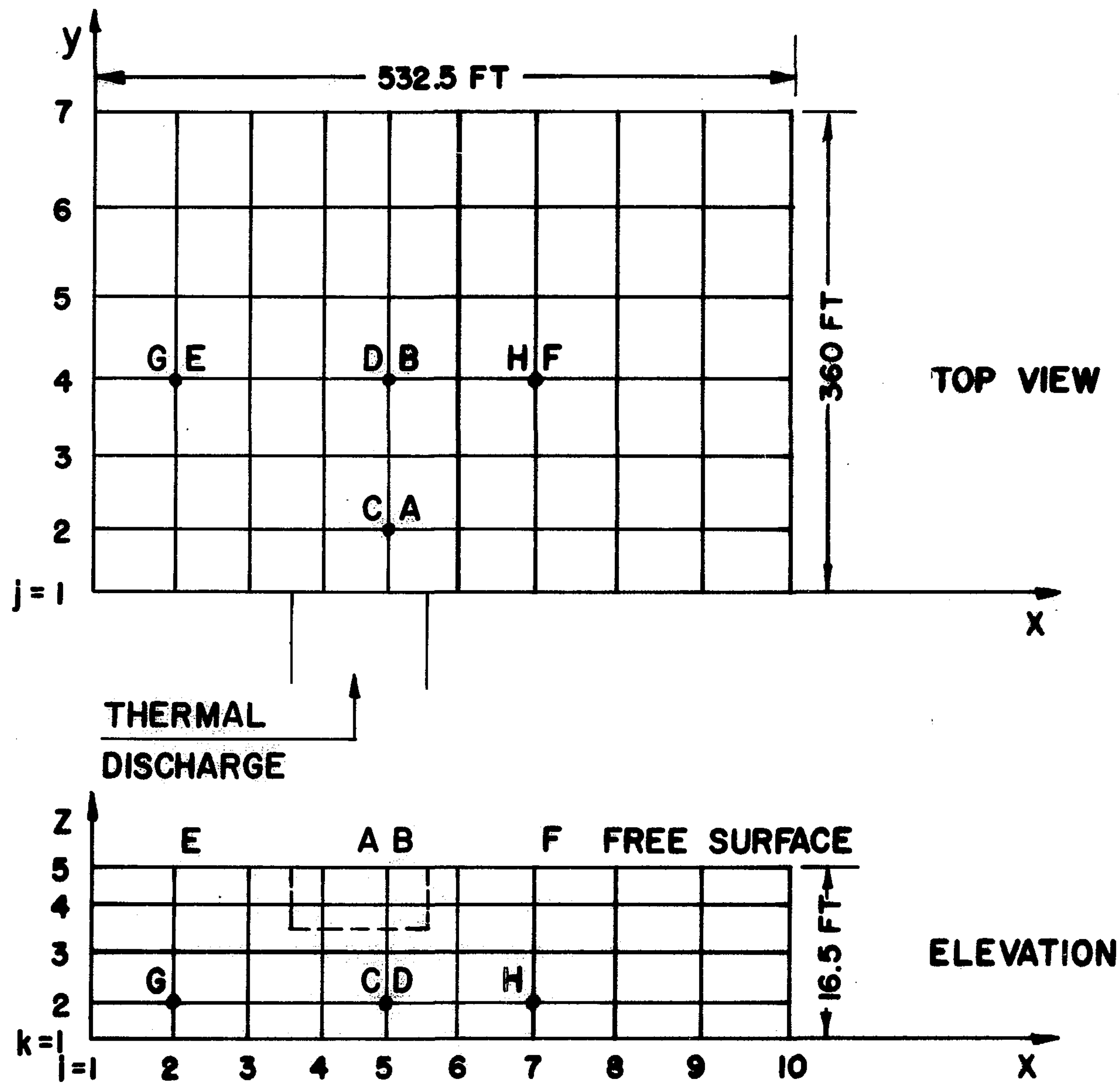


FIG 7. GRID WORK WITH VARIABLE MESH SIZE SUPERIMPOSED ON THE WATER BODY

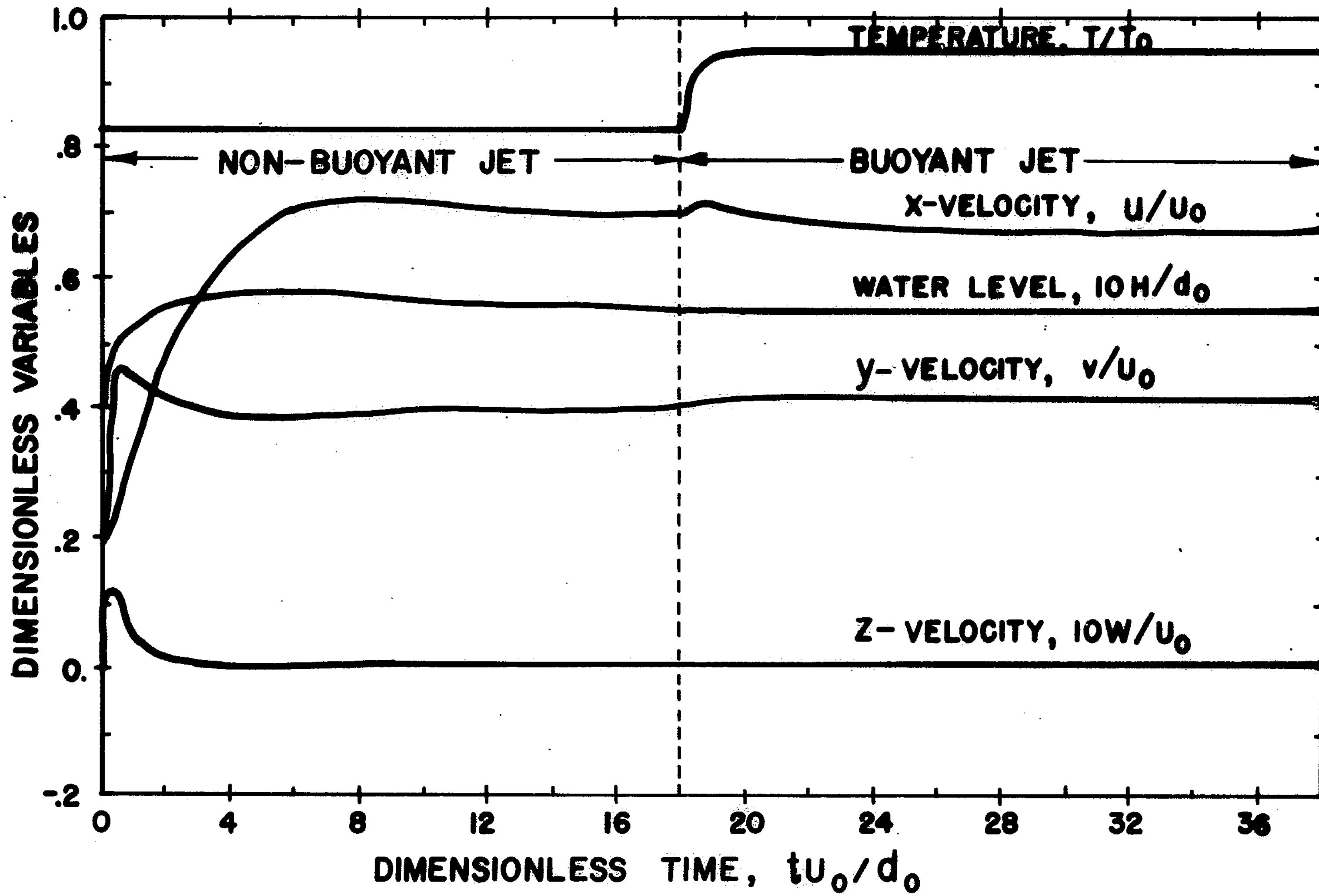


FIG 8. TIME HISTORIES OF VARIABLES AT POINT "A" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

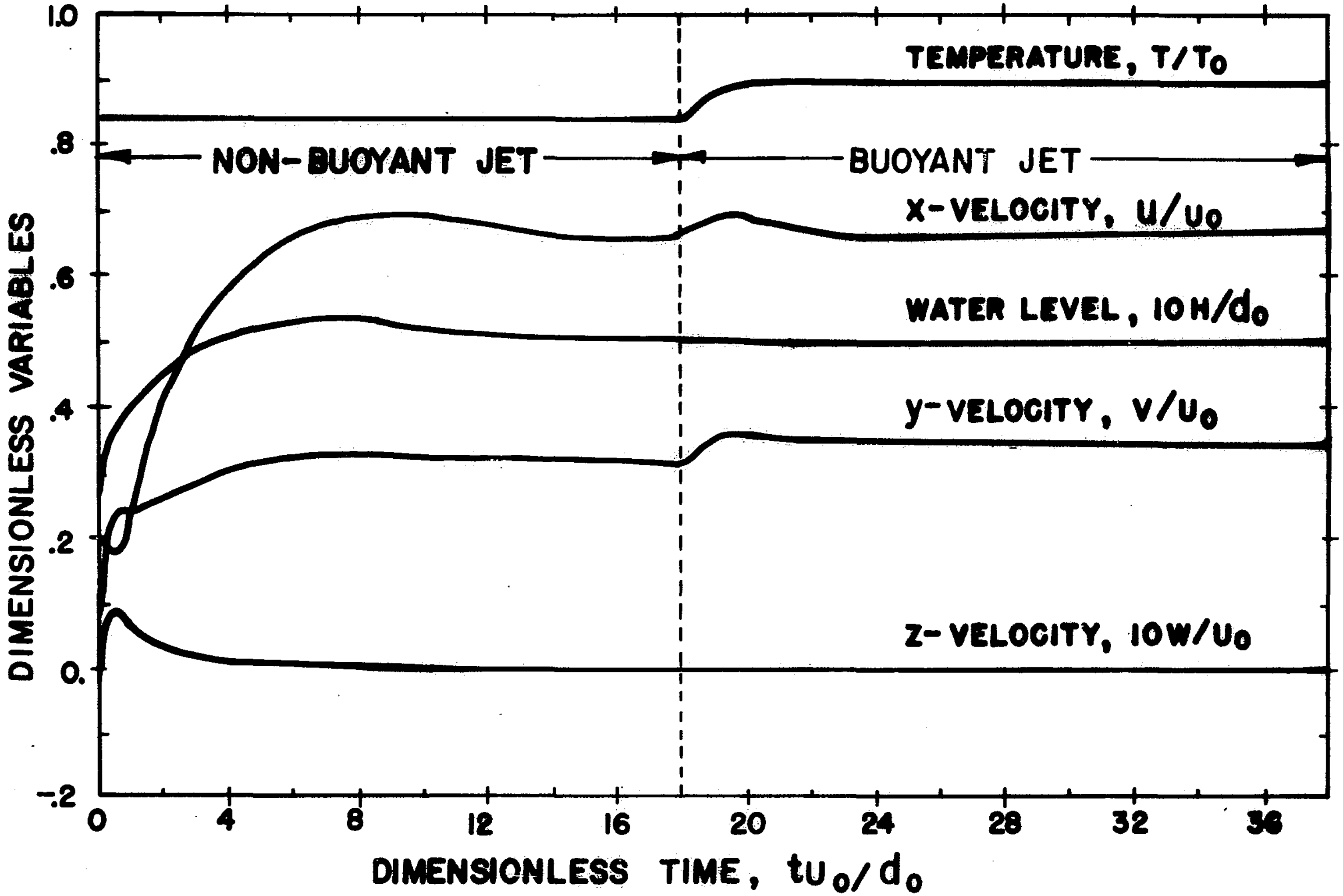


FIG 9. TIME HISTORIES OF VARIABLES AT POINT "B" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

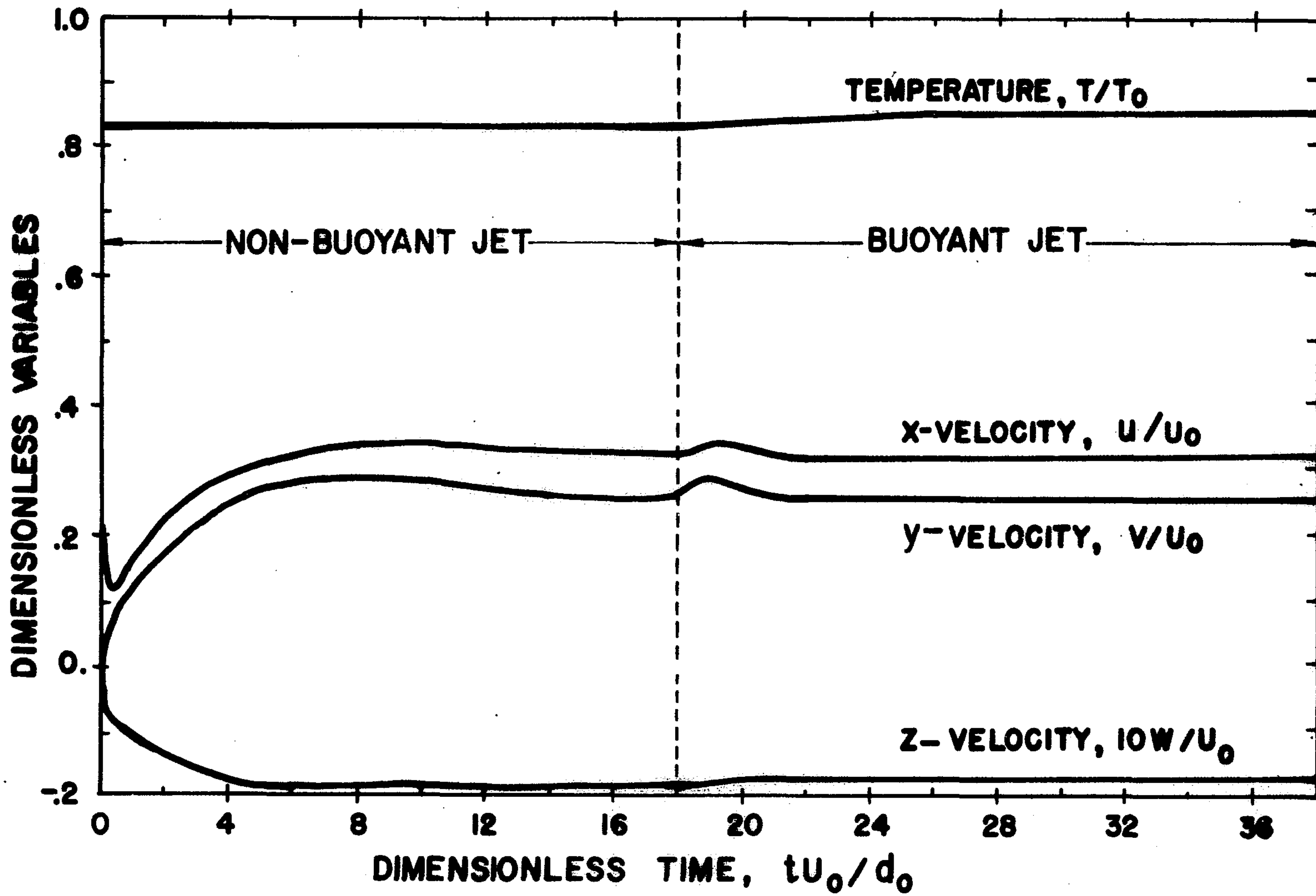


FIG 10. TIME HISTORIES OF VARIABLES AT POINT "C" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

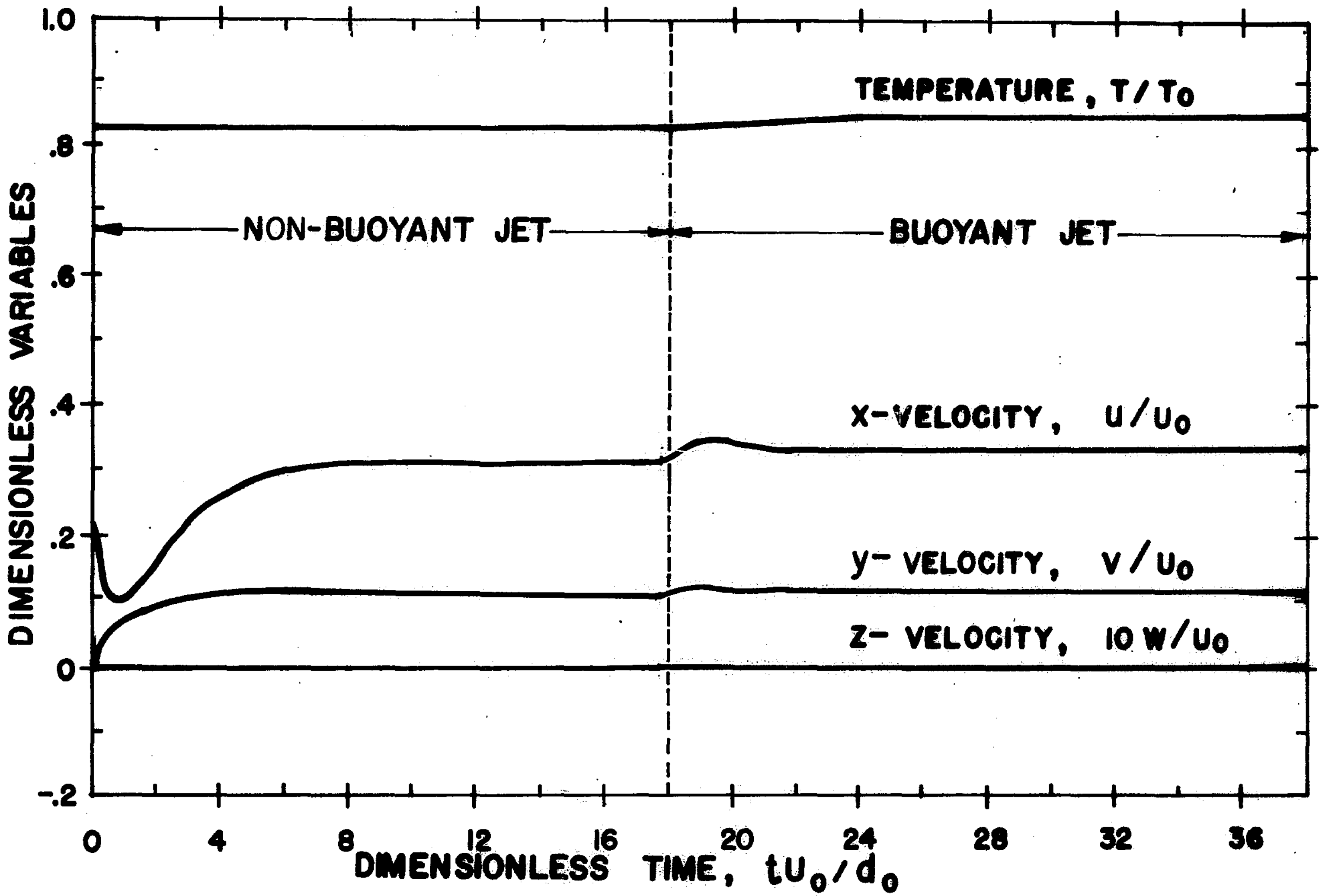


FIG 11. TIME HISTORIES OF VARIABLES AT POINT "D" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

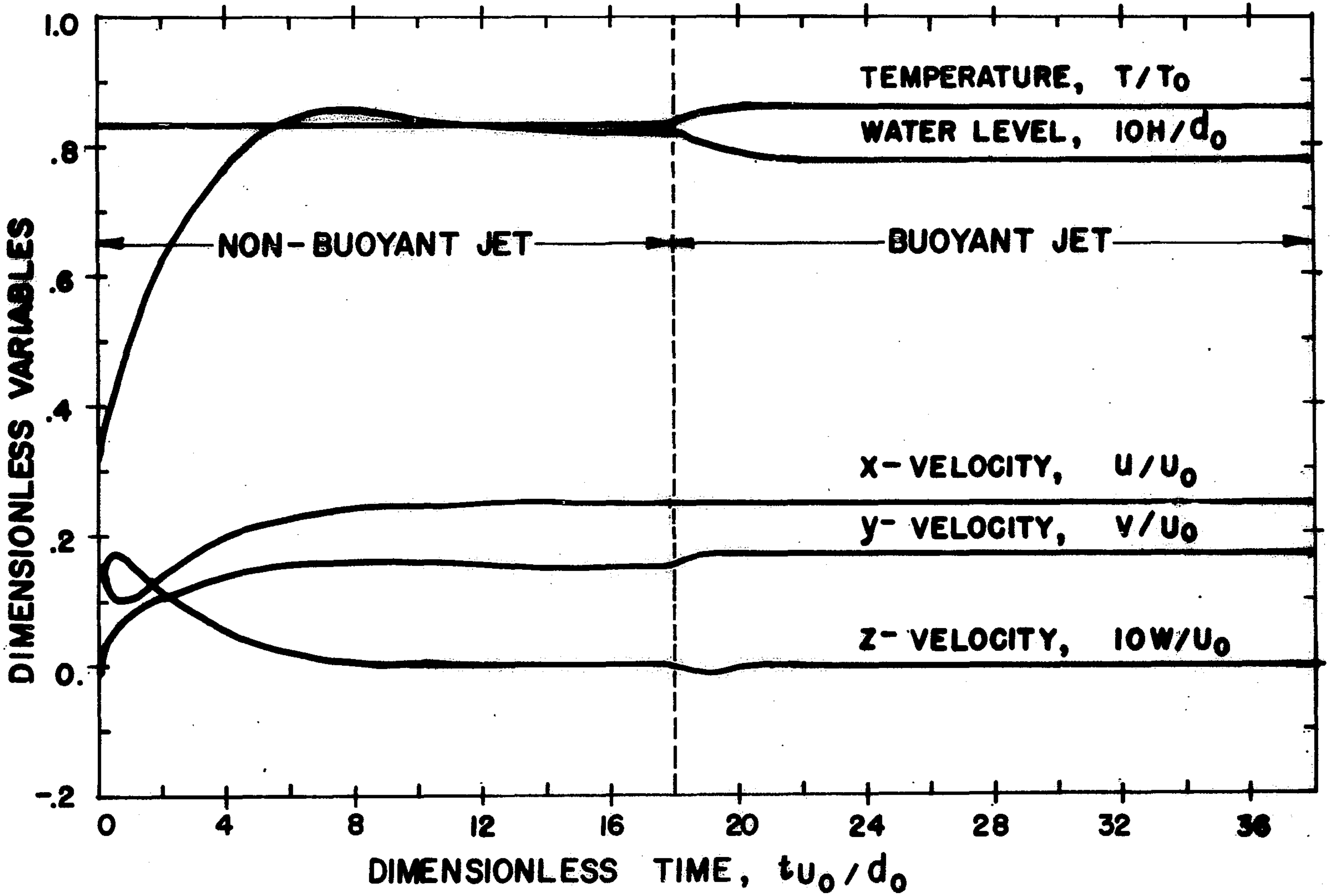


FIG 12. TIME HISTORIES OF VARIABLES AT POINT "E" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

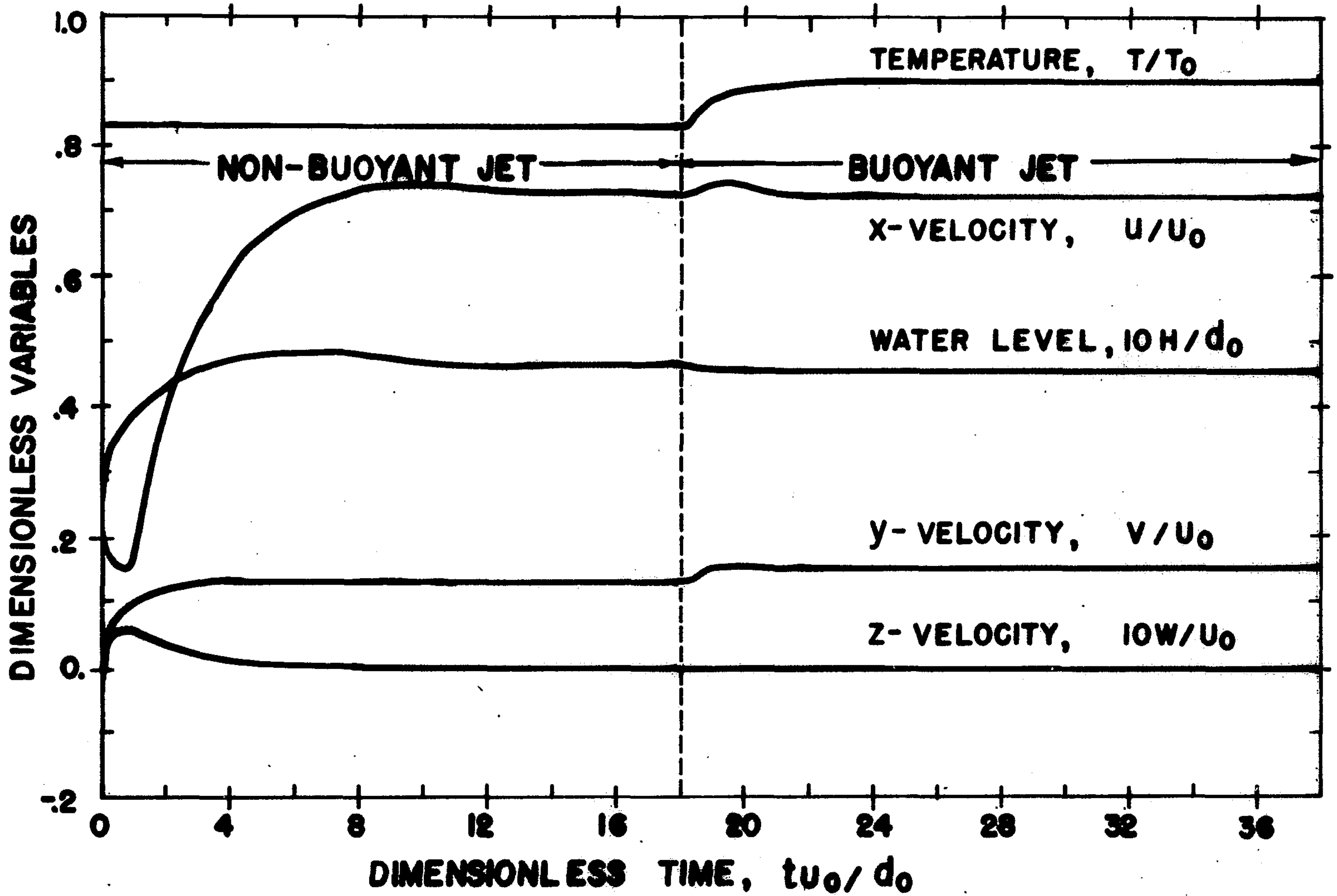


FIG 13. TIME HISTORIES OF VARIABLES AT POINT "F" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

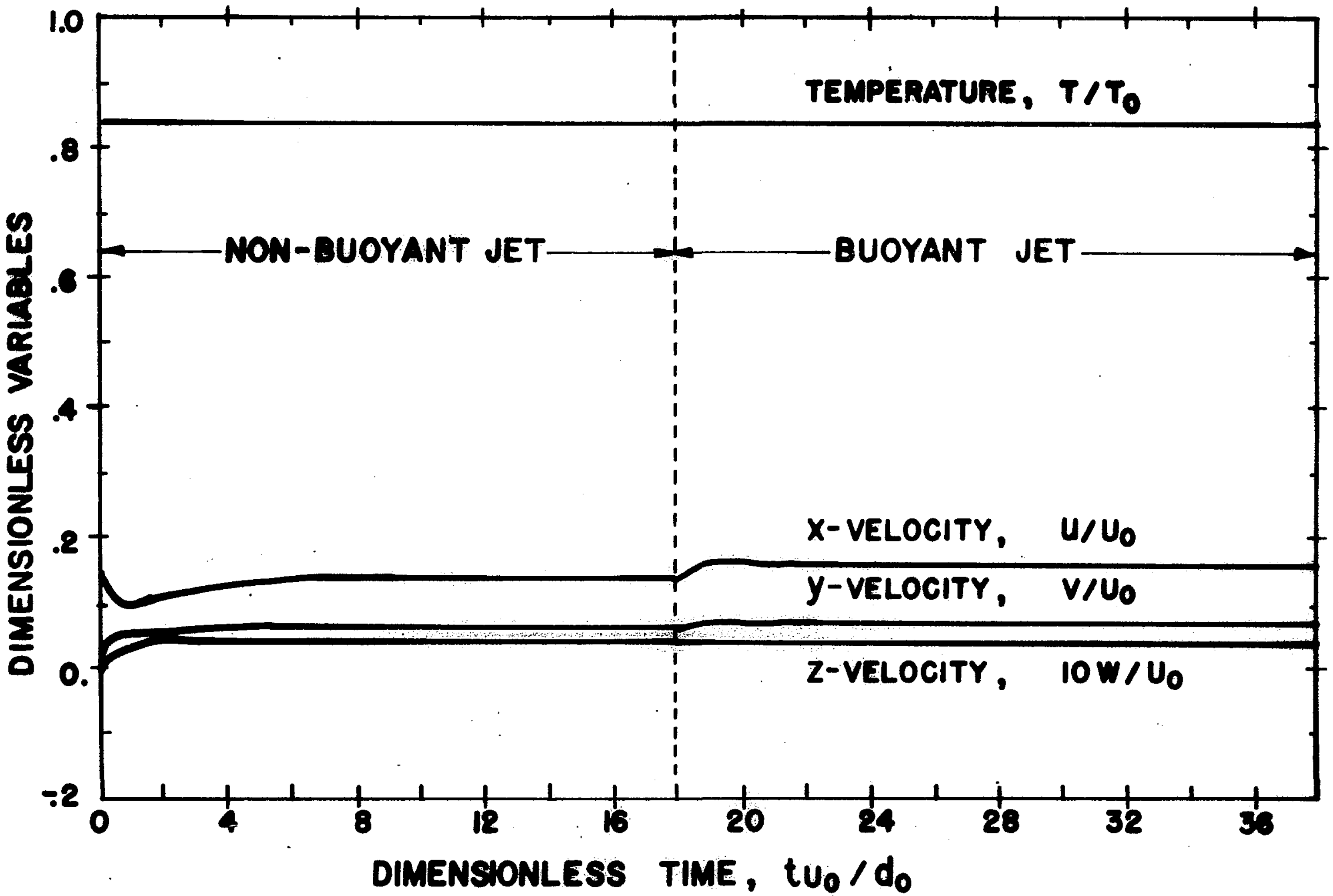


FIG 14. TIME HISTORIES OF VARIABLES AT POINT "G" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

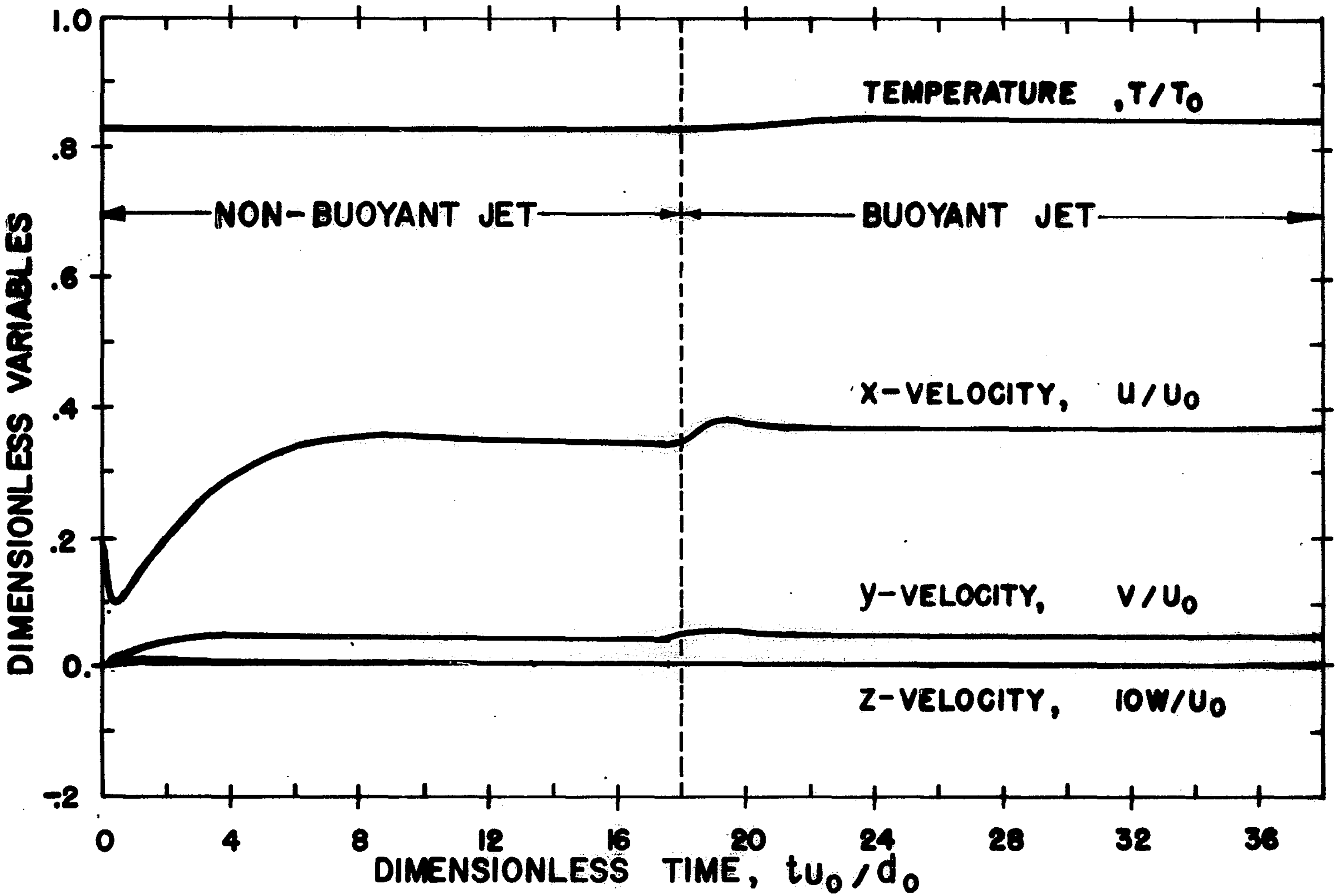


FIG 15. TIME HISTORIES OF VARIABLES AT POINT "H" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS

in turn affects the field flow conditions as time progresses. Since the total head associated with the jet and river flow entries are constant on a short term basis, an increase in the water-level must be accomplished with a corresponding decrease in the field velocity. The disturbance caused by the incoming jet travel from the discharge point both upstream and downstream of the river. This demonstrates the propagation of the surface gravity waves (surge waves). As time progresses, these disturbances reach the river exit and are reflected back upstream. This demonstrates the reflection of the surface gravity waves from the river exit. However, on a long term basis, a further increase in the water-level in the neighborhood of the incoming jet causes additional horizontal velocity components for further downstream points which will gradually affect the downstream flow. These flow patterns are observed both in axial and transversal direction as discussed hereunder:

a) The increase in the water-level initially decelerates the axial flow on a short term basis. With a further increase in the water-level, the axial flow at various points accelerates to their final steady state values which are larger for the downstream points and smaller for the upstream points from the incoming jet position as expected. These effects can be clearly observed in u/U_0 curves for points E, B, and F, in Figs. 12, 9, and 13, where the transient start with a small dip in the flow rate followed by a steep rise settling to its final steady state value.

b) Similarly, the transversal flow generated by the incoming jet, increases the water-level, which in turn, decelerates the transversal flow. However, since, unlike the axial flow patterns, the boundary condition in the transversal direction (solid wall) does not accommodate flow, the transversal flow reaches its final steady state value without undergoing large swings observed in the axial flow curves. These effects can be clearly observed in v/U_0 curves for points A and B in Figs. 8 and 9, where the transversal flow after the initial rise undergoes a reduction in magnitude followed by a mild rise settling to its final steady state values.

At the end of transients discussed above, the increase in transversal flow caused by the incoming jet is accommodated by an increase in the axial flow. For this reason, the water-level reaches a steady state value which in turn results in zero vertical velocity and constant axial and transversal flow, observed in Figs. 8 and 9, as the end of non-buoyant analysis is approached.

Examining the flow at 12.75 feet below the water surface, the axial flow exhibit a pattern similar to that of the water surface except that the results are further accentuated due to the bottom boundary condition. Figs. 11, 14 and 15 show that the flow transients u/U_0 start with a relatively large dip followed by relatively smaller rise as it approaches its final steady state value. The effect of the non-slip bottom boundary condition is to reduce the final steady state value to a quantity below that of the surface and to enlarge the initial dip. Furthermore, the shear initiated by the incoming jet creates transversal flow components v/U_0 at this level as observed in Figs. 10 and 11. The transversal flow reduces from the top to the bottom. At the end of transients discussed above, the increase in transversal flow caused by the incoming jet cannot be fully accommodated by an increase in the axial flow in cells near the solids boundaries. For this reason, a downward velocity component develops which induces a vertical downward velocity in the upper cells, observed in w/U_0 curve in Fig. 10. This downward flow extends in a few cells from the jet entrance and is changed to an upwards flow in cells in the center of the flow field.

5.4 Three Dimensional Buoyant Jet in a Cross Current

The problem of three-dimensional buoyant jet in a cross current is next analyzed. The geometric and hydraulic input data for this case is exactly similar to the non-buoyant jet case detailed in Fig. 7 and Table 6. The water body is assumed to be initially under the steady state conditions reached in the non-buoyant case discussed earlier when the thermal discharge temperature is suddenly increased from 75^o to 90^oF.

The dynamic buoyant jet problem is analyzed in a manner similar to the non-buoyant jet case. However, in view of the introduction of thermal effects, the water body becomes stratified as discussed hereunder. The time histories of the dynamic variables are shown, for points A, B, C, D, E, F, G, and H marked in Fig. 7, as a continuation of the non-buoyant time history plots in Figs. 8 to 15. Examination of these plots shows the following features:

From the circulation viewpoint, the effect of the heated thermal discharge entering the flow field can be decomposed into two components: 1) unheated discharge entering the flow field studied in the non-buoyant case problem; and 2) thermal effect of the discharge considered as a heated wall studied earlier under the topic of ponds partially heated from the side. According to the above decomposition, the velocity time history plots in Figs. 8 to 15 for buoyant case should be the sum of the non-buoyant case and the corresponding heated wall problem.

Examining the flow at the water surface at points A and B, in front of the incoming jet, the transversal velocity for the buoyant case shows a small increase as compared with the non-buoyant results due to the natural circulation caused by the heated wall. This additional transversal velocity induces an axial velocity transient similar in nature to the non-buoyant case. However, since the magnitude of the transversal velocity transient is small, the resultant axial velocity transient would also be small. This behavior can be clearly observed in u/U_0 and v/U_0 curves in Figs. 8 and 9 which also show the temperature transients T/T_0 for both points A and B. A similar pattern can also be observed at the surface points E and F in Figs. 12 and 13 as well as the points C, G, and H located at 12.75 feet below the water surface in Figs. 10, 14 and 15.

At the end of the transients, the coldest and the hottest regions are at the river and the thermal discharge entrances respectively.

For this reason, the strongest natural circulation patterns would develop mainly between these coldest and hottest regions. A comparison of the surface velocity vectors at points A and E and the velocity vectors at 12.75 feet depth for buoyant and non-buoyant cases confirm the existence of this natural circulation pattern. The surface velocity field at the end of the transient is shown in Fig. 16. The slowing down of the inlet flow near the thermal discharge entrance and the turning of the thermal discharge and incoming flows as a result of their interaction can be clearly seen in this figure.

The thermal plume effect is shown in Figs. 17 to 22. Isotherms are plotted for the water surface as well as the vertical planes in the longitudinal and transversal direction at times 60 and 500 seconds after the initiation of the heated discharge. An examination of these plots indicate the following features:

1) The stratification pattern is three dimensional in nature and shows the expansion of the thermal plume in all directions. In view of the prevailing advective currents this expansion is much more intense toward the downstream as compared to the other directions (vertical and upstream) as expected.

2) The three distinctive regions which constitute the characteristic behavior of a stratified water body (epilimnion, thermocline and hypolimnion) can be clearly observed in Fig. 22.

3) These results provide the water temperature rise and the rate of temperature rise needed for the assessment of the extent of thermal pollution in the water body.

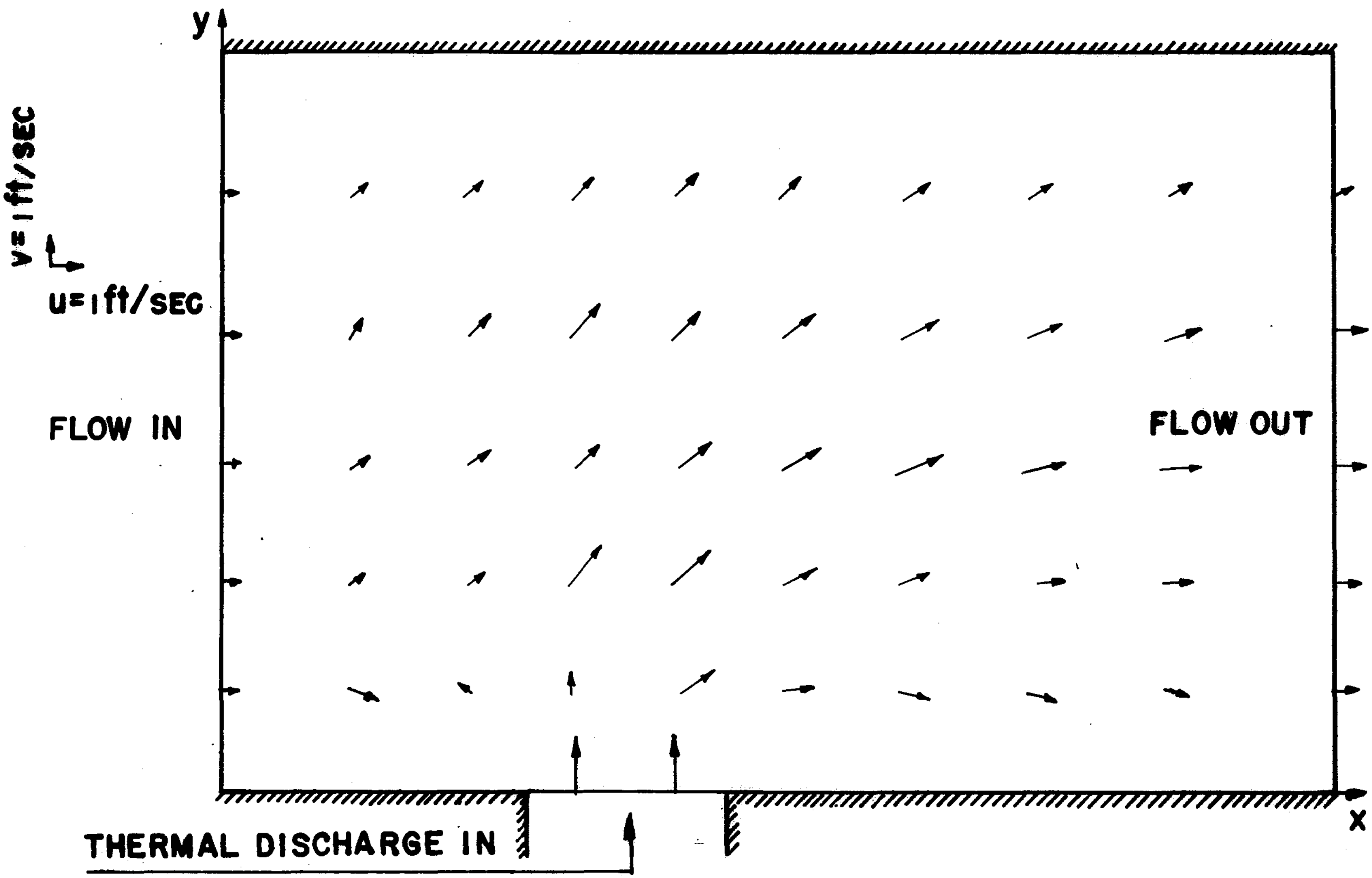


FIG 16. SURFACE VELOCITY FIELD AT t = 500 SEC. IN THE WATER SUBJECTED TO A BUOYANT JET

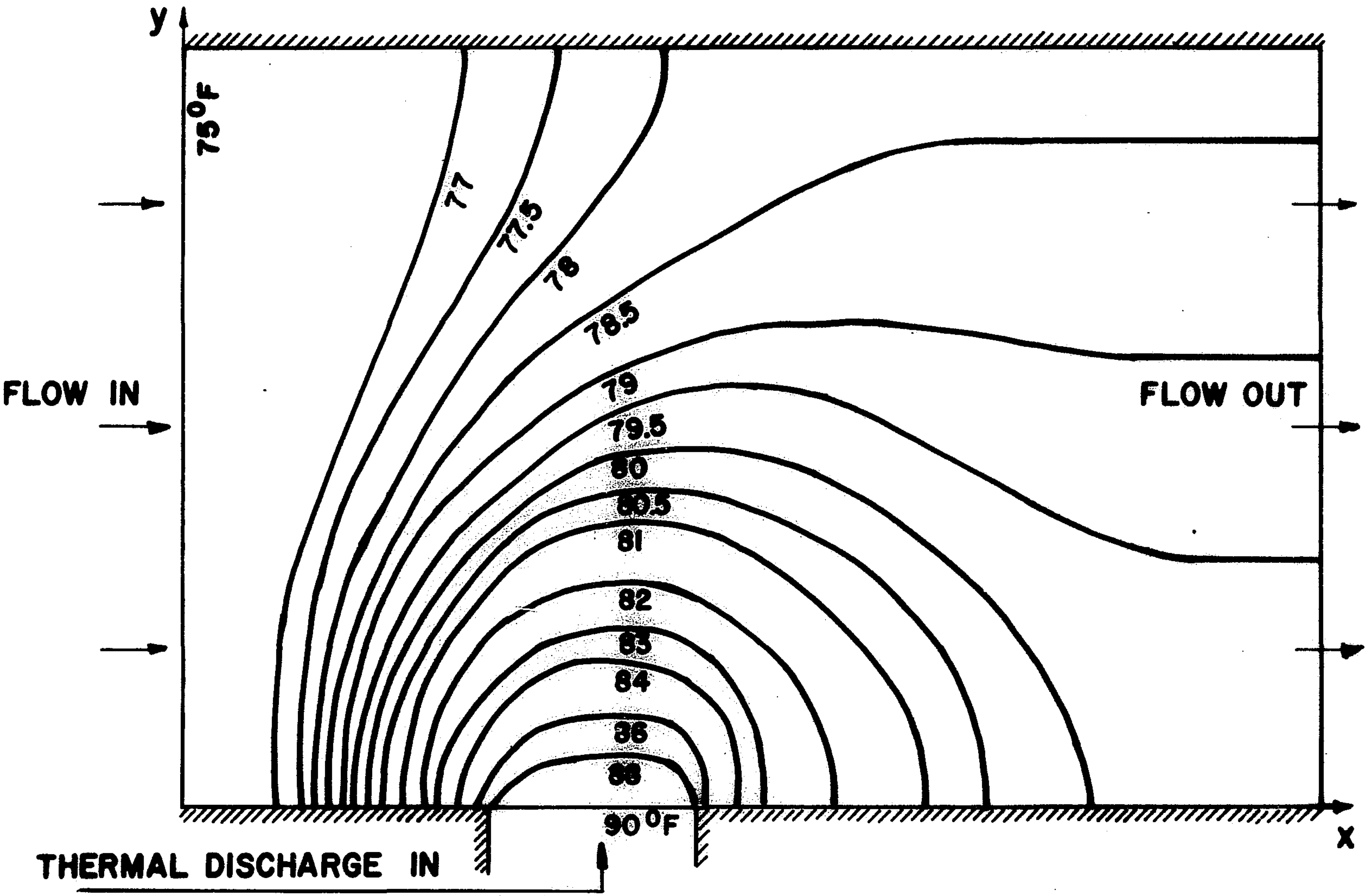


FIG 17. SURFACE ISOTHERMS AT t = 60 SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET

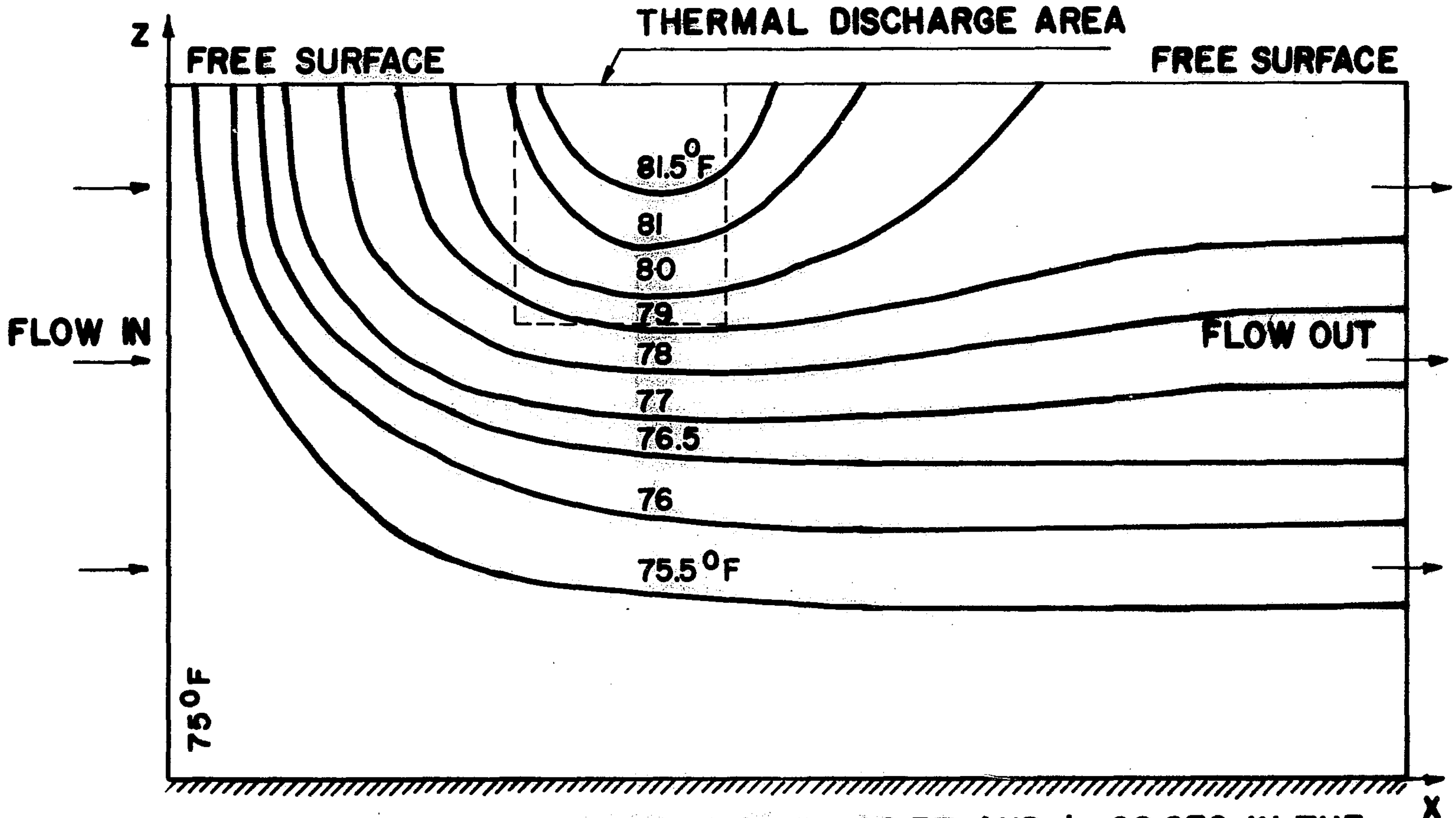


FIG 18. VERTICAL ISOTHERMS AT y = 100 FT. AND t = 60 SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET

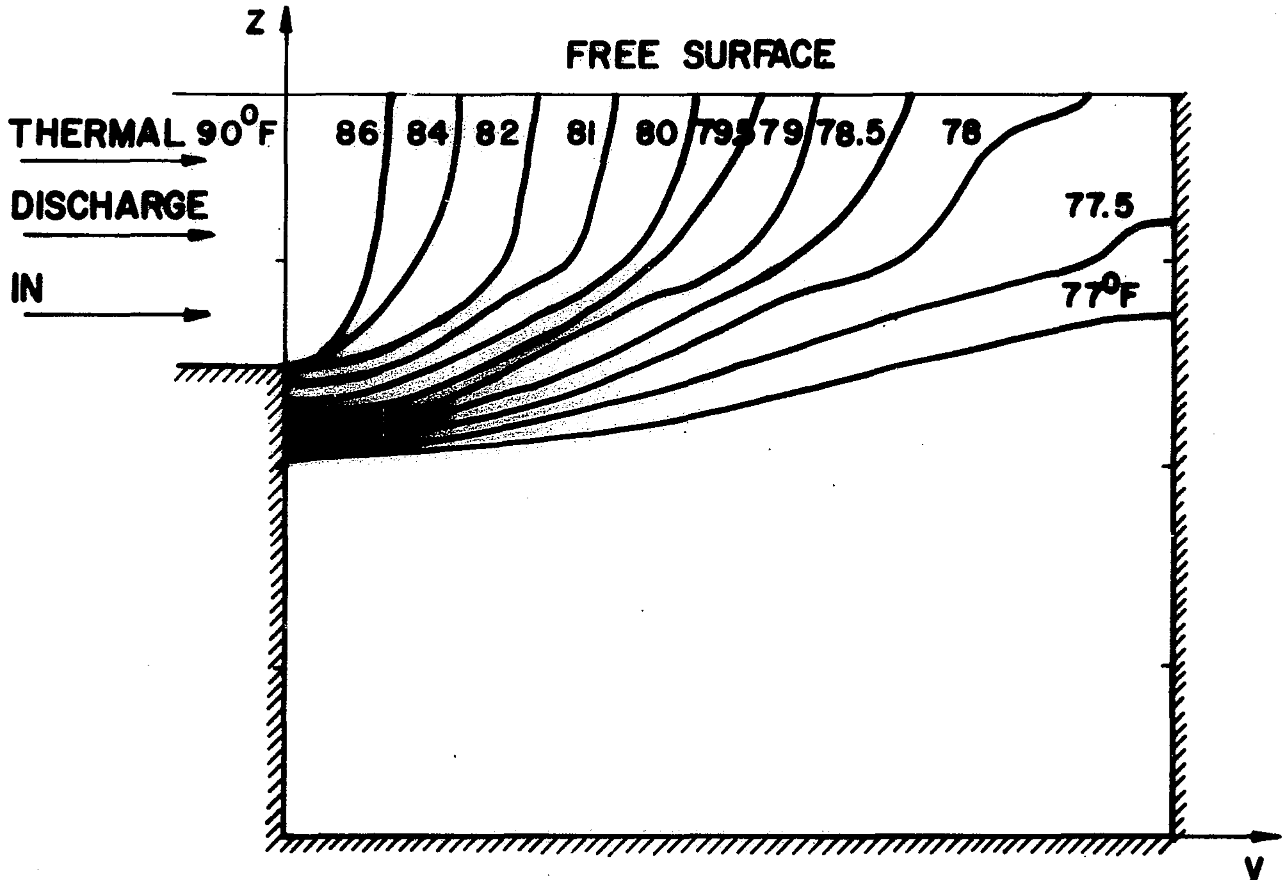


FIG 19. VERTICAL ISOTHERMS AT X = 222.5 FT. AND t = 60 SEC. IN THE WATR BODY SUBJECTED TO A BUOYANT JET

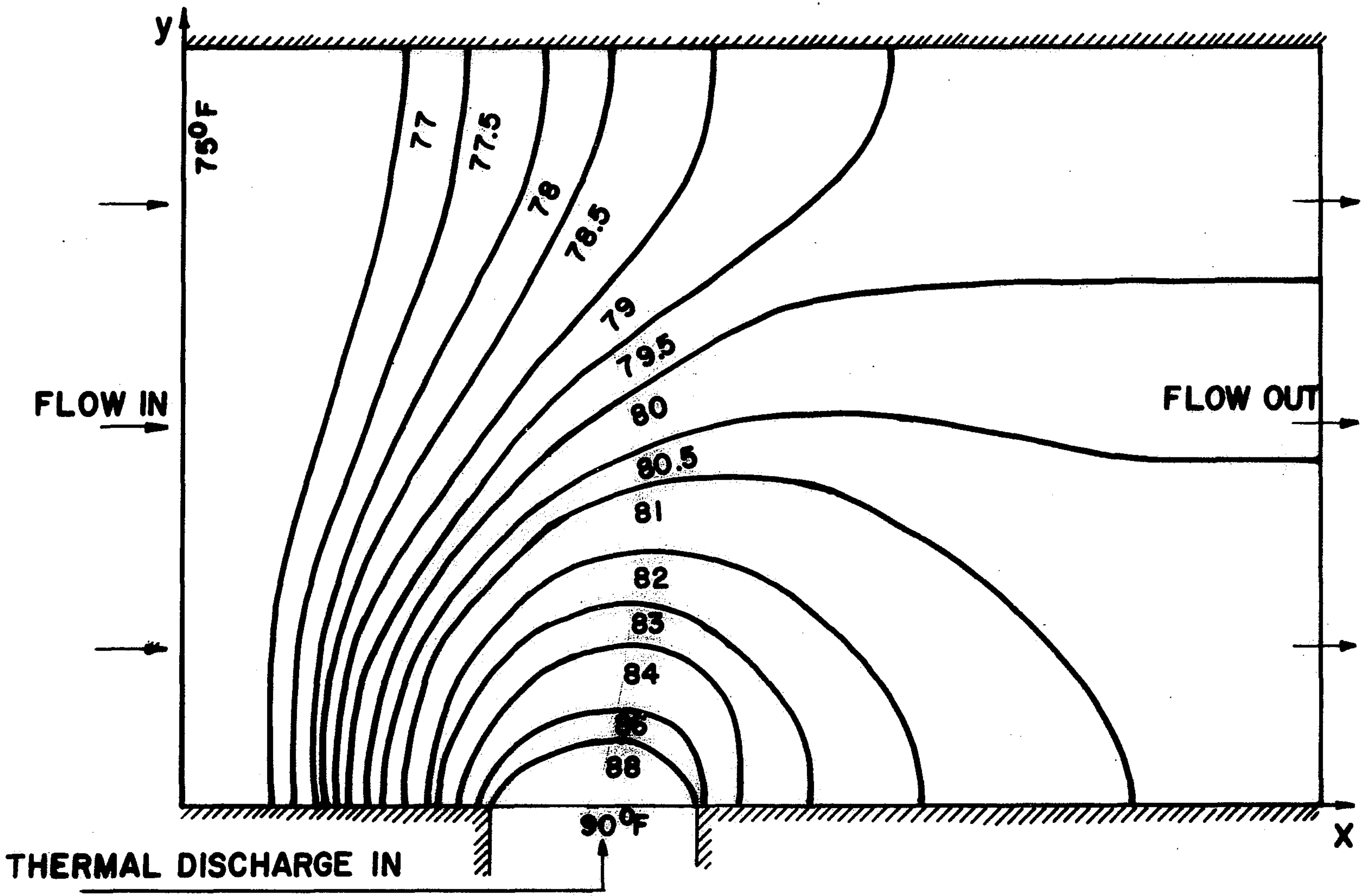


FIG 20. SURFACE ISOTHERMS AT $t = 500$ SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET

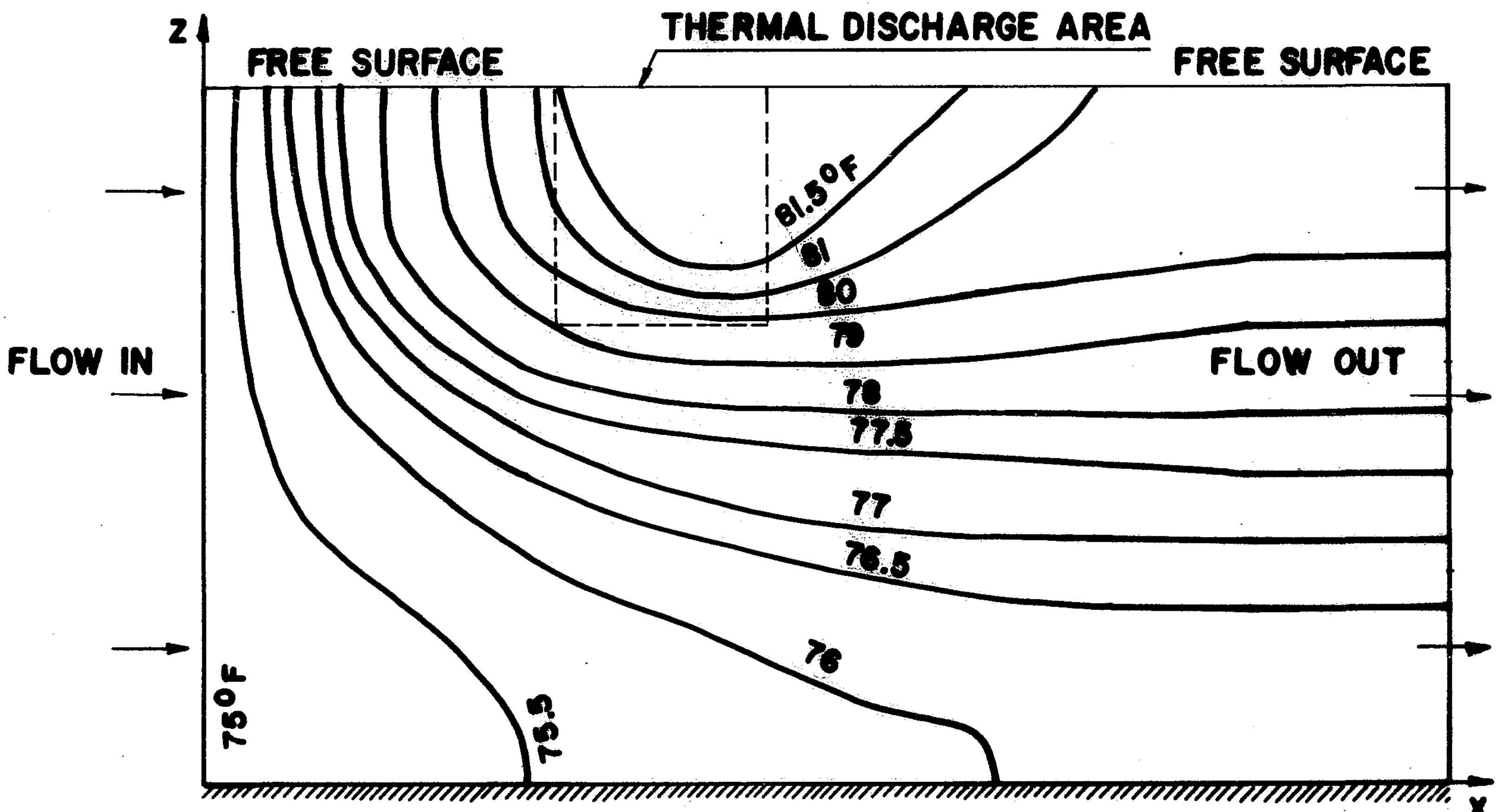


FIG 21. VERTICAL ISOTHERMS AT $y = 100$ FT. AND $t = 500$ SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET

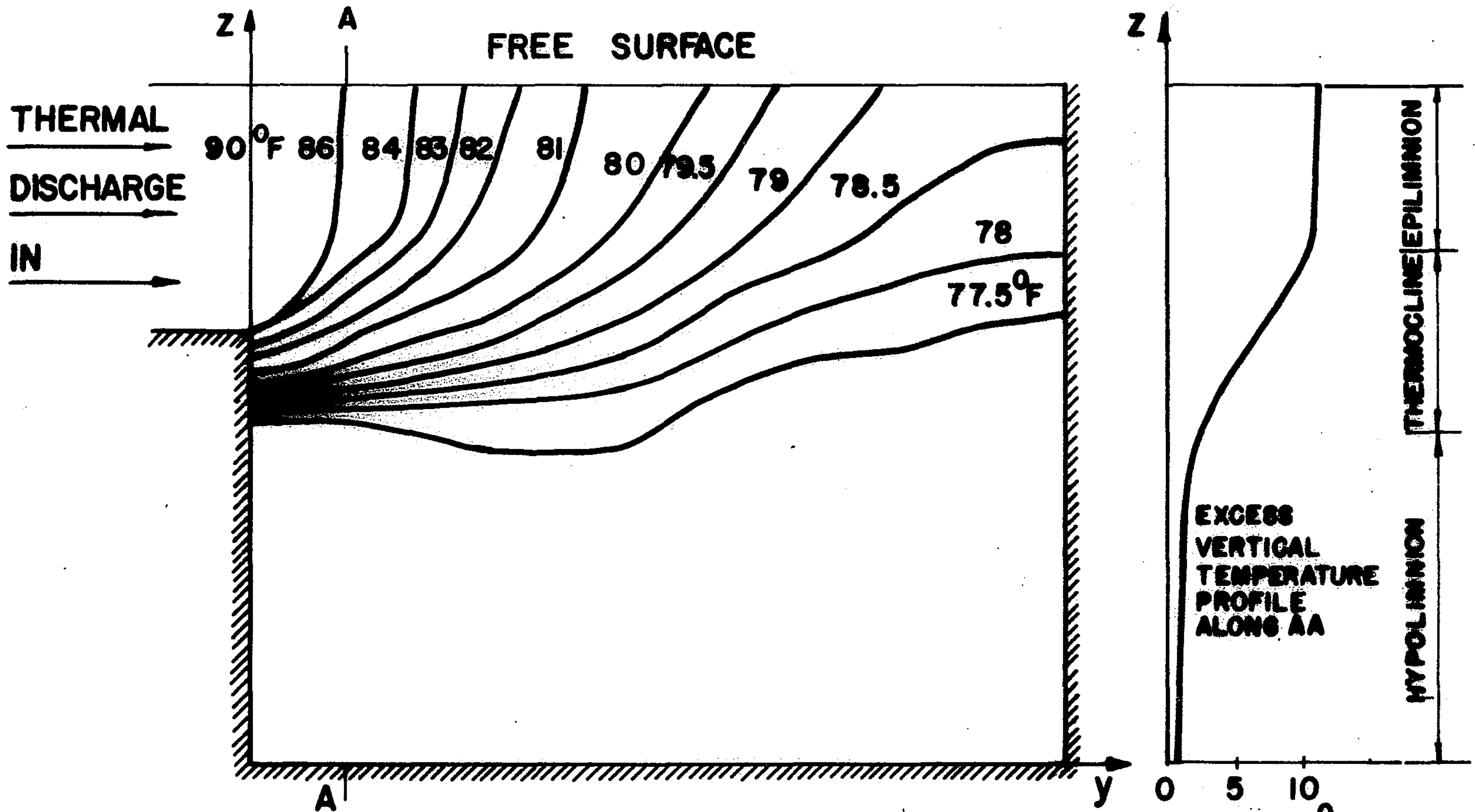


FIG 22. VERTICAL ISOTHERMS AT X=222.5 FT. AND t=500 SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET

As a further verification of the validity of the results, a mass and energy balance was performed on the water body as shown in Table 8. This table shows that conservation equations are satisfied with a good accuracy. The small magnitude of the surface heat transfer verifies the often used assumption that, at high values of equilibrium temperature used herein for the purpose of near field studies, the contribution of heat exchange to the atmosphere is insignificant and may be neglected in simplified analysis.

TABLE 8. MASS AND ENERGY BALANCE

Mass and Energy Balance	Incoming Flow	Thermal Discharge	Outgoing Flow	Surface Flow	% Error
Mass Balance, in 10^6 lbm/sec	0.12252	0.08464	0.20748	0.0	0.15
Energy Balance, in 10^8 Btu/sec	0.09189	0.07454	0.16204	0.15096 $\times 10^{-4}$	0.18

6. CONCLUSIONS AND RECOMMENDATIONS

An improved three-dimensional analytical model for the mathematical description of a large rectangular water body subjected to a thermal discharge has been developed and presented herein. The improvement results from the facts that:

A) Compliance with the conservation of mass and momentum is obtained by the introduction of two flow regions in the entire flow field: a) the water-level region containing a portion of water near the free surface in which the water-level rises or falls, as the case may be, during the dynamic solution; and b) the sub-water-level region located under the water-level region which remains totally filled with fluid at all times during the transient.

B) A different set of differential equations for the conservation of mass, momentum, and energy is applied to the cells located in the water-level and sub-water-level flow regions.

C) The pressure distribution is obtained accurately by combining the momentum and continuity equations and not by the hydrostatic approximation.

D) A detailed numerical stability analysis is performed which provides accurate criteria for the selection of the space and time increments.

The analysis, programmed on a digital computer, is generally capable of predicting velocity, pressure and temperature distribution in water bodies subjected to thermal discharge. Particularly, the present study can be employed to determine the temperature rise and rate of temperature rise needed for the assessment of the extent of thermal pollution in water bodies.

Further studies should be undertaken with the following objectives:

a) To develop a three-dimensional analytical model for the mathematical description of actual rivers with arbitrary bottom and side configurations.

b) To develop accurate correlations for momentum and thermal eddy diffusivities and to incorporate these correlations in the digital program.

c) To provide an experimental verification for the thermal discharge problem.

7. NOMENCLATURE

a — Terms of marix involved in the solution of the pressure equations

$a_x, a_y, a_z, b_x, b_y, b_z, c_x, c_y, c_z$ — Weighting factors for derivatives

C_p — Specific heat of water

C_{fx} — Skin coefficient along x-axis

C_{fy} — Skin coefficient along y-axis

d_0 — Half widht of thermal discharge

d_x, d_y, d_z — Thermal eddy diffusivities

D_x, D_y, D_z — Sum of thermal conductivity plus thermal eddy diffusivity as defined by equations (A-19)

D_h — Horizontal eddy diffusivity of heat

D_v — Vertical eddy diffusivity of heat

$e_x, e_y, e_z, f_x, f_y, f_z$ — Weighting factors for pressure

E — Equilibrium temperature of water

F_0 — Froude number

g — Gravitational acceleration

H — Water-level height

K — Heat exchange coefficient for water-air interface

n — Normal to the solid boundary

p — Pressure

R_0 — Reynolds number

S_0 — Stanton number

T_0 — Thermal discharge temperature

T — Local water temperature

t — Time

NOMENCLATURE (Cont'd)

U_0	Thermal discharge velocity
u	Horizontal velocity components in x-direction
v	Horizontal velocity components in y-direction
w	Vertical velocity components in z-direction
W_x	Wind velocity components in x-direction
W_y	Wind velocity components in y-direction
x,y,z	Cartesian coordinate system
Δx_i	Dimension of element i in x-direction
Δy_j	Dimension of element j in y-direction
Δz_k	Dimension of element k in z-direction

Greek Symbols

$\alpha_x, \alpha_y, \alpha_z$	Momentum eddy diffusivities
$\alpha, \beta, \gamma, \delta$	Coefficients in Von-Neumann stability analysis
Δ	A quantity defined by equation (99)
ξ	A quantity defined by equation (66b)
λ^2	Norm ratio
ν	Viscosity
ν_0	Kinematic viscosity of thermal discharge
ν_x', ν_y', ν_z'	Sum of viscosity and momentum eddy diffusivity as defined by equation (A-19)
ν_h	Horizontal eddy viscosity
ν_v	Vertical eddy viscosity
ρ	Local density of water

NOMENCLATURE (Cont'd)

Greek Symbols (Cont'd)

ρ_a Air density

ρ_0 Thermal discharge density

ω Relaxation factor

ω_{opt} Optimum value of relaxation factor

∇^2 Laplacian operator $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Superscripts

* Refers to non-dimensional quantities

n Refers to updated value

. Refers to a small variation

\- Refers to time average

' Refers to fluctuating components used in Appendix 1 and to steady-state components used in Appendix 2

Subscripts

0 Refers to thermal discharge

i,j,k Refers to element numbers

max Refers to maximum

s Refers to surface

rm Refers to components of matrix "a"

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APPENDIX 1. Derivation of Governing Equation (1) through (15)

To derive the governing equations for the sub-water-level region, the Navier-Stokes equations for incompressible flow (31) are used in conjunction with the Boussinesq's approximation which neglects the fluid density variation in all terms involved in the momentum equations except in the body force term as follows:

Momentum Equations

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\,\frac{\partial p}{\partial x} + \nu\nabla^2 u \tag{A-1}$$

$$\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} + w\,\frac{\partial v}{\partial z} = -\frac{1}{\rho_0}\,\frac{\partial p}{\partial y} + \nu\nabla^2 v \tag{A-2}$$

$$\frac{\partial w}{\partial t} + u\,\frac{\partial w}{\partial x} + v\,\frac{\partial w}{\partial y} + w\,\frac{\partial w}{\partial z} = -\frac{1}{\rho_0}\,\frac{\partial p}{\partial z} + \nu\nabla^2 w - \frac{\rho g}{\rho_0} \tag{A-3}$$

Energy Equation

$$\frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{y} + w\,\frac{\partial T}{\partial z} = k\nabla^2 T \tag{A-4}$$

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{A-5}$$

In describing a turbulent flow in mathematical terms, it is convenient to separate the flow into a mean motion and a fluctuating, or eddying motion. Denoting the time-average of the u-component of velocity by $\bar{u}$ and its velocity fluctuation by u' and using a similar notation for the other variables, one can express the following relations for the velocity components, temperature and pressure:

$$u=\bar{u} + u'; \; v=\bar{v} + v'; \; w=\bar{w} + w'; \; T=\bar{T} + T'; \; p=\bar{p} + p' \qquad \text{(A-6)}$$

By definition, the time-average of all fluncuating quantities is equal to zero. For example,

$$\bar{u}' = 0, \; \bar{v}' = 0, \; \bar{w}' = 0, \; \bar{T}' = 0, \; \bar{p}' = 0; \; \frac{\partial \bar{u}'}{\partial x} = 0, \qquad \text{(A-7)}$$

Upon forming the time-average of equations (A-1) through (A-5), one obtains

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} + \nu\nabla^2\bar{u} - \left(\frac{\partial \bar{u}'^2}{\partial x} + \frac{\partial \bar{u}'\bar{v}'}{\partial y} + \frac{\partial \bar{u}'\bar{w}'}{\partial z}\right) \qquad \text{(A-8)}$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial y} + \nu\nabla^2\bar{v} - \left(\frac{\partial \bar{u}'\bar{v}'}{\partial x} + \frac{\partial \bar{v}'^2}{\partial y} + \frac{\partial \bar{v}'\bar{w}'}{\partial z}\right) \qquad \text{(A-9)}$$

$$\frac{\partial \bar{w}}{\partial t} + \bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial z} + \nu\nabla^2\bar{w} - \left(\frac{\partial \bar{u}'\bar{w}'}{\partial x} + \frac{\partial \bar{v}'\bar{w}'}{\partial y} + \frac{\partial \bar{w}'^2}{\partial z}\right) - \frac{\rho g}{\rho_0} \qquad \text{(A-10)}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} + \bar{w}\frac{\partial \bar{T}}{\partial z} = k\nabla^2\bar{T} - \left(\frac{\partial \bar{u}'\bar{T}'}{\partial x} + \frac{\partial \bar{v}'\bar{T}'}{\partial y} + \frac{\partial \bar{w}'\bar{T}'}{\partial z}\right) \qquad \text{(A-11)}$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad \text{(A-12)}$$

Employing the eddy diffusivity concept,

$$-\bar{u}'^2 = \alpha_x \frac{\partial\bar{u}}{\partial x} + \alpha_x \frac{\partial\bar{u}}{\partial x}$$

$$-\bar{u}'\bar{v}' = \alpha_y \frac{\partial\bar{u}}{\partial y} + \alpha_x \frac{\partial\bar{v}}{\partial x}$$

$$-\bar{u}'\bar{w}' = \alpha_z \frac{\partial\bar{u}}{\partial z} + \alpha_x \frac{\partial\bar{w}}{\partial x}$$

$$-\bar{v}'^2 = \alpha_y \frac{\partial\bar{v}}{\partial y} + \alpha_y \frac{\partial\bar{v}}{\partial y}$$

$$-\bar{v}'\bar{u}' = \alpha_x \frac{\partial\bar{v}}{\partial x} + \alpha_y \frac{\partial\bar{u}}{\partial y}$$

$$-\bar{v}'\bar{w}' = \alpha_z \frac{\partial\bar{v}}{\partial z} + \alpha_y \frac{\partial\bar{w}}{\partial y}$$

$$-\bar{w}'\bar{u}' = \alpha_x \frac{\partial\bar{w}}{\partial x} + \alpha_z \frac{\partial\bar{u}}{\partial z}$$

$$-\bar{w}'\bar{v}' = \alpha_y \frac{\partial\bar{w}}{\partial y} + \alpha_z \frac{\partial\bar{v}}{\partial z}$$

$$-\bar{w}'^2 = \alpha_z \frac{\partial\bar{w}}{\partial z} + \alpha_z \frac{\partial\bar{w}}{\partial z}$$

$$-\bar{u}'\bar{T}' = d_x \frac{\partial\bar{T}}{\partial x}$$

$$-\bar{v}'\bar{T}' = d_y \frac{\partial\bar{T}}{\partial y}$$

$$-\bar{w}'\bar{T}' = d_z \frac{\partial\bar{T}}{\partial z} \qquad \text{(A-13)}$$

after substituting equations (A-13) into equations (A-8) through (A-11), one obtains the following equations for incompressible turbulent flow:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} + \nu_x\frac{\partial^2 \bar{u}}{\partial x^2} + \nu_y\frac{\partial^2 \bar{u}}{\partial y^2} + \nu_z\frac{\partial^2 \bar{u}}{\partial z^2} \qquad \text{(A-14)}$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial y} + \nu_x\frac{\partial^2 \bar{v}}{\partial x^2} + \nu_y\frac{\partial^2 \bar{v}}{\partial y^2} + \nu_z\frac{\partial^2 \bar{v}}{\partial z^2} \qquad \text{(A-15)}$$

$$\frac{\partial \bar{w}}{\partial t} + \bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial z} + \nu_x\frac{\partial^2 \bar{w}}{\partial x^2} + \nu_y\frac{\partial^2 \bar{w}}{\partial y^2} + \nu_z\frac{\partial^2 \bar{w}}{\partial z^2} - \frac{\rho g}{\rho_0} \qquad \text{(A-16)}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} + \bar{w}\frac{\partial \bar{T}}{\partial z} = D_x\frac{\partial^2 \bar{T}}{\partial x^2} + D_y\frac{\partial^2 \bar{T}}{\partial y^2} + D_z\frac{\partial^2 \bar{T}}{\partial z^2} \qquad \text{(A-17)}$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad \text{(A-18)}$$

where

$$\nu_x = \nu + \alpha_x$$

$$\nu_y = \nu + \alpha_y$$

$$\nu_z = \nu + \alpha_z$$

$$D_x = k + \alpha_x$$

$$D_y = k + \alpha_y$$

$$D_z = k + \alpha_z \qquad \text{(A-19)}$$

In the present analysis, the horizontal components of diffusivities are considered to be equal, i.e.,

$$\nu_h = \nu_x = \nu_y$$

$$\nu_v = \nu_z$$

$$D_h = D_x = D_y$$

$$D_v = D_z \tag{A-20}$$

The magnitude of momentum eddy viscosities and thermal eddy diffusivities (α_x, α_y, α_z, d_x, d_y, d_z) are usually so large in comparison with their molecular counterparts (ν, k) that the contributions of the latter are neglected. For practical calculations, it is satisfactory to assume that the horizontal and vertical momentum eddy viscosities are equal to the horizontal and vertical thermal eddy diffusivities respectively such that (14, 5, 13)

$$\nu_h = D_h$$

$$\nu_v = D_v \tag{A-21}$$

Liggett (19), Wada (39), and Lerman (17) are among the recent investigators of eddy diffusivity concept. Based on their studies, the following average values for the eddy viscosities in the horizontal and vertical directions, ν_h and ν_v, are used in this analysis

$$\nu_h = 1350 \ ft^2/sec$$

$$\nu_v = .4 \ ft^2/sec$$

Considering the above simplifications, equations (A-14) through (A-18) become identical with equations (1) through (5) except that, for convenience, the superscript (-) has been dropped in the latter equations.

APPENDIX 2. Verification of Stability Analysis

The purpose of this appendix is to demonstrate the validity of the rule which applies to an explicit computational scheme as discussed in Section 4. This rule is verified by developing the stability criteria for the energy equation (5) using the two following methods:

a) Setting the coefficients multiplying each of the temperature values equal to a positive quantity (3)

b) Von-Neumann stability analysis (1)

In order to apply stability analysis to a nonlinear equation, the equation must be first linearized. To linearize the equation, one assumes that the temperature T consists of a steady-state component T' and a small variation about steady-state $\dot{T}$ such that

$$T = T' + \dot{T}$$

By substituting these quantities in the energy equation, and considering the velocity components constant over the integration time step, one will obtain the following linearized equation:

$$\frac{\partial \dot{T}}{\partial t} = - u \frac{\partial \dot{T}}{\partial x} - v \frac{\partial \dot{T}}{\partial y} - w \frac{\partial \dot{T}}{\partial z} + D_h \left(\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial y^2}\right) + D_v \frac{\partial^2 \dot{T}}{\partial z^2} \qquad \text{(B-1)}$$

where superscript dot (·) indicates linearized quantities. By applying a forward two points finite difference approximation to the temperature time derivative and centered two and three points finite difference approximations to the first and second spatial differences respectively, and denoting $\dot{T}_{i,j,k}$ by $\dot{T}$ for convenience, one obtains

$$\frac{\dot{T}^{n} - \dot{T}}{\Delta t} = -u \frac{\dot{T}_{i+1} - \dot{T}_{i-1}}{2\Delta x} - v \frac{\dot{T}_{j+1} - \dot{T}_{j-1}}{2\Delta y} - w \frac{\dot{T}_{k+1} - \dot{T}_{k-1}}{2\Delta z} + \frac{\dot{T}_{i+1} - 2\dot{T} + \dot{T}_{i-1}}{\Delta x^2}$$

$$+ D_h \frac{\dot{T}_{j+1} - 2\dot{T} + \dot{T}_{j-1}}{\Delta x^2} + D_v \frac{\dot{T}_{k+1} - 2\dot{T} + \dot{T}_{k-1}}{\Delta x^2} \qquad \text{(B-2)}$$

solving for $\dot{T}^{n}$

$$\dot{T}^{n} = \dot{T}_{i-1} \left(\frac{u\Delta t}{2\Delta x} + \frac{D_h \Delta t}{\Delta x^2}\right) + \dot{T} \left(1 - \frac{2\Delta t D_h}{\Delta x^2} - \frac{2\Delta t D_h}{\Delta y^2} - \frac{2\Delta t D_v}{\Delta z^2}\right.$$

$$+ \dot{T}_{i+1} \left(- \frac{u\Delta t}{2\Delta x} + \frac{D_h \Delta t}{\Delta x^2}\right) + \dot{T}_{j-1} \left(\frac{v\Delta t}{2\Delta y} + \frac{D_h \Delta t}{\Delta y^2}\right)$$

$$+ \dot{T}_{j+1} \left(- \frac{v\Delta t}{2\Delta y} + \frac{D_h \Delta t}{\Delta y^2}\right) + \dot{T}_{k-1} \left(\frac{w\Delta t}{2\Delta z} + \frac{D_v \Delta t}{\Delta z^2}\right) + \dot{T}_{k+1} \left(- \frac{w\Delta t}{2\Delta z} + \frac{D_v \Delta t}{\Delta z^2}\right)$$

(B-3)

a) Employing the rule of positive coefficients, which applies to explicit computational scheme discussed previously, the following stability criteria will result from coefficient of $\dot{T}_{i+1}$, $\dot{T}_{i-1}$

$$\Delta x \leqslant \frac{2D_h}{|u|} \qquad \text{(B-4)}$$

similarly, from $\dot{T}_{j+1}$, $\dot{T}_{j-1}$, $\dot{T}_{k+1}$, $\dot{T}_{k-1}$ coefficients, one obtains

$$\Delta y \leqslant \frac{2D_h}{|v|} \qquad \Delta z \leqslant \frac{2D_v}{|w|} \qquad \text{(B-5)}$$

Finally, from the coefficient of $\dot{T}$, the following temporal condition results

$$\Delta t \leqslant 1/(2D_h/\Delta x^2 + 2D_h/\Delta y^2 + 2D_v/\Delta z^2) \qquad \text{(B-6)}$$

If $D_h = D_v$ and $\Delta x = \Delta y = \Delta z$, the expression for Δt simplifies to

$$\Delta t \leqslant \frac{\Delta x^2}{6D_h} \qquad \text{(B-7)}$$

b) Von-Neumann stability analysis.

According to the Von-Neumann stability analysis approach (1)

$$T^{\,n} = e^{\delta t} e^{i\alpha x} e^{i\beta y} e^{i\gamma z} \qquad \text{(B-8)}$$

where

$$x = \sum_{\ell=1}^{\ell} \Delta x_\ell$$

$$y = \sum_{m=1}^{m} \Delta y_m$$

$$z = \sum_{p=1}^{p} \Delta z_p$$

$$t = n\Delta t \qquad \text{(B-9)}$$

Substituting the above quantities into equation (B-3), and dividing by $e^{\delta t} e^{i\alpha x} e^{i\beta y} e^{i\gamma z}$, the following results:

$$e^{\delta\Delta t} = \left(\frac{u\Delta t}{2\Delta x} + \frac{D_h\Delta t}{\Delta x^2}\right) \bar{e}^{i\alpha\Delta t} + \left(-\frac{u\Delta t}{2\Delta x} + \frac{D_h\Delta t}{\Delta x^2}\right) e^{i\alpha\Delta x}$$

$$+ \left(\frac{v\Delta t}{2\Delta y} + \frac{D_h\Delta t}{\Delta y^2}\right) \bar{e}^{i\beta\Delta y} + \left(- \frac{v\Delta t}{2\Delta y} + \frac{D_h\Delta t}{\Delta y^2}\right) e^{i\beta\Delta y}$$

$$+ \left(\frac{w\Delta t}{2\Delta z} + \frac{D_v\Delta t}{\Delta z^2}\right) \bar{e}^{i\gamma\Delta z} + \left(- \frac{w\Delta t}{2\Delta z} + \frac{D_v\Delta t}{\Delta z^2}\right) e^{i\gamma\Delta z}$$

$$+ \left(1 - \frac{2\Delta tD_h}{\Delta x^2} - \frac{2\Delta tD_h}{\Delta y^2} - \frac{2\Delta tD_v}{\Delta z^2}\right) \qquad \text{(B-10)}$$

For the above equation to be numerically stable, the absolute value of $e^{\delta\Delta t}$ must be set less than or equal to one. This leads to

$$\left(1-4 \frac{\Delta tD_h}{\Delta x^2} \sin^2 \frac{\alpha\Delta x}{2} - 4 \frac{\Delta tD_h}{\Delta y^2} \sin^2 \frac{\beta\Delta y}{2} - 4 \frac{\Delta tD_v}{\Delta z} \sin^2 \frac{\alpha\Delta z}{2}\right)^2 + \qquad \text{(B-11)}$$

$$\left(\frac{u\Delta t}{\Delta x} \sin \alpha\Delta x + \frac{v\Delta t}{\Delta y} \sin \alpha\Delta y + \frac{w\Delta t}{\Delta z} \sin \alpha\Delta z \right)^2 \leq 1 \qquad \text{(B-12)}$$

Expanding and assuming that $\Delta x = \Delta y = \Delta z$, and $D_h = D_v$, the following will be obtained:

$$1 + \left(144 \frac{\Delta t^2 D_h^2}{\Delta x^4} - 36 \frac{u^2\Delta t^2}{\Delta x^2}\right) \sin^4 \frac{\alpha\Delta x}{2} + \left(36 \frac{u^2\Delta t^2}{\Delta x^2} - 24 \frac{\Delta tD_h}{\Delta x^2}\right) \sin^2 \frac{\alpha\Delta x}{2} \leq 1 \qquad \text{(B-13)}$$

In order for the above inequality to be less or equal one, the following inequalities must hold.

$$144 \frac{\Delta t^2 D_h^2}{\Delta x^4} - 36 \frac{u^2\Delta t^2}{\Delta x^2} \leq \varepsilon^2 \qquad \text{(B-14)}$$

$$36\,\frac{u^2\Delta t^2}{\Delta x^2} - 24\,\frac{\Delta t D_h}{\Delta x^2} = -\,\varepsilon^2 \tag{B-15}$$

where ε^2 is very small number. From equation (B-14), the spatial stability criterion may be written as

$$144\,\frac{\Delta t^2 D_h^2}{\Delta x^4} \geqslant 36\,\frac{u^2\Delta t^2}{\Delta x^2}$$

or

$$\Delta x \leqslant \frac{2D_h}{|u|} \tag{B-16}$$

Adding equations (B-14) and (B-15), the upper limit for Δt becomes:

$$\Delta t \leqslant \frac{\Delta x^2}{6D_h} \tag{B-17}$$

These results are in agreement with those obtained in part (a) of this appendix.

APPENDIX 3. Calculation of the First and Second Derivatives at the Boundary and in the Field[1]

The calculation of the first derivative in the flow field is based on a quadratic approximation. Let $q(x)$ be defined by a parabolic equation

$$q(x) = ax^2 + bx + c \qquad \text{(C-1)}$$

shown in Fig. 23. Values of the function at nodal points x_1, x_2, and x_3, are q_1, q_2, and q_3, respectively. Therefore

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \qquad \text{(C-2)}$$

combining equations (C-1) and (C-2) yields

$$q(x) = \{x^2 \quad x \quad 1\} \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \qquad \text{(C-3)}$$

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[1]Nomenclature used in this Appendix is defined within the text and not in Section 7

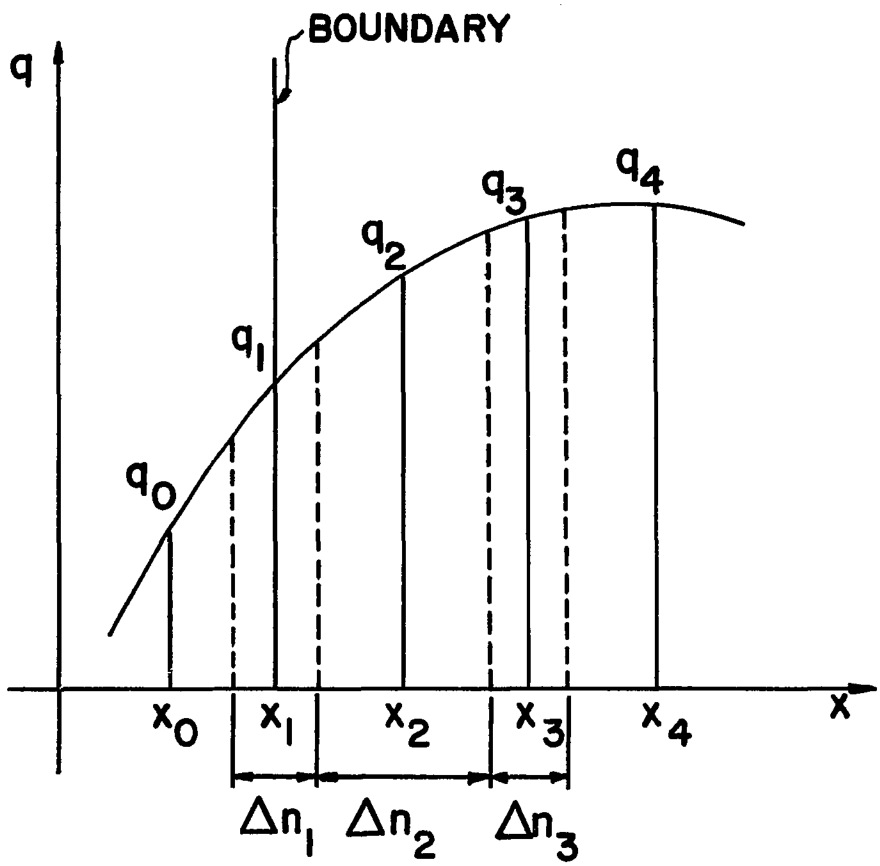


FIG. 23 CALCULATION OF FIRST AND SECOND DERIVATIVES

First Derivative at x_1

The derivative at x_1 is given by

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = \{2x_1 \quad 1 \quad 0\} \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{C-4}$$

Inverting the matrix, yields

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = \frac{\{2x_1 \quad 1 \quad 0\}}{\det \begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix}} \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_3^2 - x_2^2 & x_1^2 - x_3^2 & x_2^2 - x_1^2 \\ x_3x_2(x_2-x_3) & x_3x_1(x_3-x_1) & x_1x_2(x_1-x_2) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{C-5}$$

After multiplication, one obtains

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = \frac{(x_3-x_2)\,(x_2+x_3-2x_1)\,q_1 - (x_3-x_1)^2\,q_2 + (x_1-x_2)^2\,q_3}{(x_2-x_1)\,(x_3-x_2)\,(x_1-x_3)} \tag{C-6}$$

In many instances the value of the first derivative is known at the boundary, i.e.,

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = C \tag{C-7}$$

For this boundary condition, a relationship between q_1, q_2, and q_3 may be obtained by first defining the element boundaries (dotted lines on Fig. 23) and observing that each nodal point is located at the midpoint between its boundary lines. Thus,

$$x_2-x_1 = \frac{1}{2}(\Delta n_1 + \Delta n_2) \tag{C-8}$$

$$x_3-x_2 = \frac{1}{2}(\Delta n_2 + \Delta n_3) \tag{C-9}$$

$$x_1-x_3 = -\frac{1}{2}(\Delta n_1 + 2\Delta n_2 + \Delta n_3) \tag{C-10}$$

Defining

$$\Delta n_{12} = \Delta n_1 + \Delta n_2 \tag{C-11}$$

and

$$\sigma = \frac{\Delta n_1 + \Delta n_2}{\Delta n_2 + \Delta n_3} \tag{C-12}$$

Combining the equations (C-8) through (C-12) gives the value of the derivative at the boundary

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = - \frac{2}{\Delta n_{12}(1+\sigma)} \left[(1 + 2\sigma)\, q_1 - (1+\sigma)^2\, q_2 + \sigma^2\, q_3 \right] \tag{C-13}$$

or

$$\left.\frac{dq(x)}{dx}\right|_{x=x_1} = \frac{q_2 - q_1}{x_2 - x_1} \left(\frac{1 + 2\sigma}{1 + \sigma}\right) + \frac{q_3 - q_2}{x_3 - x_2} \left(\frac{-\sigma}{1+\sigma}\right) \tag{C-14}$$

which means that the slope at $x=x_1$ is equal to the weighted average of the slopes in intervals $x_2 - x_1$ and $x_3 - x_2$. It should be noted that for equally spaced increments, the slope in the interval $x_2 - x_1$ is weighted by 1.5, while the slope in the interval $x_3 - x_2$ is weighted by - 0.5.

Using equations (C-7) and (C-14), yields a relationship between q_1, q_2, and q_3

$$- \frac{2}{\Delta n_{12}(1+\sigma)} \left[(1+2\sigma)\, q_1 - (1+\sigma)^2\, q_2 + \sigma^2\, q_3 \right] = C \tag{C-15}$$

from which the value of the variable at the boundary can be determined in terms of the values within the flow region.

$$q_1 = \frac{1}{1+2\sigma} \left[(1+\sigma)^2\, q_2 - \sigma^2\, q_3 - \frac{1}{2}\, \Delta n_{12}\, (1+\sigma)\, C \right] \tag{C-16}$$

<u>First Derivative at x_2</u>

The value of the first derivative in the field is calculated by differentiating equation (C-3) and setting $x=x_2$

$$\left.\frac{dq(x)}{dx}\right|_{x=x_2} = \{2x_2 \quad 1 \quad 0\} \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{C-17}$$

Inverting the matrix yields,

$$\left.\frac{dq(x)}{dx}\right|_{x=x_2} = \frac{\{2x_2 \quad 1 \quad 0\}}{\det \begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix}} \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_3^2 - x_2^2 & x_1^2 - x_3^2 & x_2^2 - x_1^2 \\ x_3x_2(x_2-x_3) & x_3x_1(x_3-x_1) & x_1x_2(x_1-x_2) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{C-18}$$

After multiplication, one obtains

$$\left.\frac{dq(x)}{dx}\right|_{x=x_2} = \frac{(x_3-x_2)^2\, q_1 + (x_3-x_1)(2x_2-x_3-x_1)\, q_2 - (x_2-x_1)^2\, q_3}{(x_2-x_1)\,(x_3-x_2)\,(x_1-x_3)} \tag{C-19}$$

Using relationships (C-8), (C-9), and (C-10), the above equation becomes

$$\frac{dq(x)}{dx}\bigg|_{x=x_2} = -\frac{2}{\Delta n_{12}(1+\sigma)}\left[\; q_1+(\sigma^2-1)\; q_2-\sigma^2\; q_3 \;\right] \qquad \text{(C-20)}$$

or

$$\frac{dq(x)}{dx}\bigg|_{x=x_2} = \frac{q_2-q_1}{x_2-x_1}\left(\frac{1}{1+\sigma}\right) + \frac{q_3-q_2}{x_3-x_2}\left(\frac{\sigma}{1+\sigma}\right) \qquad \text{(C-21)}$$

which means that the slope at $x=x_2$ is equal to the weighted average of the slopes in intervals x_2-x_1 and x_3-x_2. It should be noted that for equally spaced increments, the slope at x_2 becomes equal to the mean value of the slopes in intervals x_2-x_1 and x_3-x_2. Furthermore, using equations (C-12), (C-10), (C-9), and (C-8), it can easily be shown that the weighting function can be expressed in terms of spatial differences as follows:

$$\frac{1}{1+\sigma} = \frac{x_3-x_2}{x_3-x_1} \qquad \text{(C-22)}$$

$$\frac{\sigma}{1+\sigma} = \frac{x_2-x_1}{x_3-x_1} \qquad \text{(C-23)}$$

<u>First Derivative at x_3</u>

The derivative at x_3 is given by

$$\left.\frac{dq(x)}{dx}\right|_{x=x_3} = \frac{\{2x_3 \quad 1 \quad 0\}}{\det \begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_1 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix}} \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_3^2 - x_2^2 & x_1^2 - x_3^2 & x_2^2 - x_1^2 \\ x_3x_2(x_2-x_3) & x_3x_1(x_3-x_1) & x_1x_2(x_1-x_2) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{C-24}$$

After multiplication, one obtains

$$\left.\frac{dq(x)}{dx}\right|_{x=x3} = \frac{-(x_3-x_2)^2\, q_1 + (x_3-x_1)^2\, q_2 + (x_1-x_2)\,(x_3-x_2+x_3-x_1)\, q_3}{(x_2-x_3)\,(x_1-x_3)\,(x_1-x_2)} \tag{C-25}$$

Using relationships (C-8), (C-9), and (C-10) and redefining

$$\Delta n_{23} = \Delta n_2 + \Delta n_3 \tag{C-26}$$

and

$$\sigma' = \frac{\Delta n_3 + \Delta n_2}{\Delta n_1 + \Delta n_2} = \frac{1}{\sigma} \tag{C-27}$$

equation (C-25) becomes

$$\left.\frac{dq(x)}{dx}\right|_{x=x_3} = \frac{2}{\Delta n_{23}(1+\sigma')} \left[(1+2\sigma')\, q_3 - (1+\sigma')^2\, q_2 + \sigma'^2\, q_1 \right] \tag{C-28}$$

or

$$\left.\frac{dq(x)}{dx}\right|_{x=x_3} = \frac{q_2-q_1}{x_2-x_1}\left(\frac{-\sigma'}{1+\sigma'}\right) + \frac{q_3-q_2}{x_3-x_2}\left(\frac{1+2\sigma'}{1+\sigma'}\right) \qquad \text{(C-29)}$$

<u>Second Derivative</u>

In a similar manner, the second derivative can be calculated by differentiating equation (C-1) twice with respect to x to obtain

$$\frac{d^2q(x)}{dx^2}\ 2\ \frac{(x_2-x_3)q_1 + (x_3-x_1)\ q_2 + (x_1-x_2)\ q_3}{(x_2-x_1)\ (x_3-x_2)\ (x_1-x_3)} \qquad \text{(C-30)}$$

or

$$\frac{d^2q(x)}{dx^2} = \frac{1}{\frac{1}{4}(\Delta n_{12}+\Delta n_{23})}\left(\frac{q_3-q_2}{x_3-x_2} - \frac{q_2-q_1}{x_2-x_1}\right) \qquad \text{(C-31)}$$

It should be noted that:

1) The above equation is exactly identical with the central finite difference form for the second derivative. This is in contrast with the first derivative which introduced weighting factors.

2) The value of the second derivative is equal at positions x_1, x_2, or x_3.

Furthermore, the coefficient of the finite difference quotients on the right hand side of equation (C-31) in terms of spatial differences becomes

$$\frac{1}{\frac{1}{4}\left(\Delta n_{12}+\Delta n_{23}\right)} = \frac{2}{x_3-x_1} \qquad \text{(C-32)}$$

PART TWO

USER'S MANUAL

1. DESCRIPTION OF THERMA DIGITAL COMPUTER PROGRAM

The program THERMA, developed for the analysis of three dimensional thermal stratification and circulation in water bodies subjected to thermal discharge, consists of a MAIN program and several subroutines as shown in Figs. 24 to 28. A tabular outline of the MAIN program and its subroutines together with their functions are presented below and followed by a more detailed description.

Program	Function
MAIN	The MAIN program reads the input data in, initializes the problem, calls the subroutines needed to solve the problem, and controls the termination of program.

Subroutine	Function
DERIV	This subroutine calculates spatial and time derivatives of variables as needed.
PRESS	This subroutine calculates the updated values of pressure.
SUM	This subroutine performs the time integration.

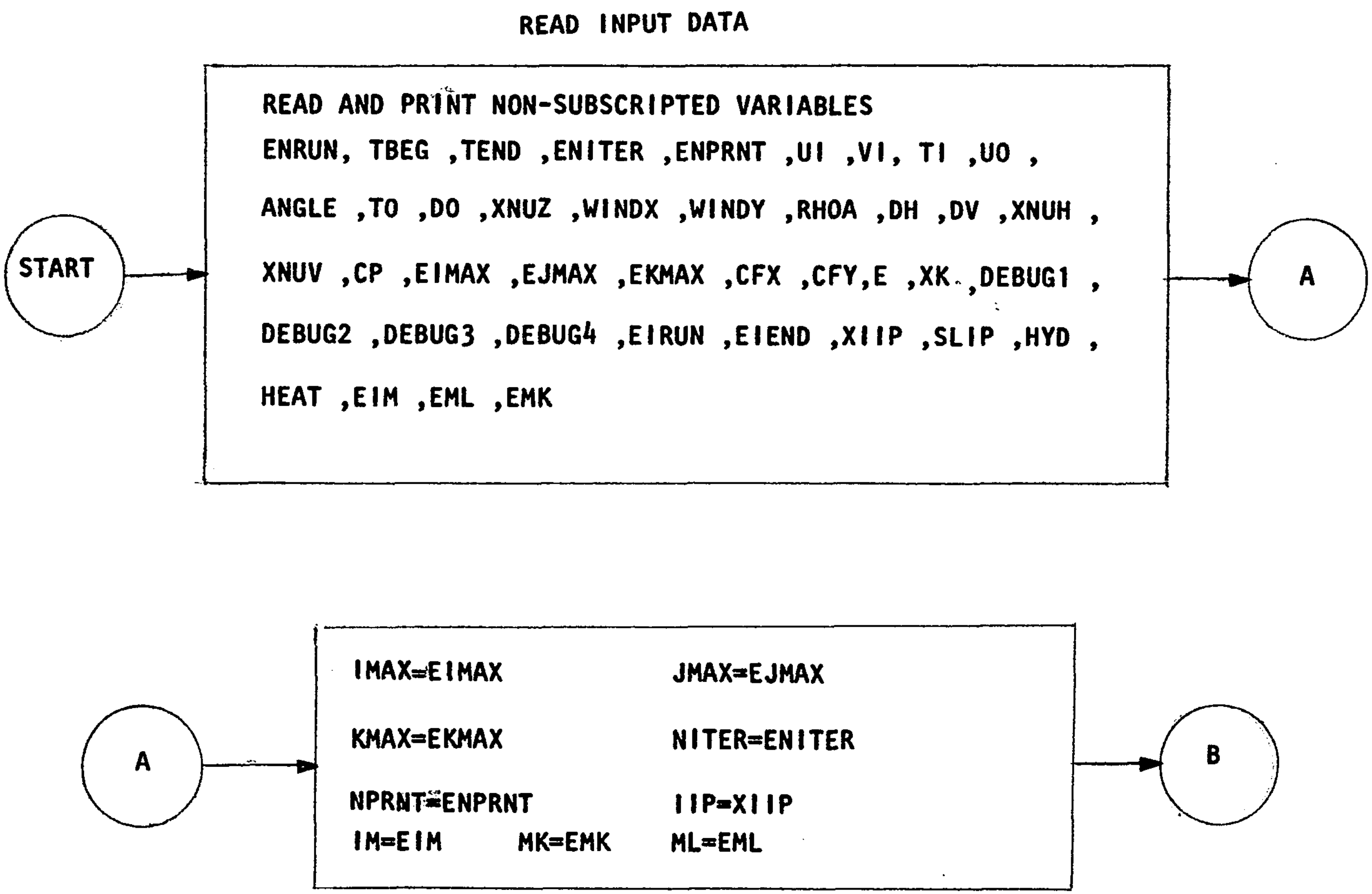


FIG 24a FLOW CHART OF MAIN PROGRAM

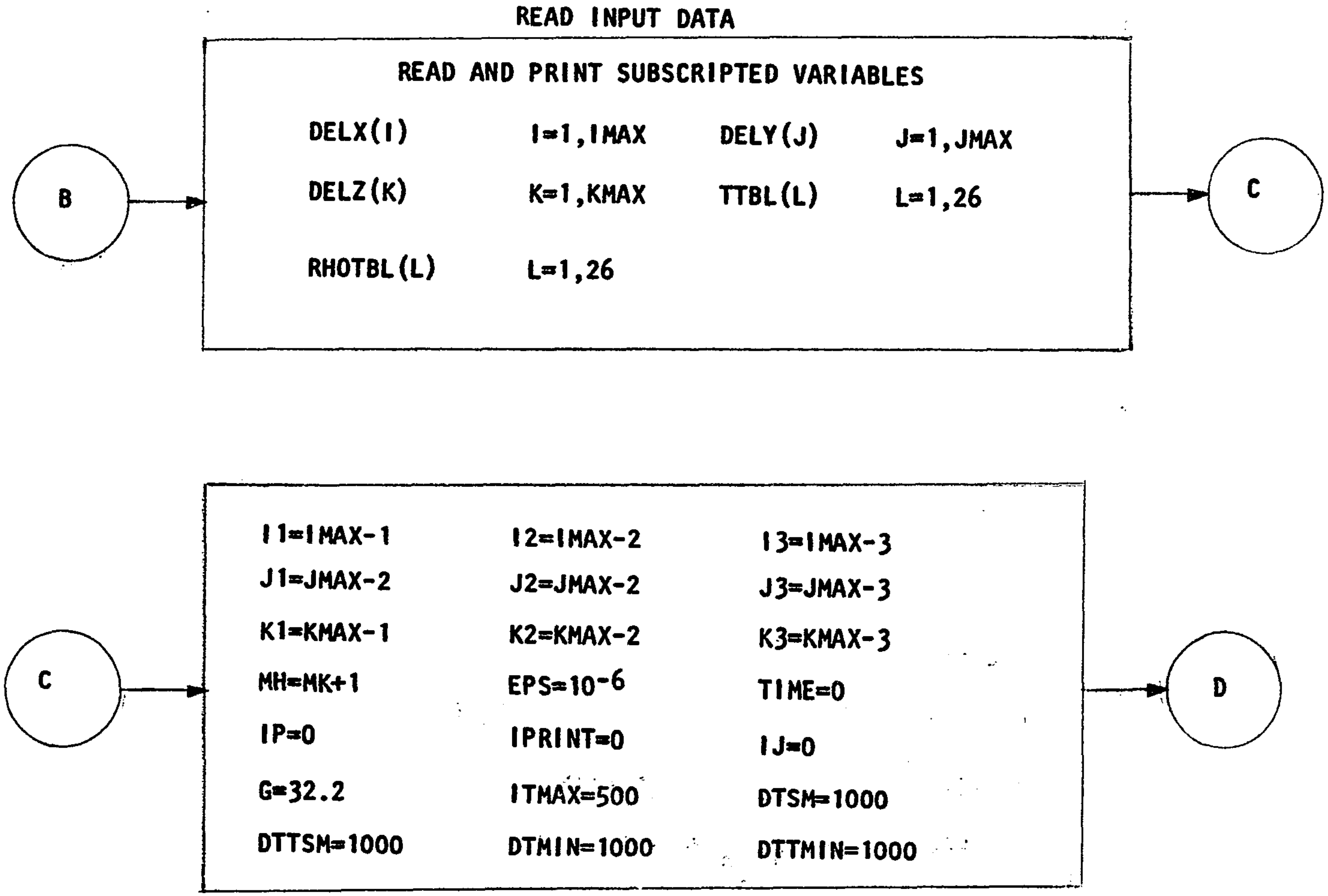


FIG 24b FLOW CHART OF MAIN PROGRAM

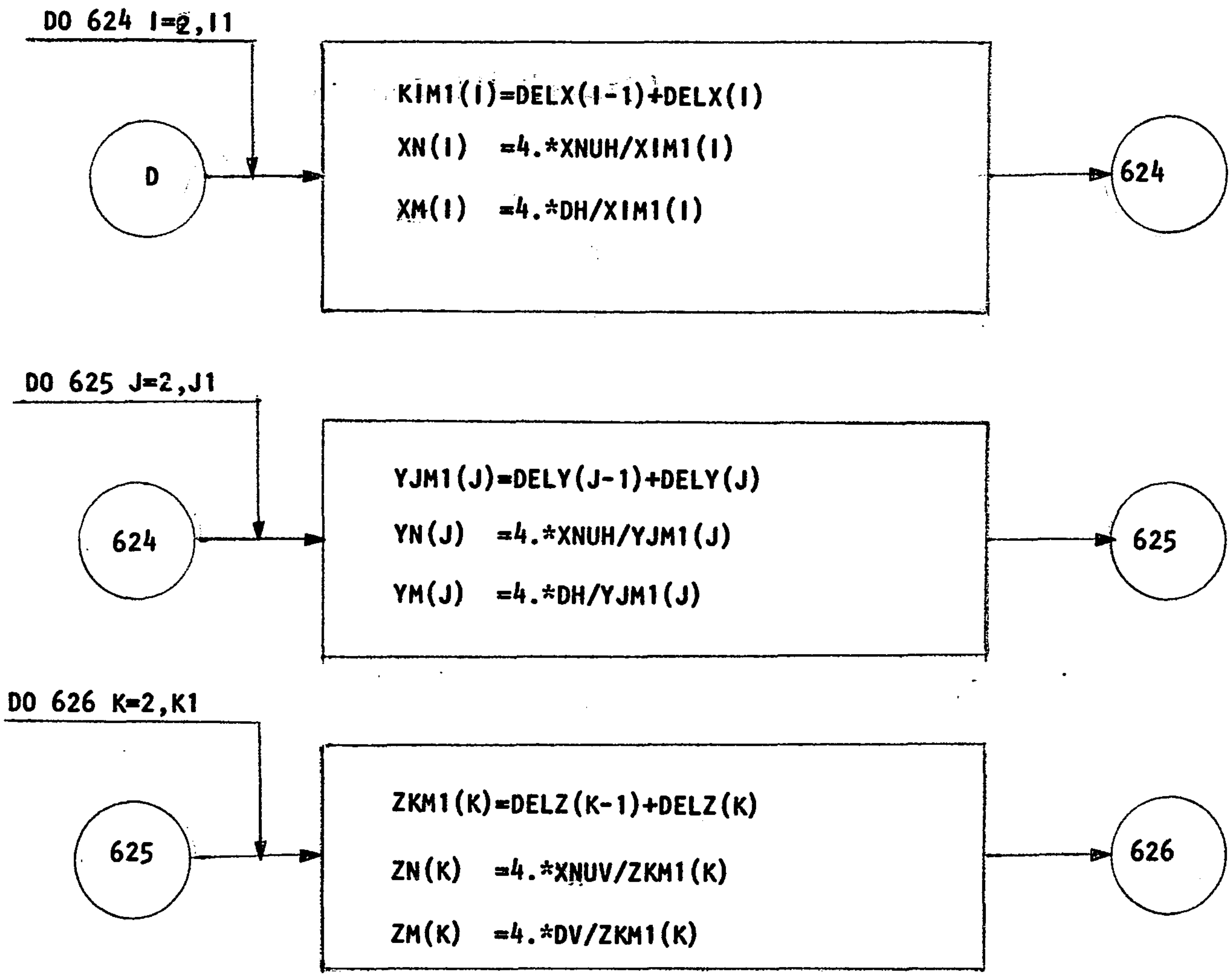


FIG 24c FLOW CHART OF MAIN PROGRAM

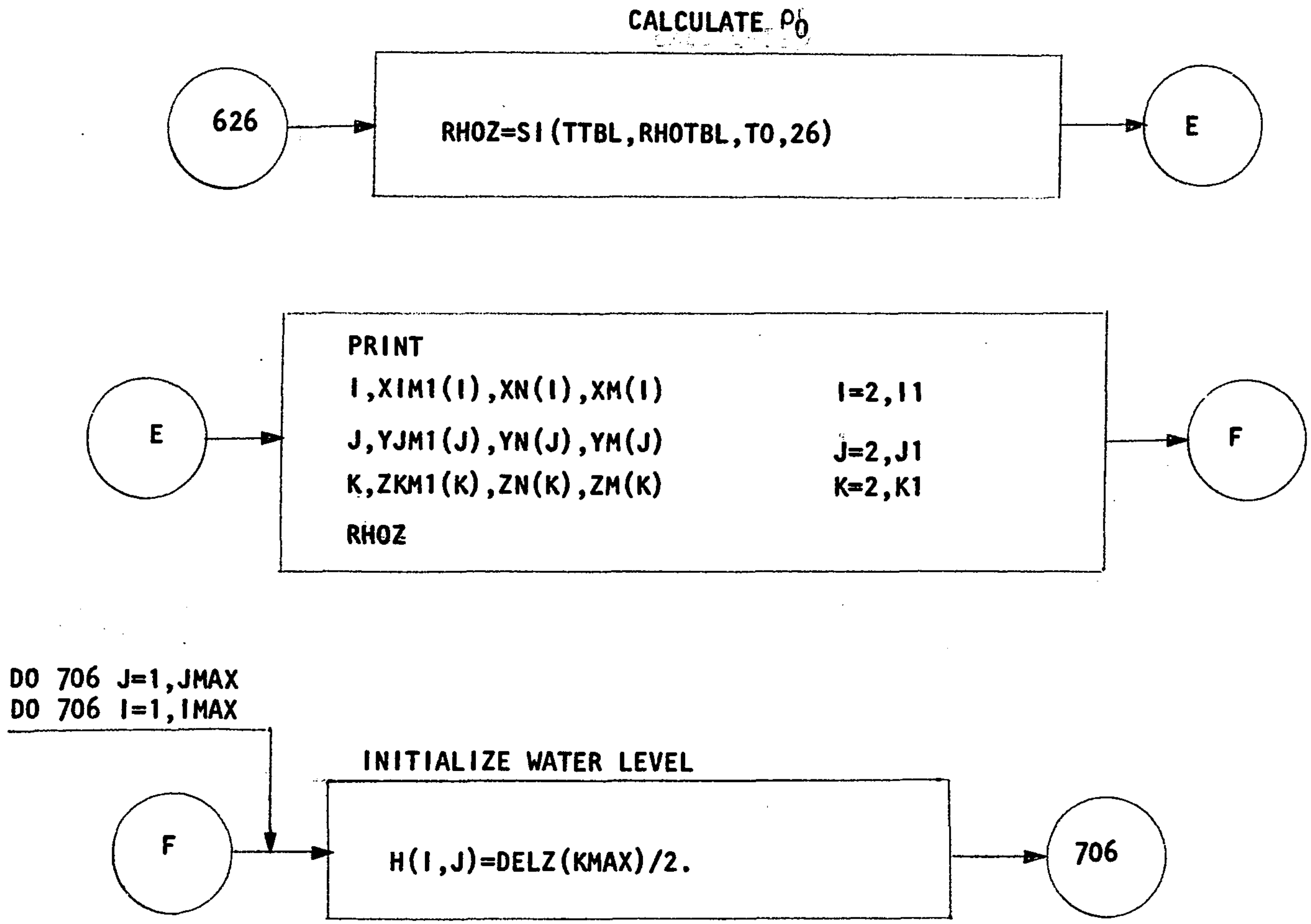


FIG 24d FLOW CHART OF MAIN PROGRAM

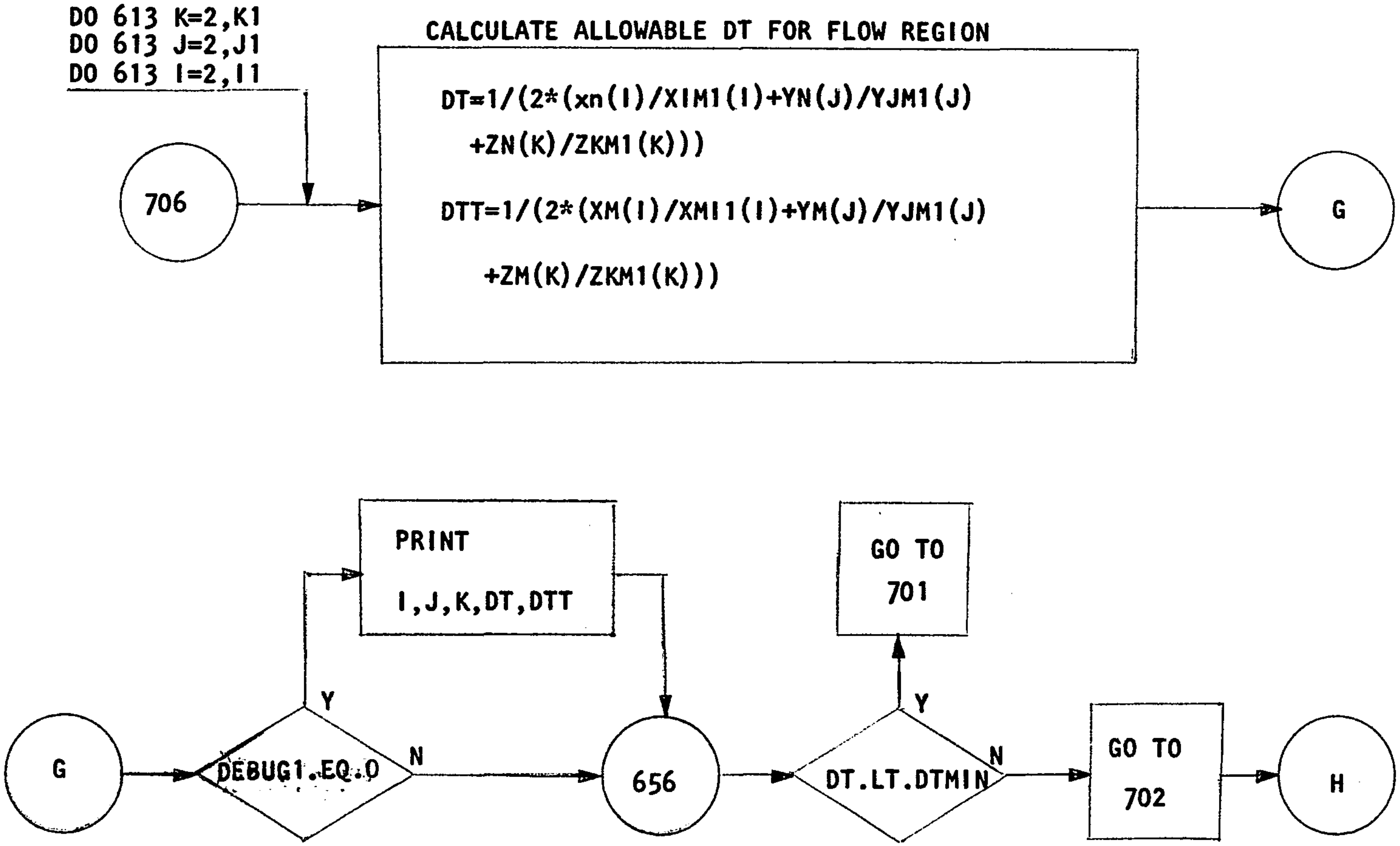


FIG 24e FLOW CHART OF MAIN PROGRAM

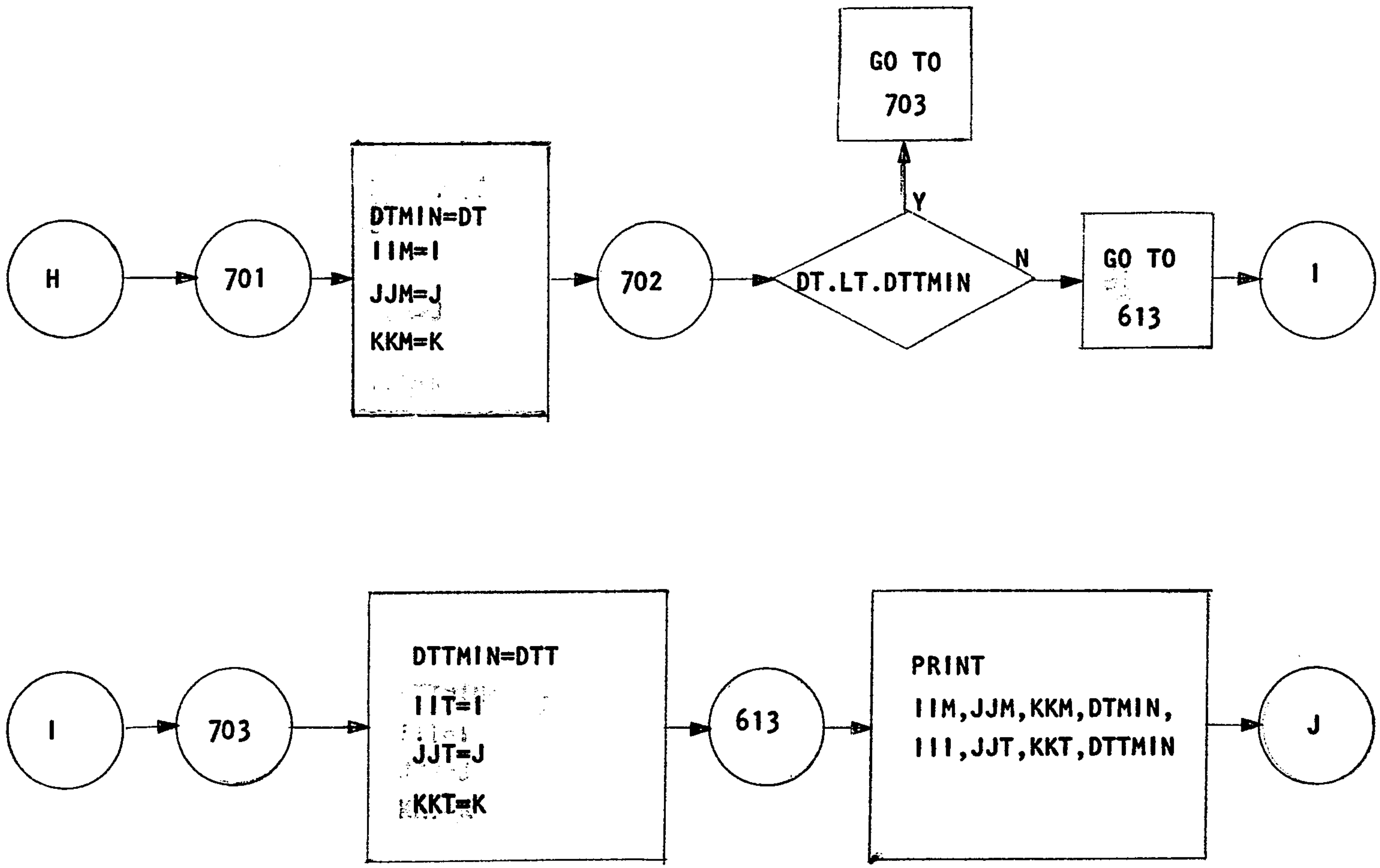


FIG 24f FLOW CHART OF MAIN PROGRAM

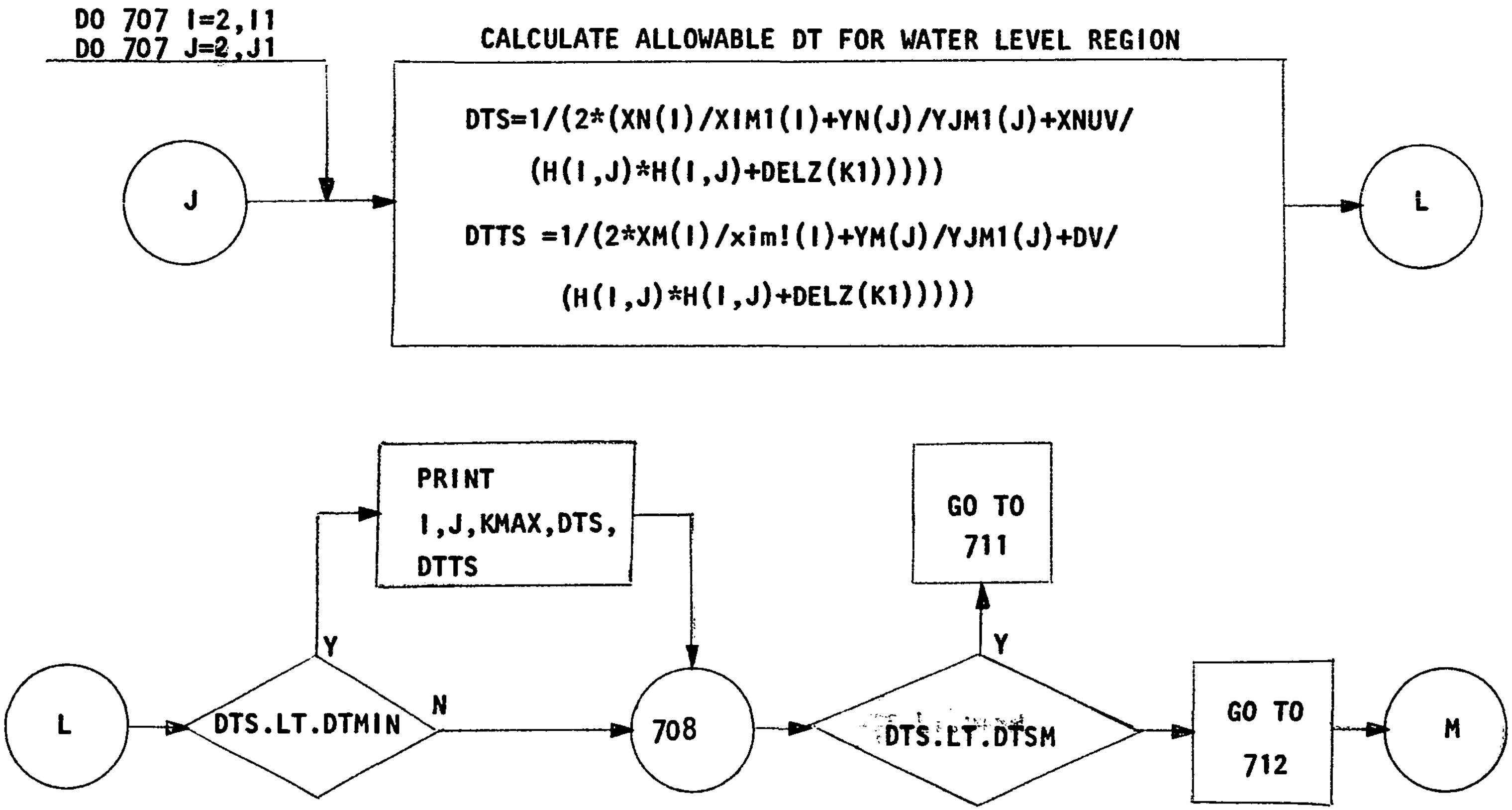


FIG 24g FLOW CHART OF MAIN PROGRAM

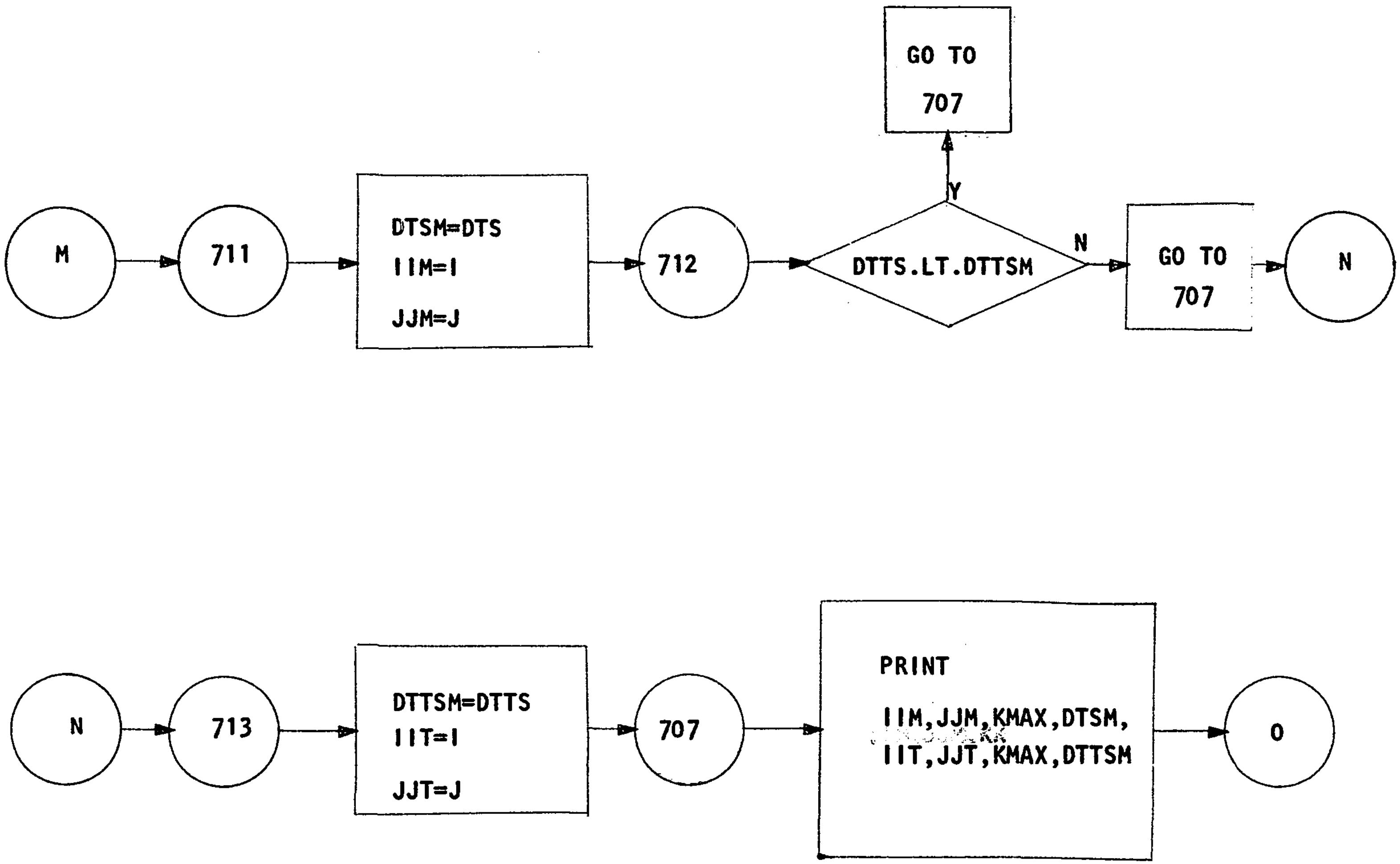


FIG 24h FLOW CHART OF MAIN PROGRAM

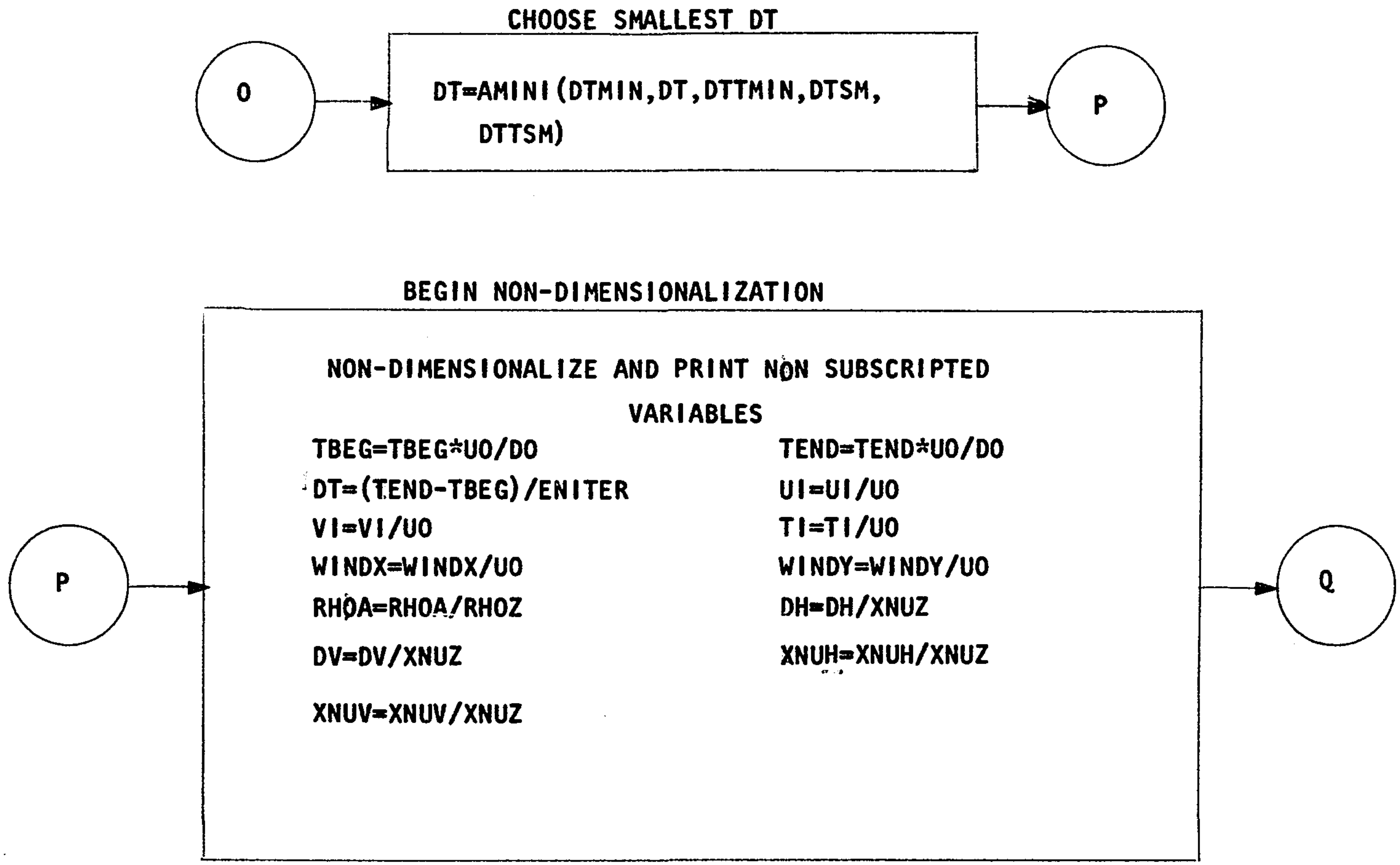
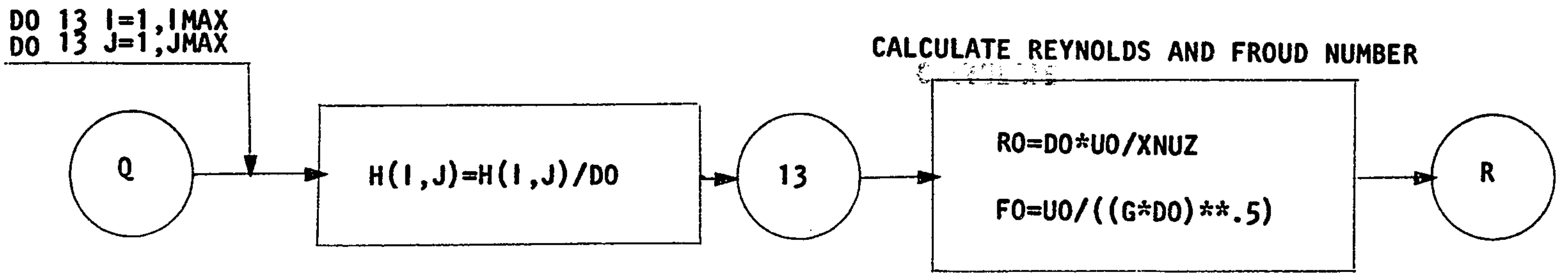


FIG 24i FLOW CHART OF MAIN PROGRAM



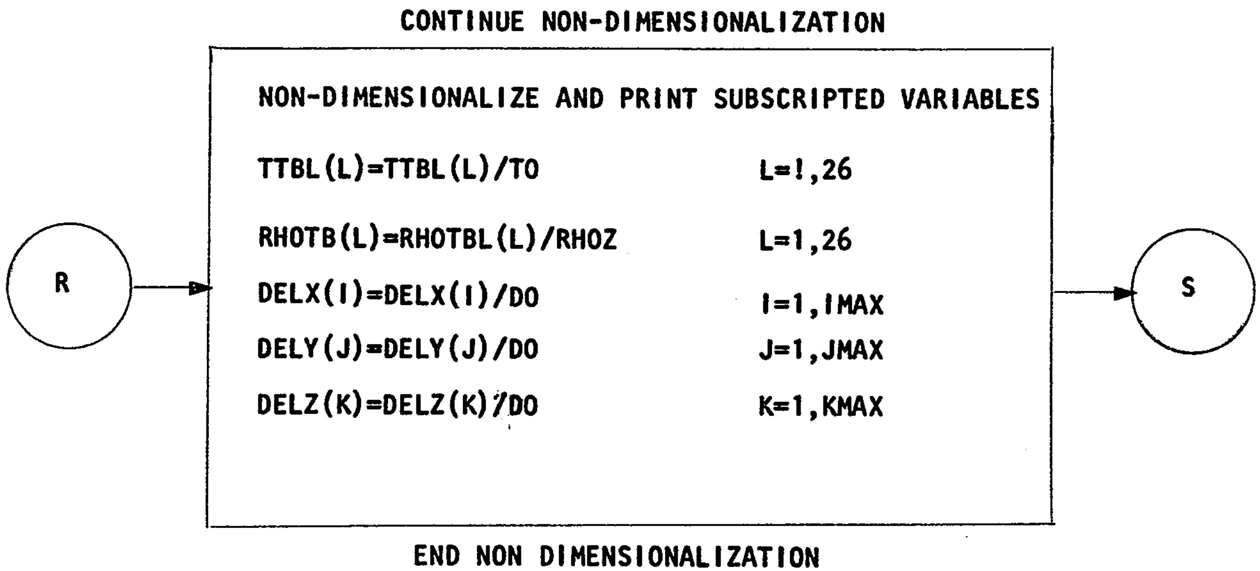


FIG 24j FLOW CHART OF MAIN PROGRAM

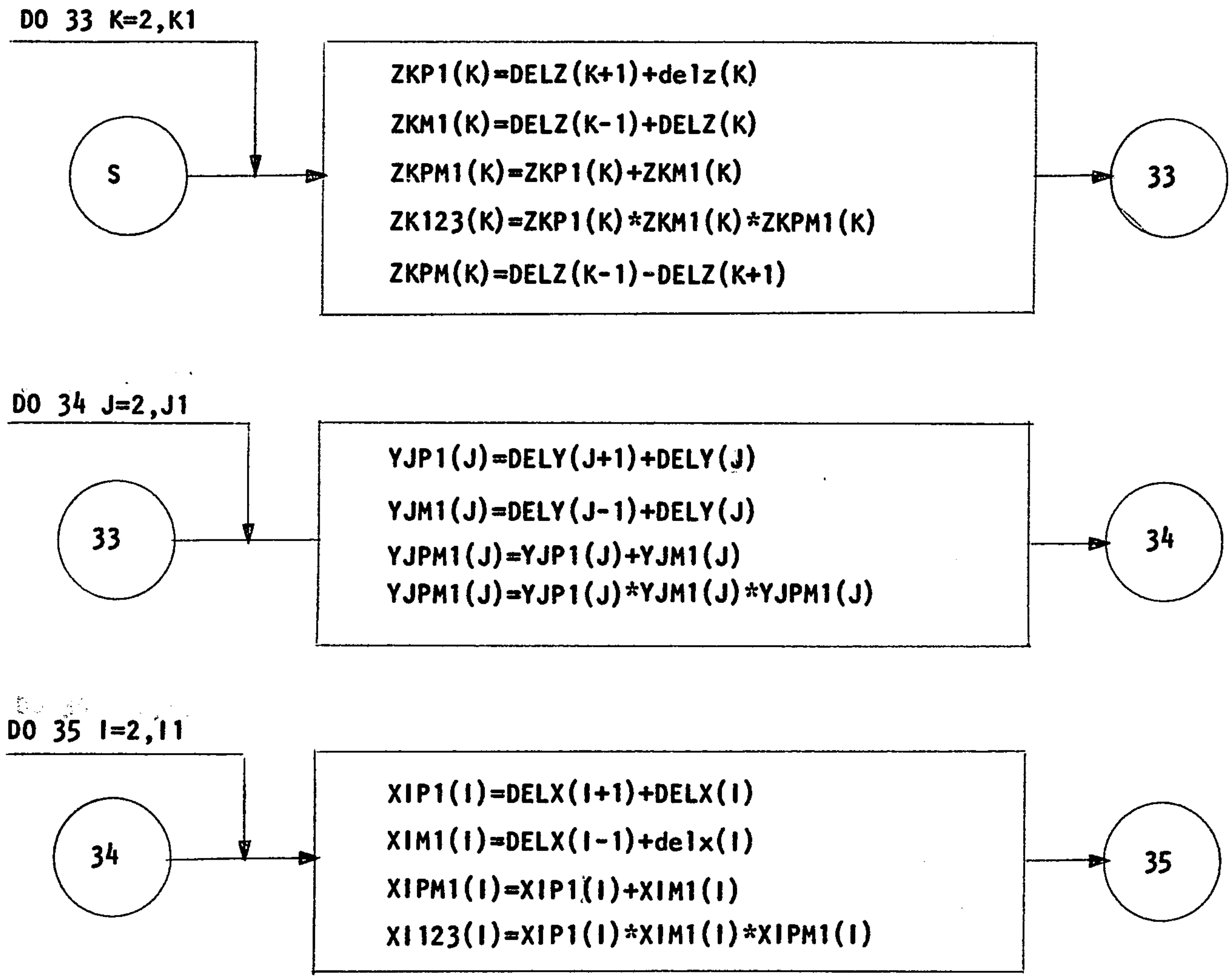


FIG 24k FLOW CHART OF MAIN PROGRAM

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COEFFICIENT OF FIRST DERIVATIVES AT THE BOUNDARIES
SIGX=(DELX(1)+DELX(2))/(DELX(2)+DELX(3))
SIGX1+(1+2*SIGX)
SIGX2=(1+SIGX)*(1+SIGX)
SIGX3=SIGX*SIGX
SIGX4=(DELX(1)+DELX(2))*(1+SIGX)/2
SIGY=(DELY(1)+dely(2))/(DELY(2)+DELY(3))
SIGY1=(1+2*SIGY)
SIGY2=(1+SIGY)*(1+SIGY)
SIGY3=SIGY*SIGY
SIGY4=(DELY(1)+DELY(2))*(1+sigy)/2
SIGPY=(DELY(JMAX)+DELY(J1))/(DELY(J1)+DELY(J2))
SIGPY1=(1+2*SIGPY)
SIGPY2=(1+SIGPY)*(1+SIGPY)
SIGPY3=SIGPY*SIGPY
SIGPY4=(DELY(JMAX)+DELY(J1))*(1+SIGPY)/2
SIGZ=(DELZ(1)+DELZ(2))/DELZ(2)+DELZ(3))
SIGZ1=(1+2*SIGZ)
SIGZ2=(1+SIGZ)*(1+SIGZ)
SIGZ3=SIGZ*SIGZ
SIGZ4=(DELZ(1)+DELZ(2))*(1+SIGZ)
```

T

FIG 241 FLOW CHART OF MAIN PROGRAM

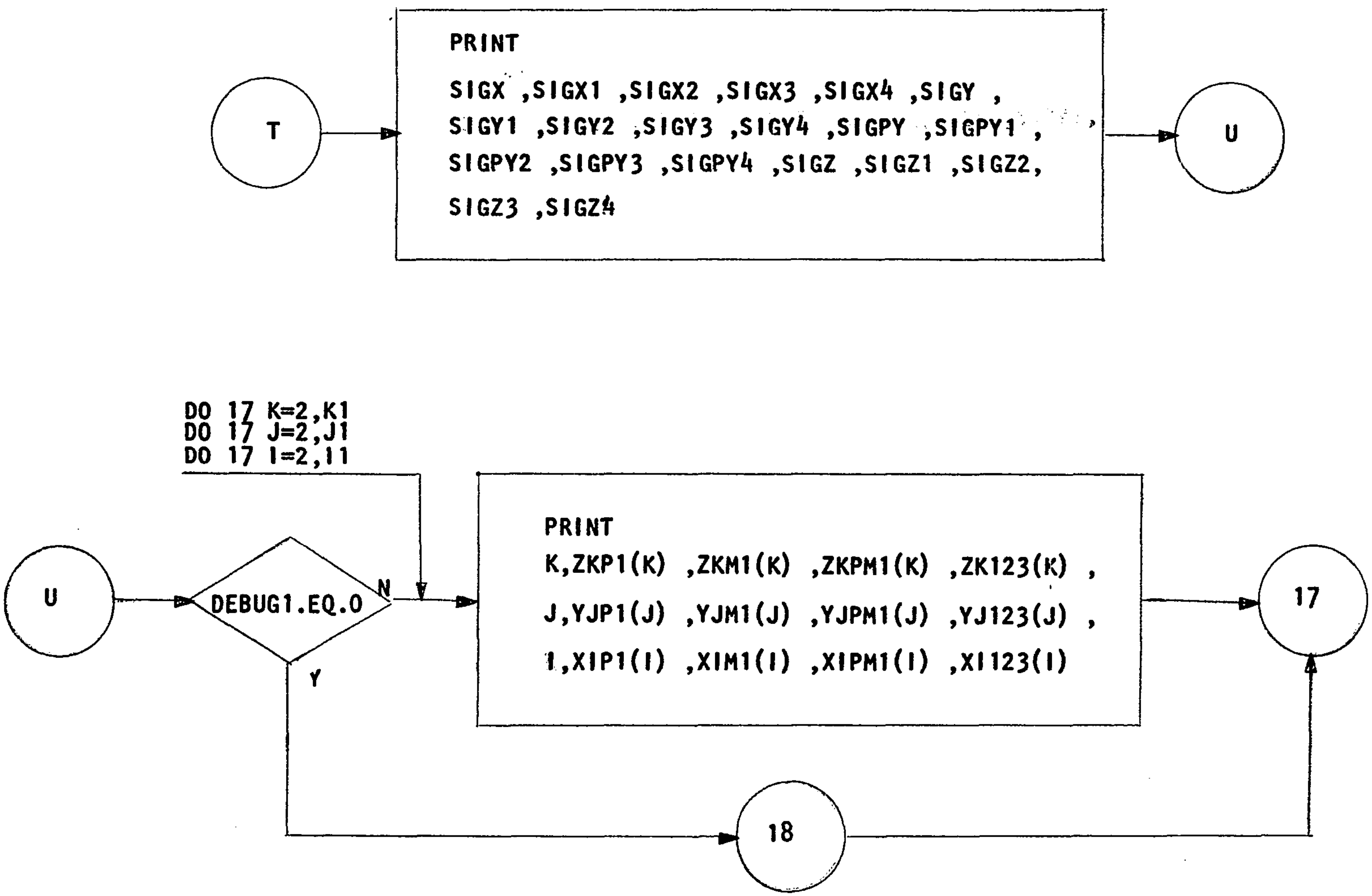


FIG 24m FLOW CHART OF MAIN PROGRAM

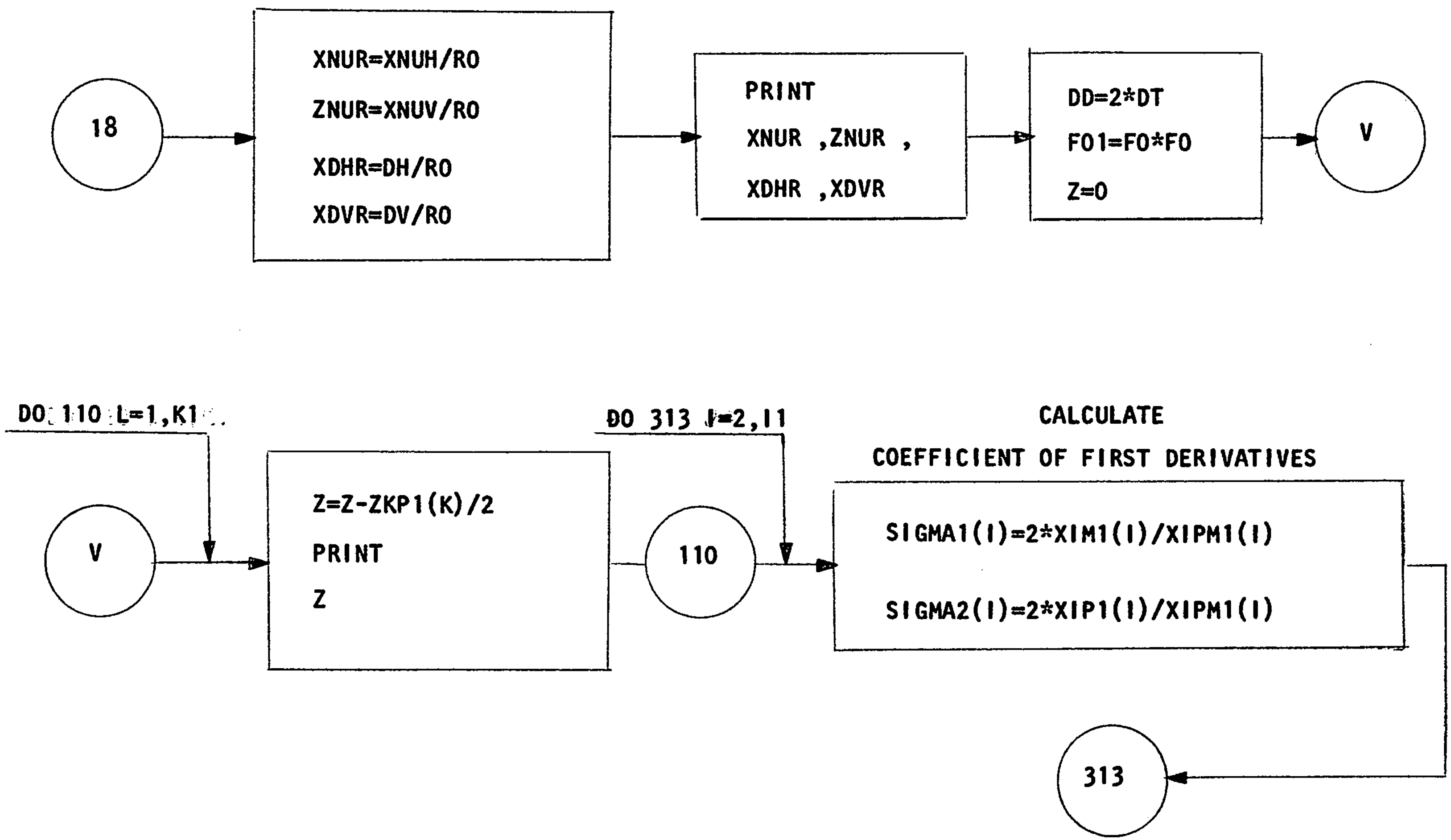


FIG 24n FLOW CHART OF MAIN PROGRAM

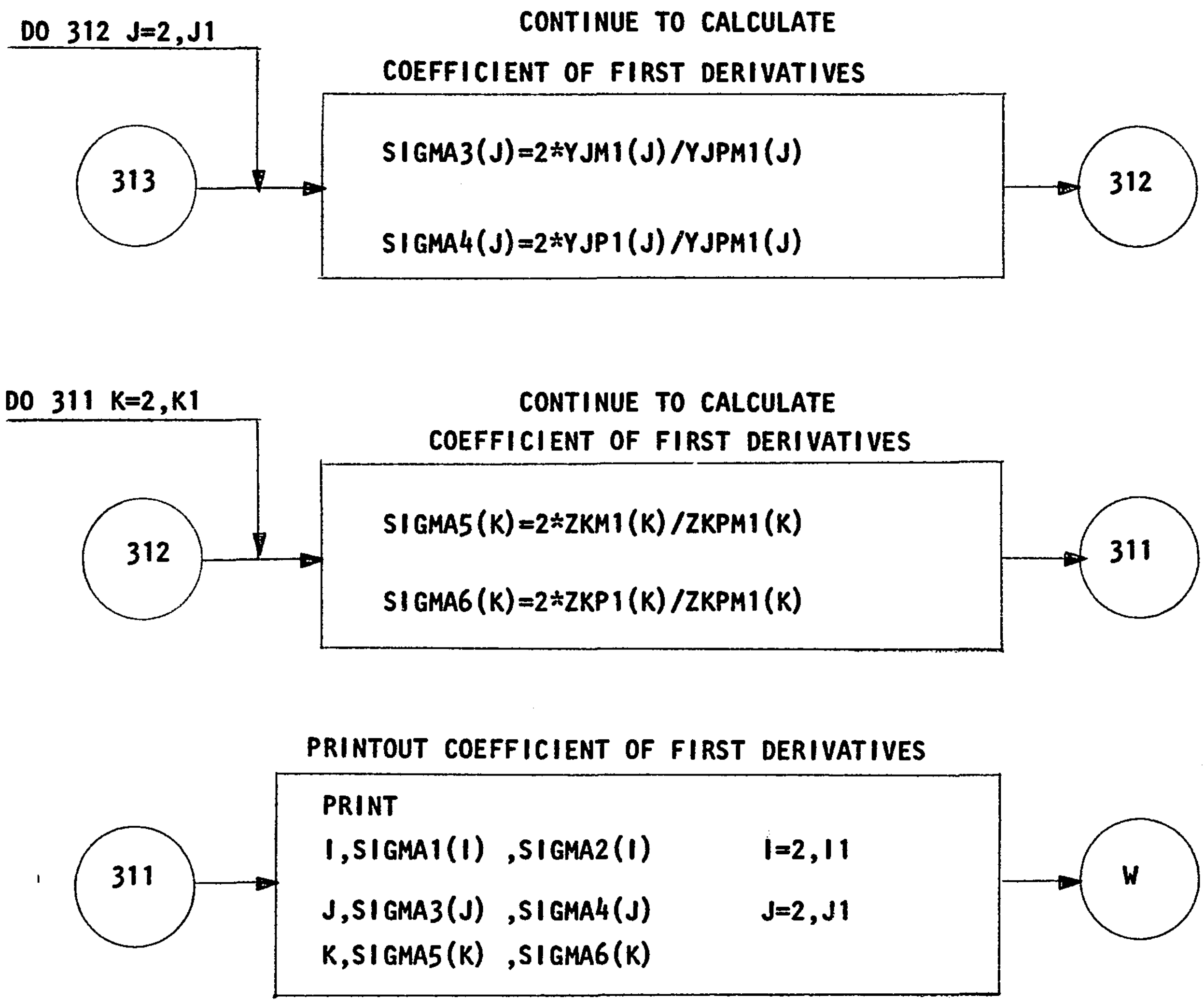


FIG 24o FLOW CHART OF MAIN PROGRAM

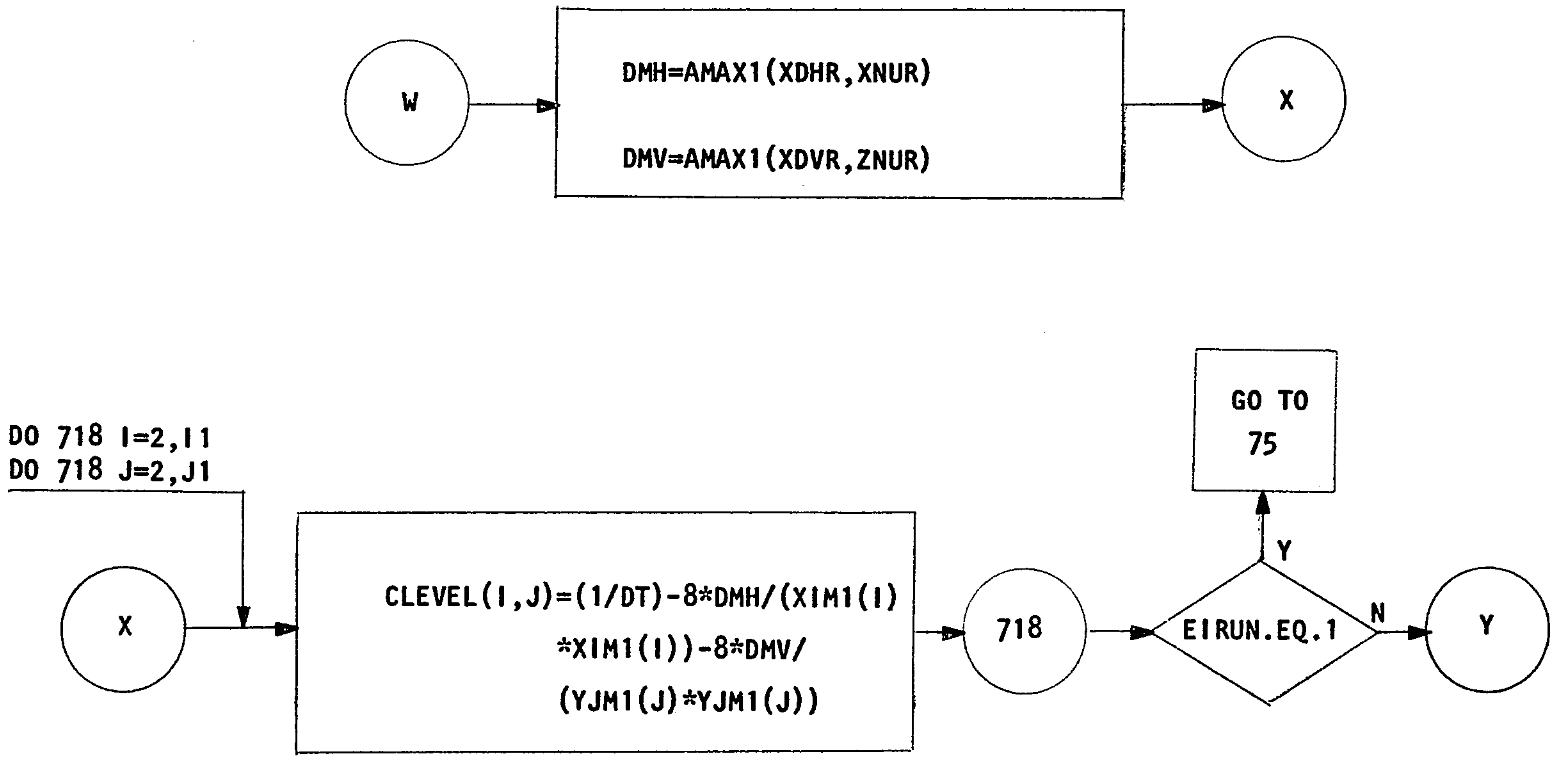
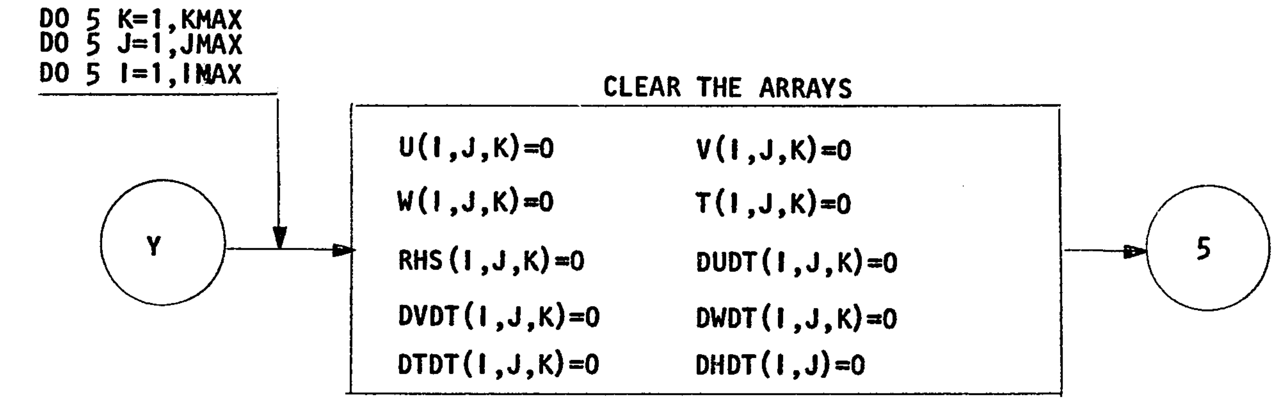


FIG 24p FLOW CHART OF MAIN PROGRAM



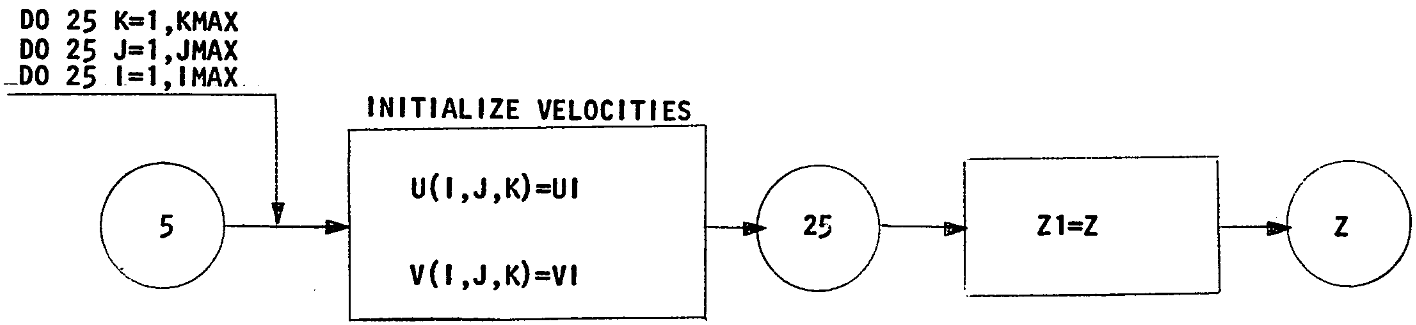


FIG 24q FLOW CHART OF MAIN PROGRAM

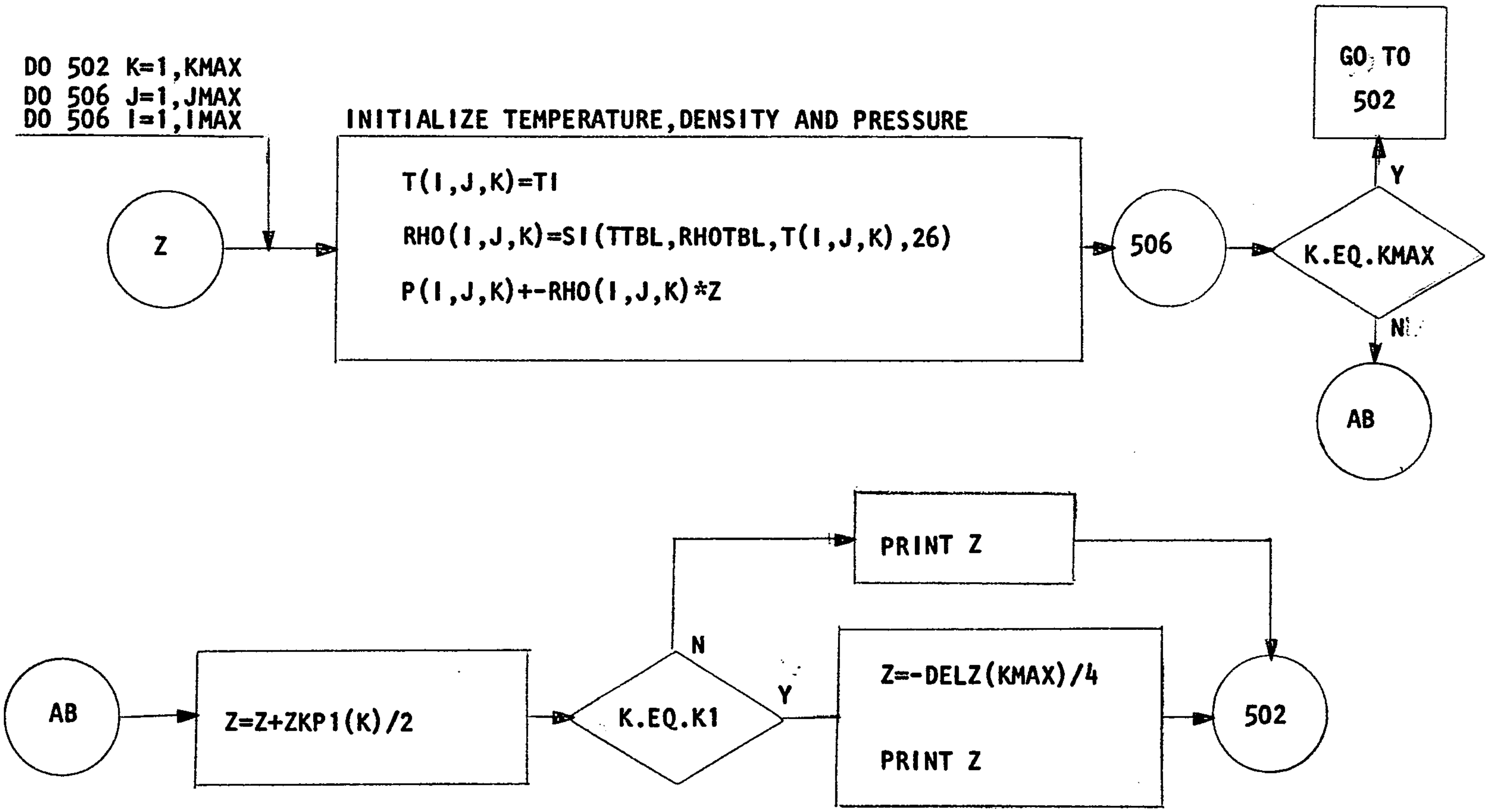


FIG 24r FLOW CHART OF MAIN PROGRAM

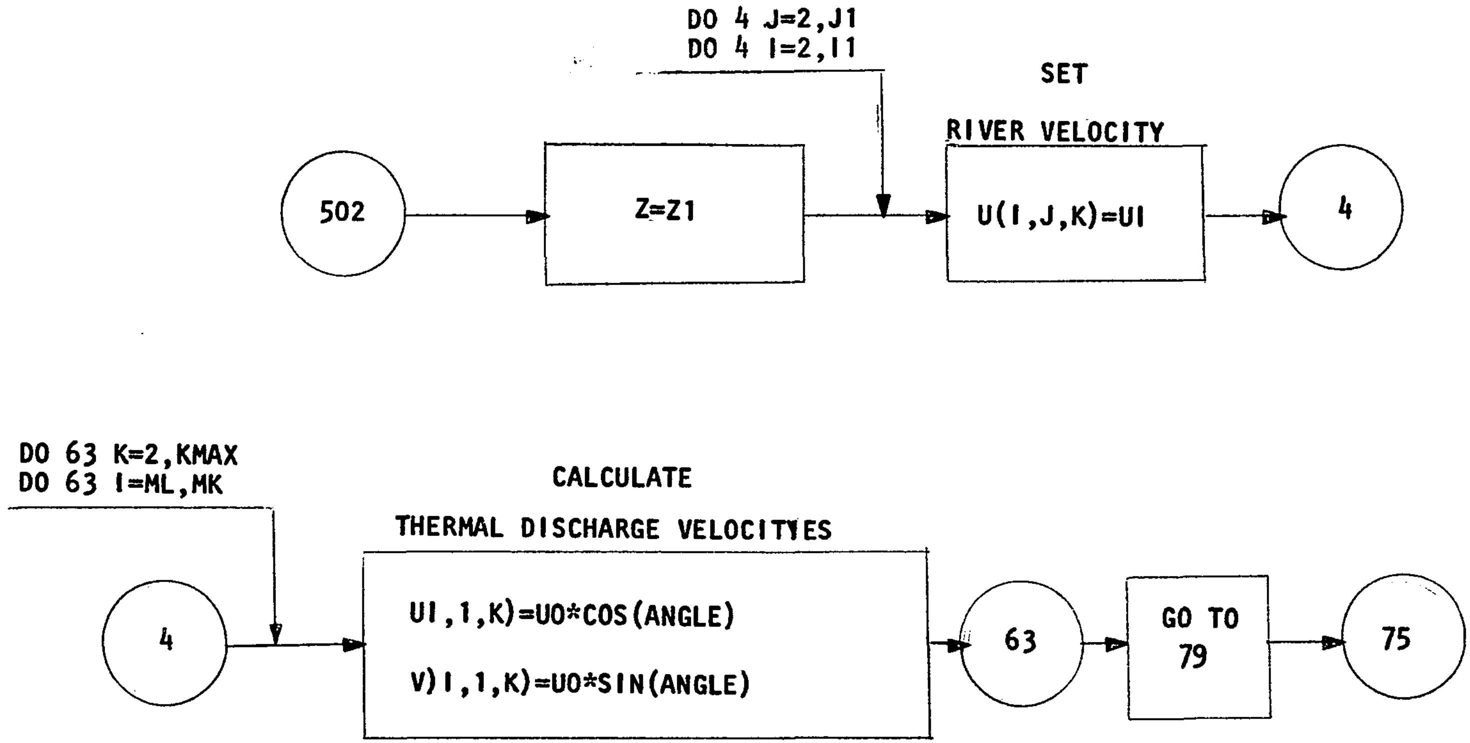


FIG 24s FLOW CHART OF MAIN PROGRAM

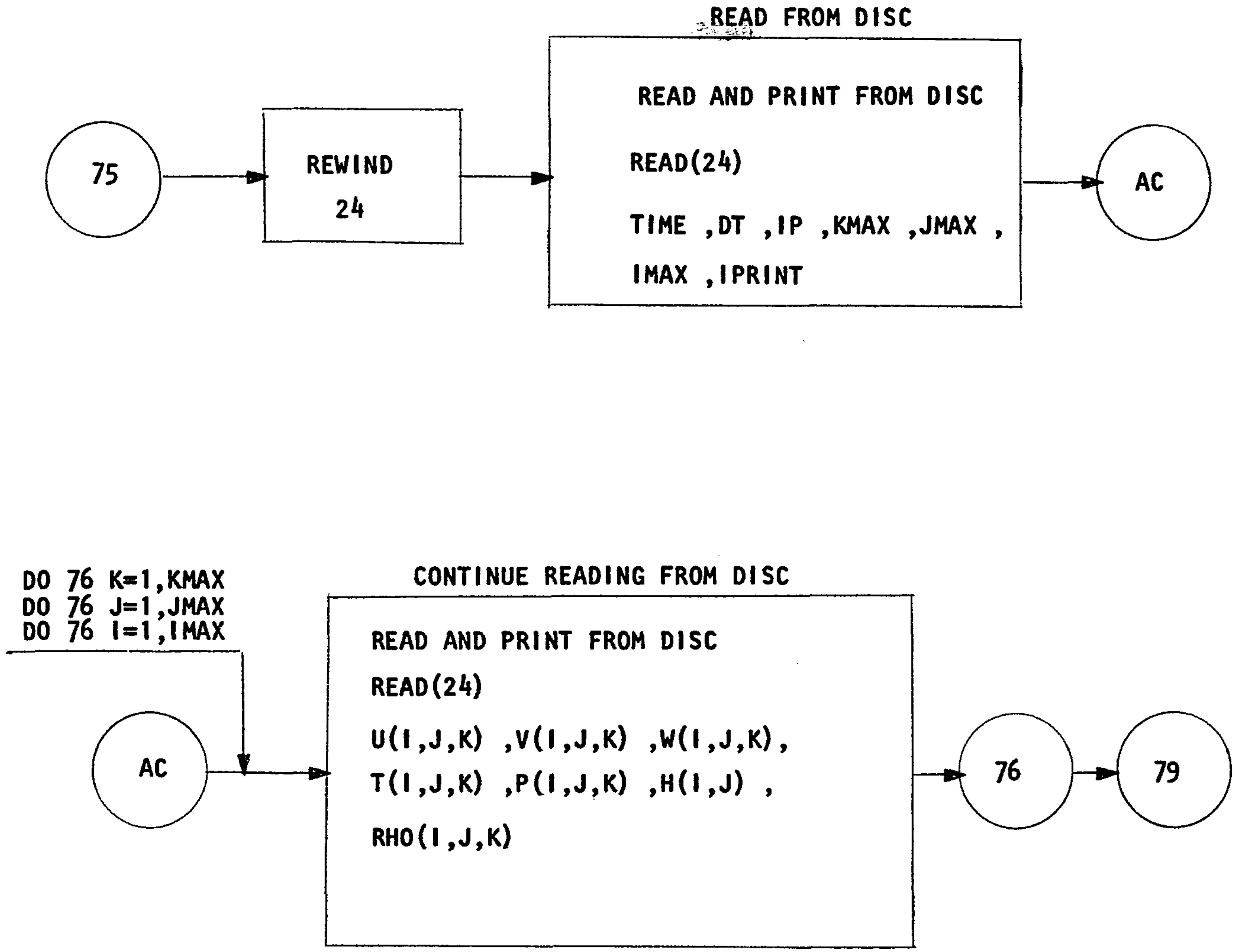


FIG 24t FLOW CHART OF MAIN PROGRAM

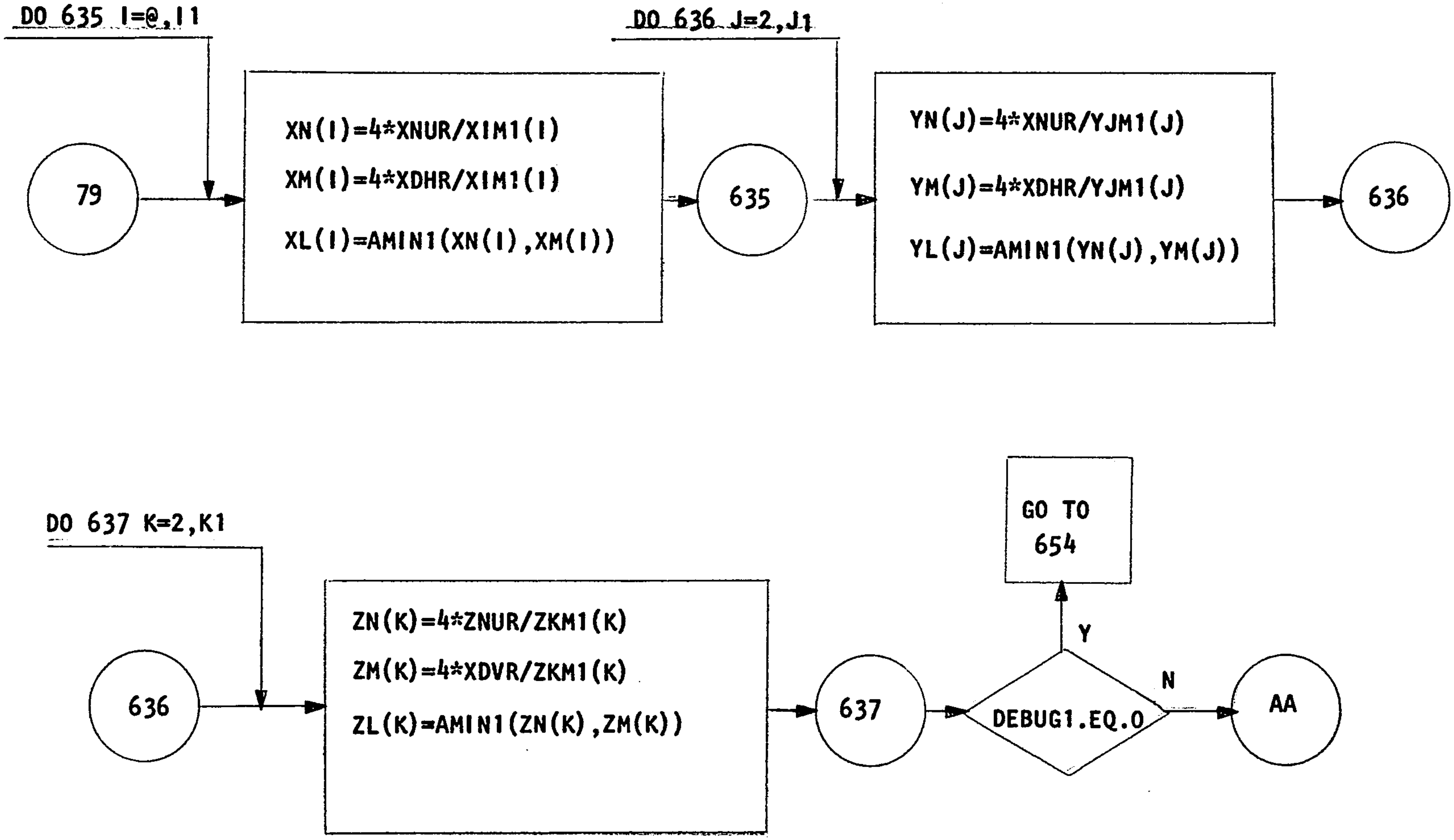
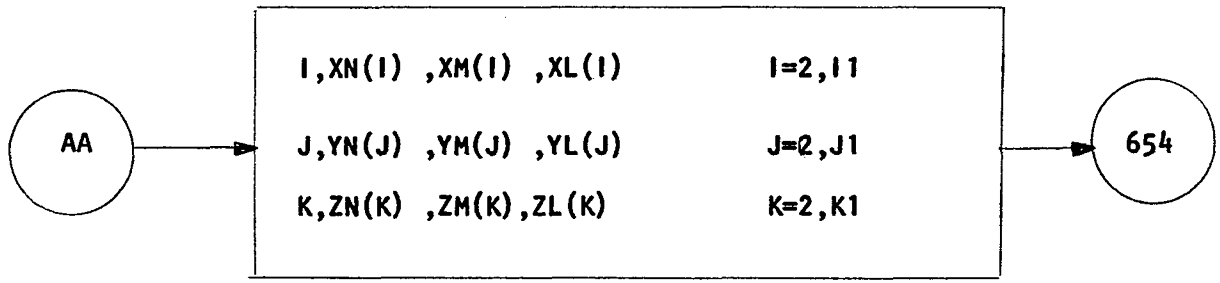


FIG 24u FLOW CHART OF MAIN PROGRAM



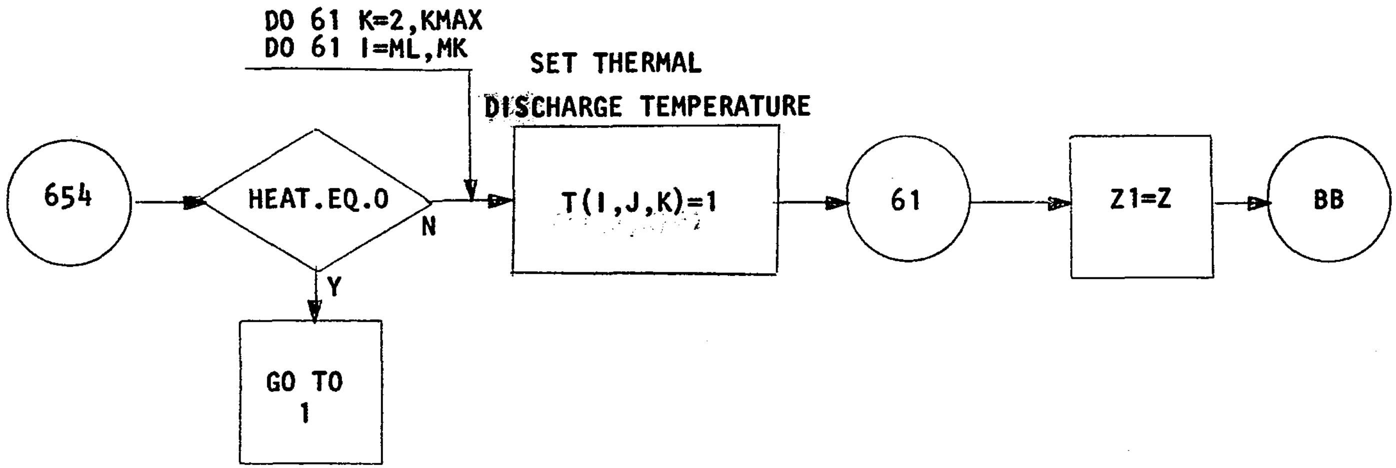


FIG 24v FLOW CHART OF MAIN PROGRAM

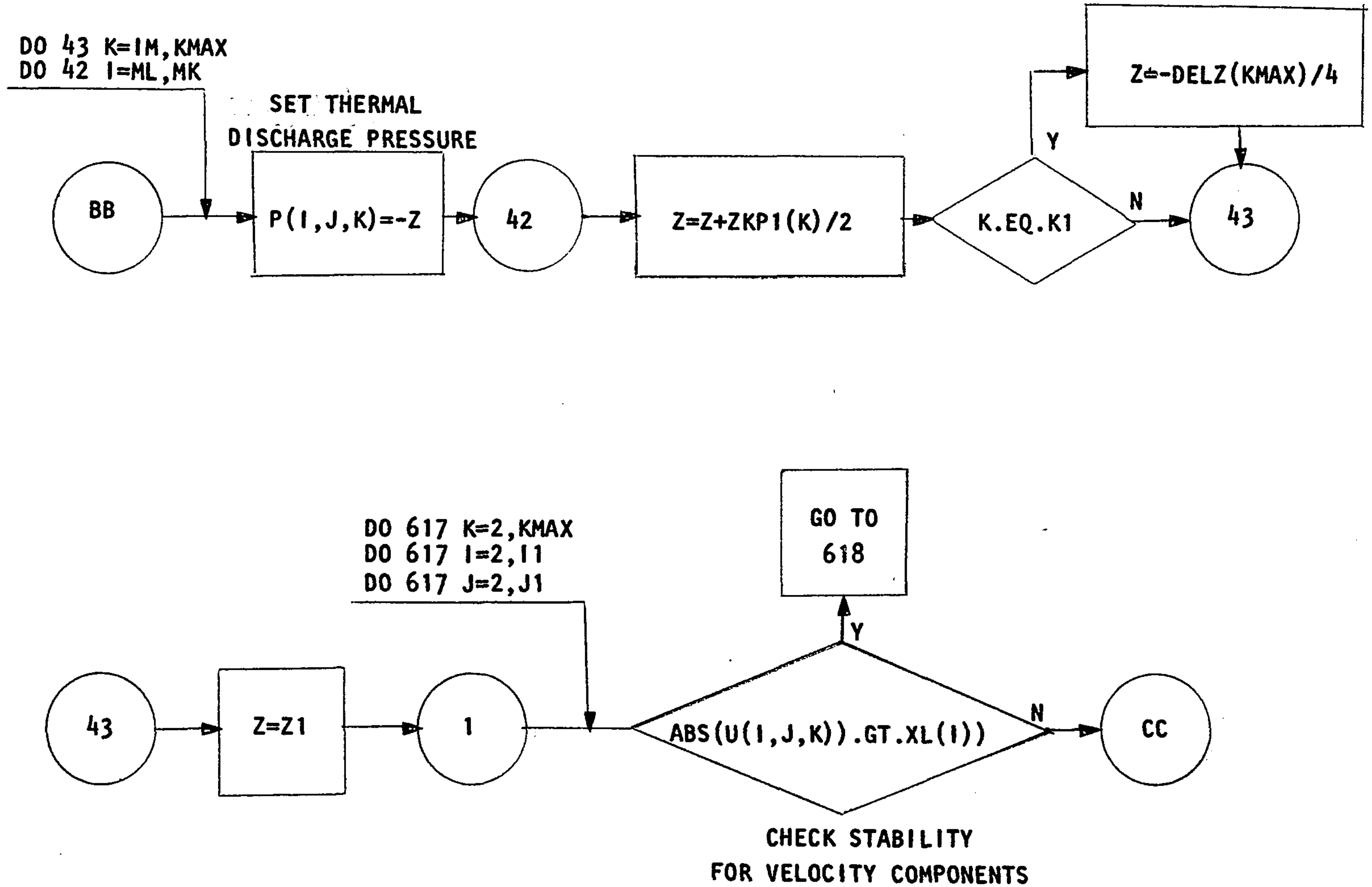


FIG 24w FLOW CHART OF MAIN PROGRAM

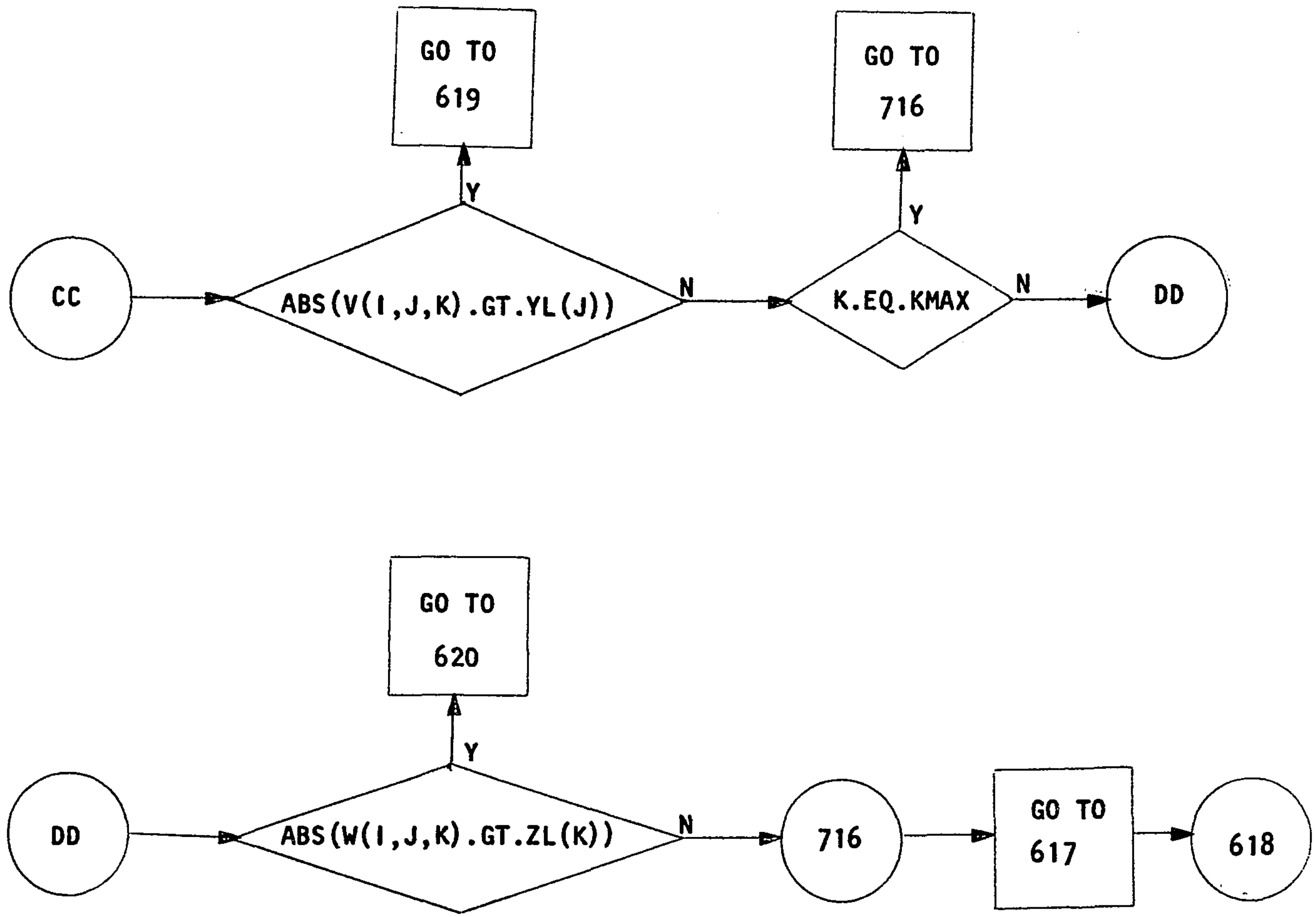


FIG 24x FLOWCHART OF MAIN PROGRAM

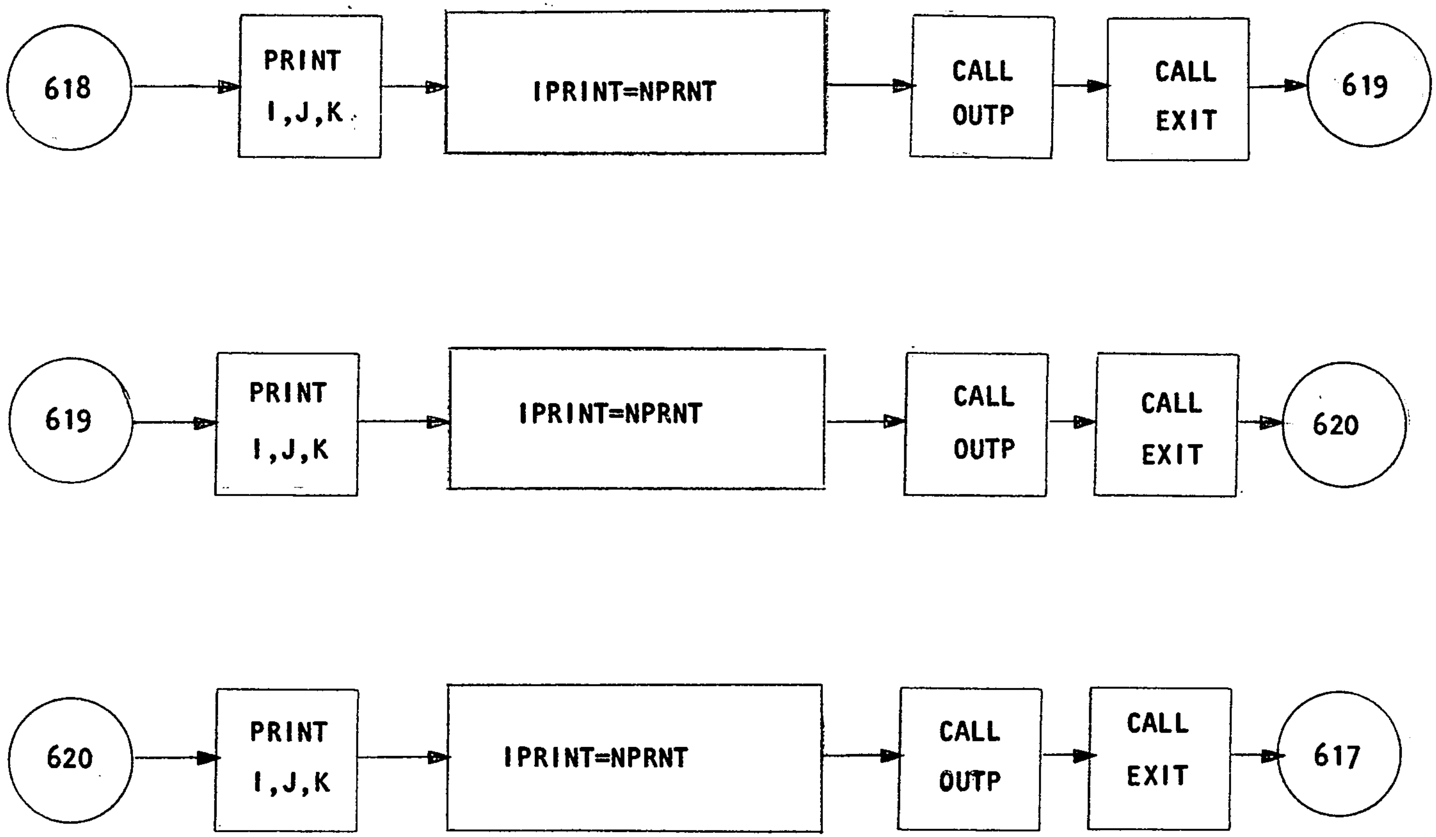


FIG 24y FLOW CHART OF MAIN PROGRAM

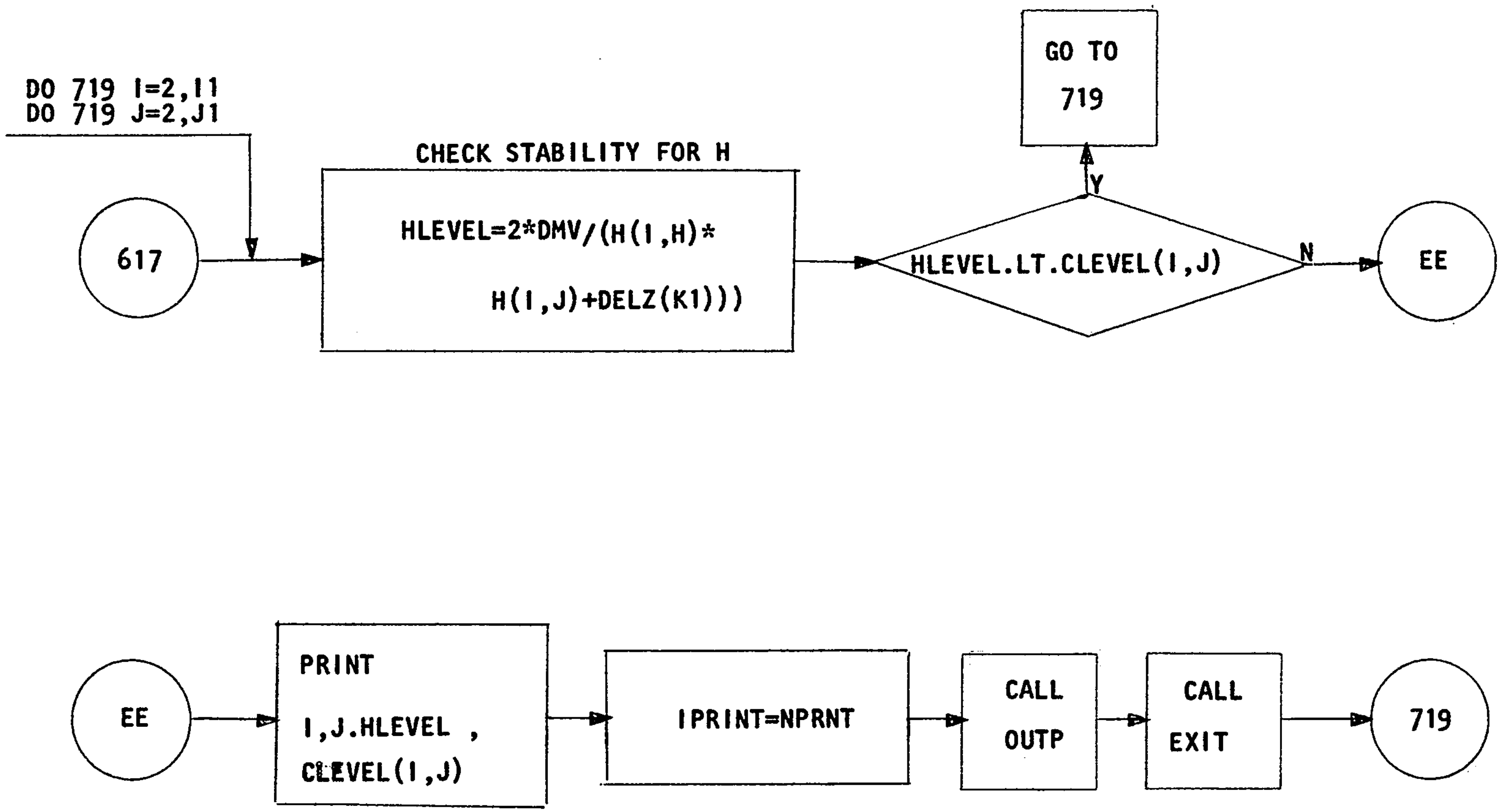


FIG 24z FLOW CHART OF MAIN PROGRAM

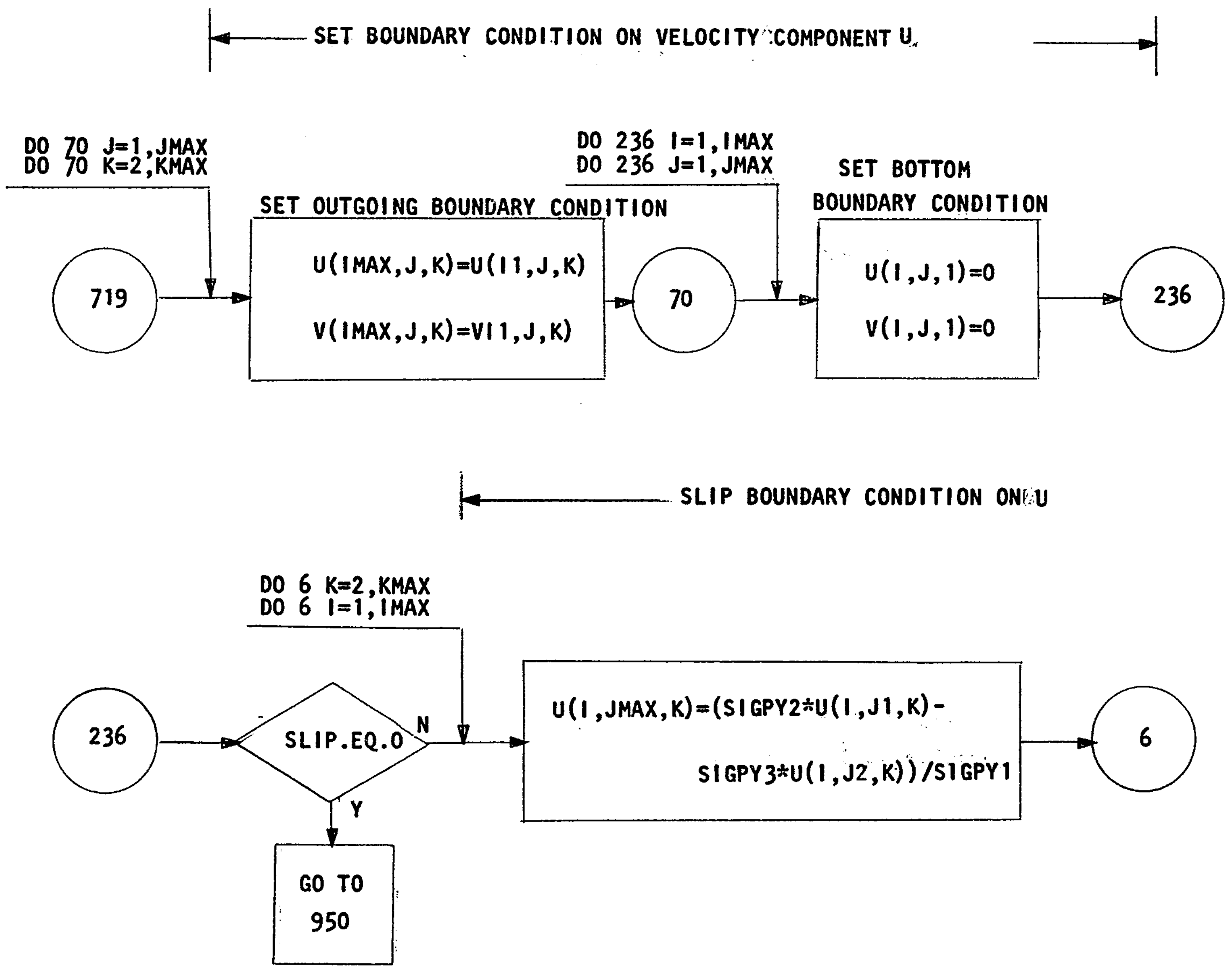


FIG 24aa FLOW CHART OF MAIN PROGRAM

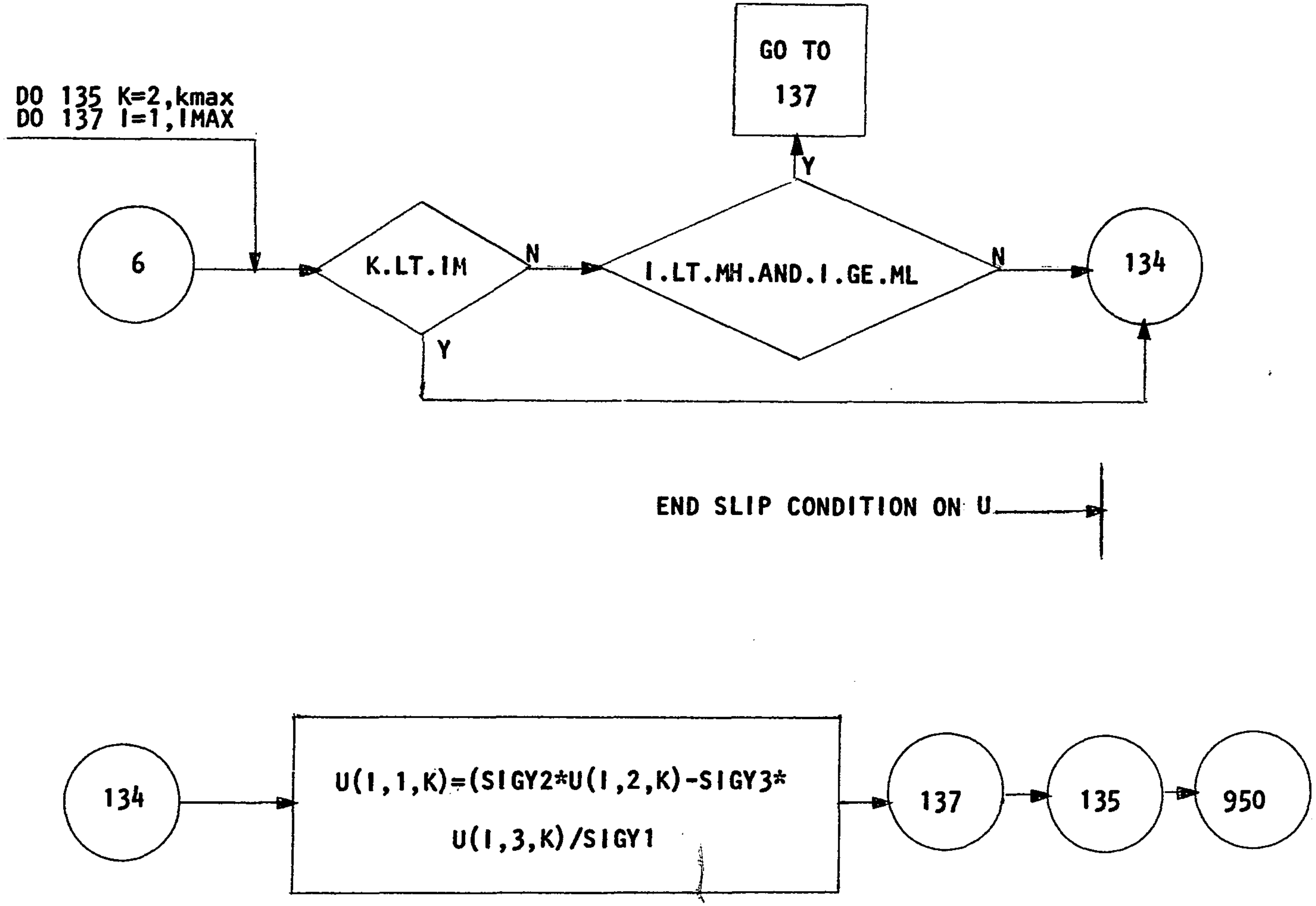


FIG 24bb FLOW CHART OF MAIN PROGRAM

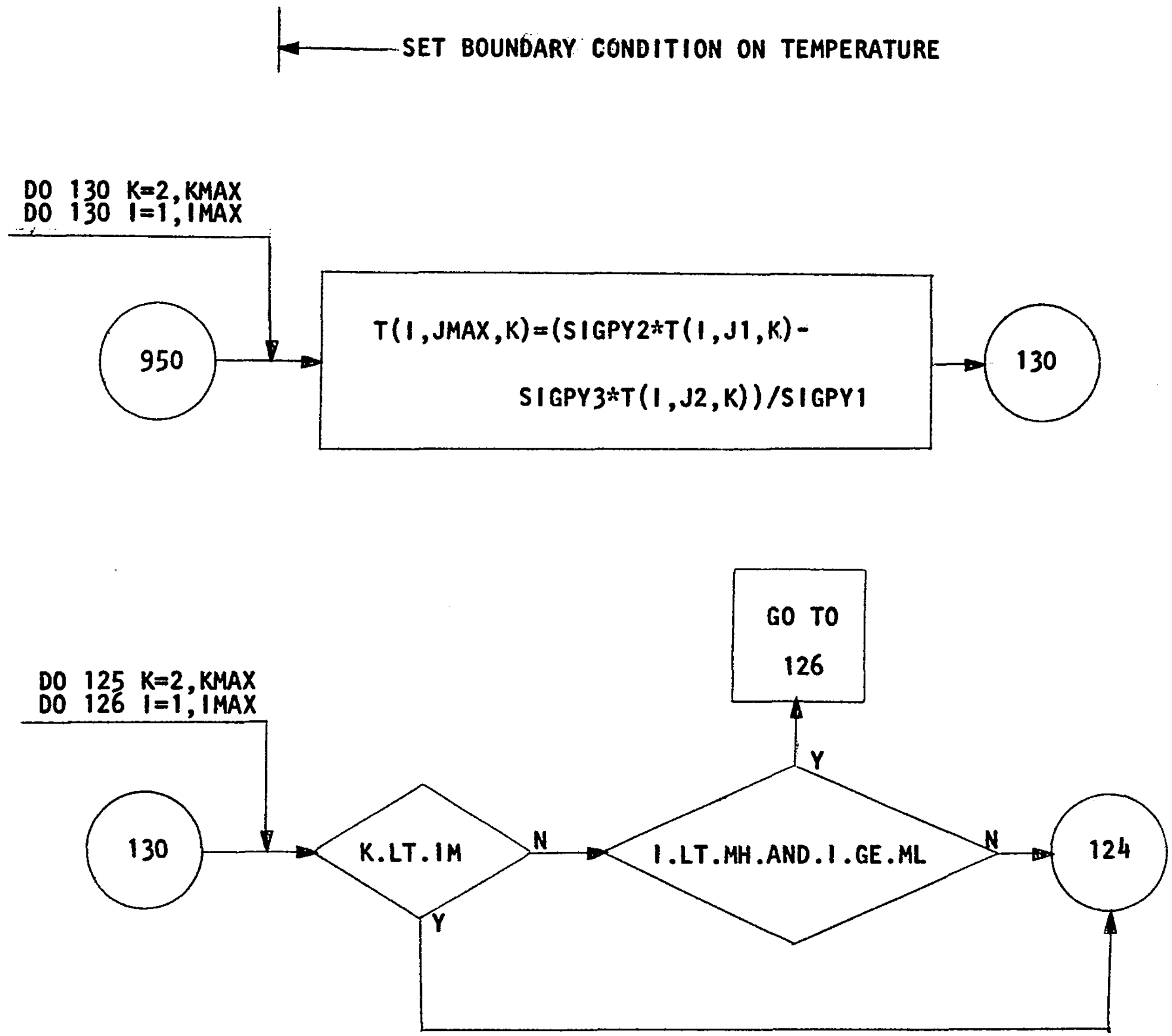


FIG 24cc FLOW CHART OF MAIN PROGRAM

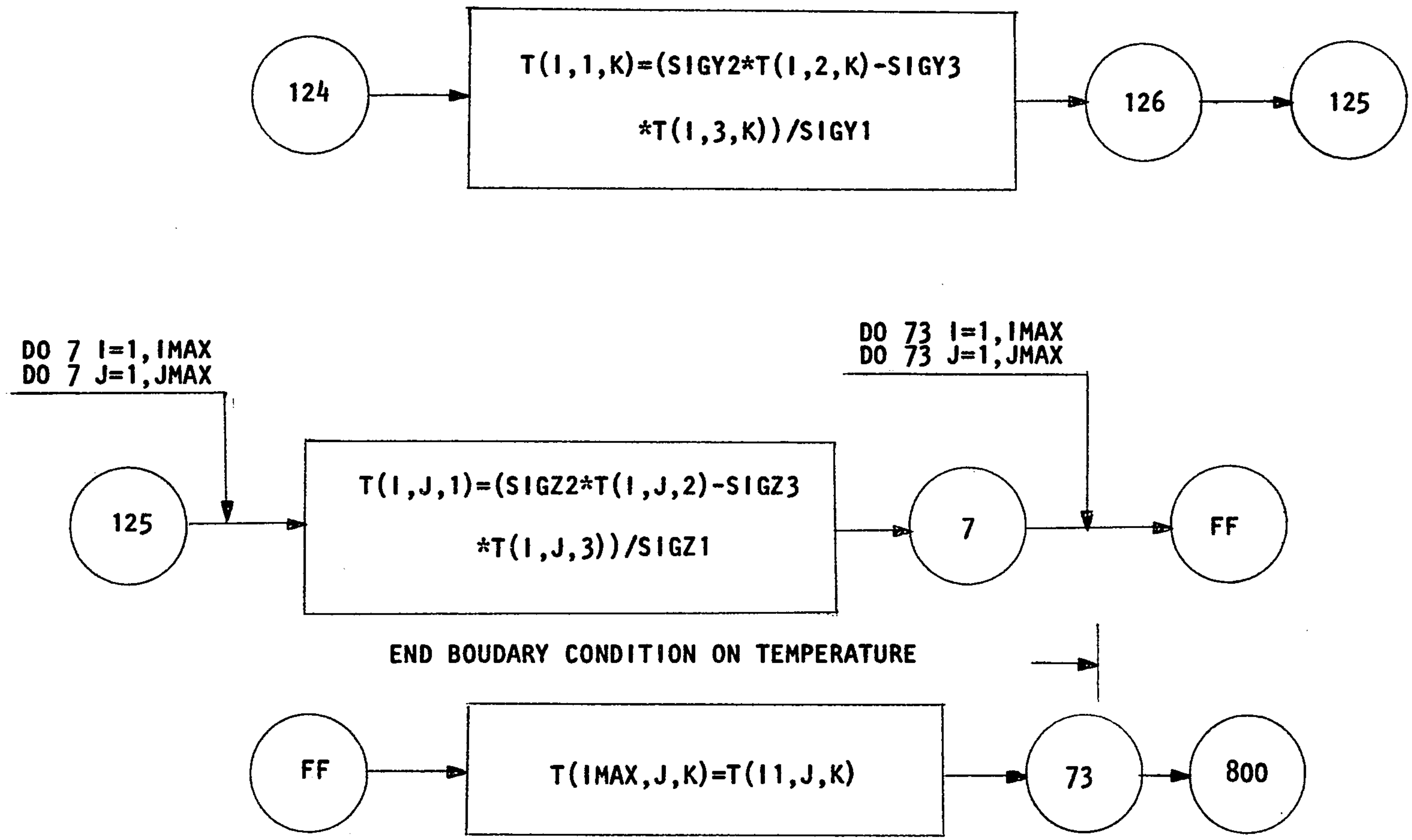


FIG 24dd FLOW CHART OF MAIN PROGRAM

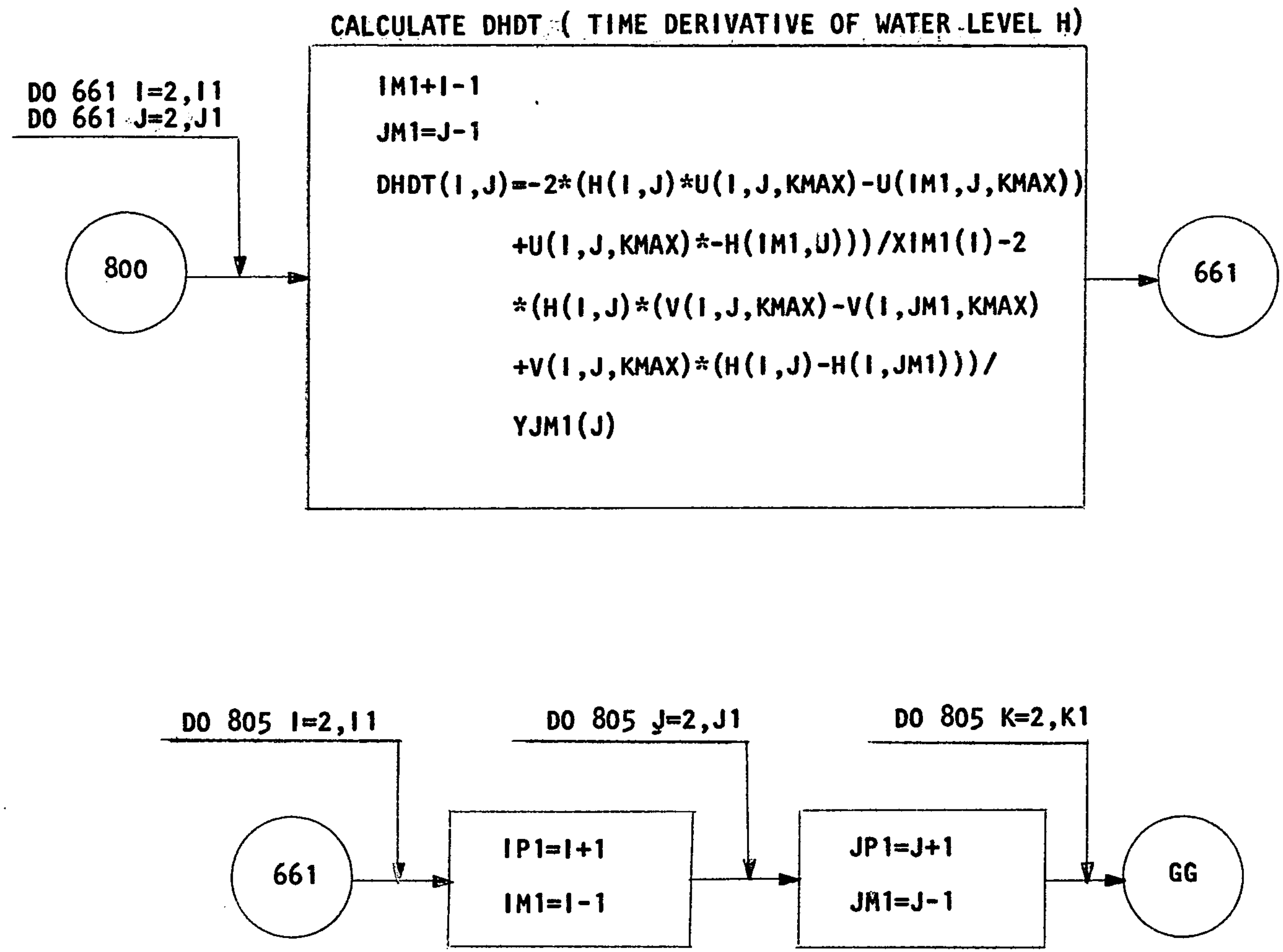


FIG 24ee FLOW CHART OF MAIN PROGRAM

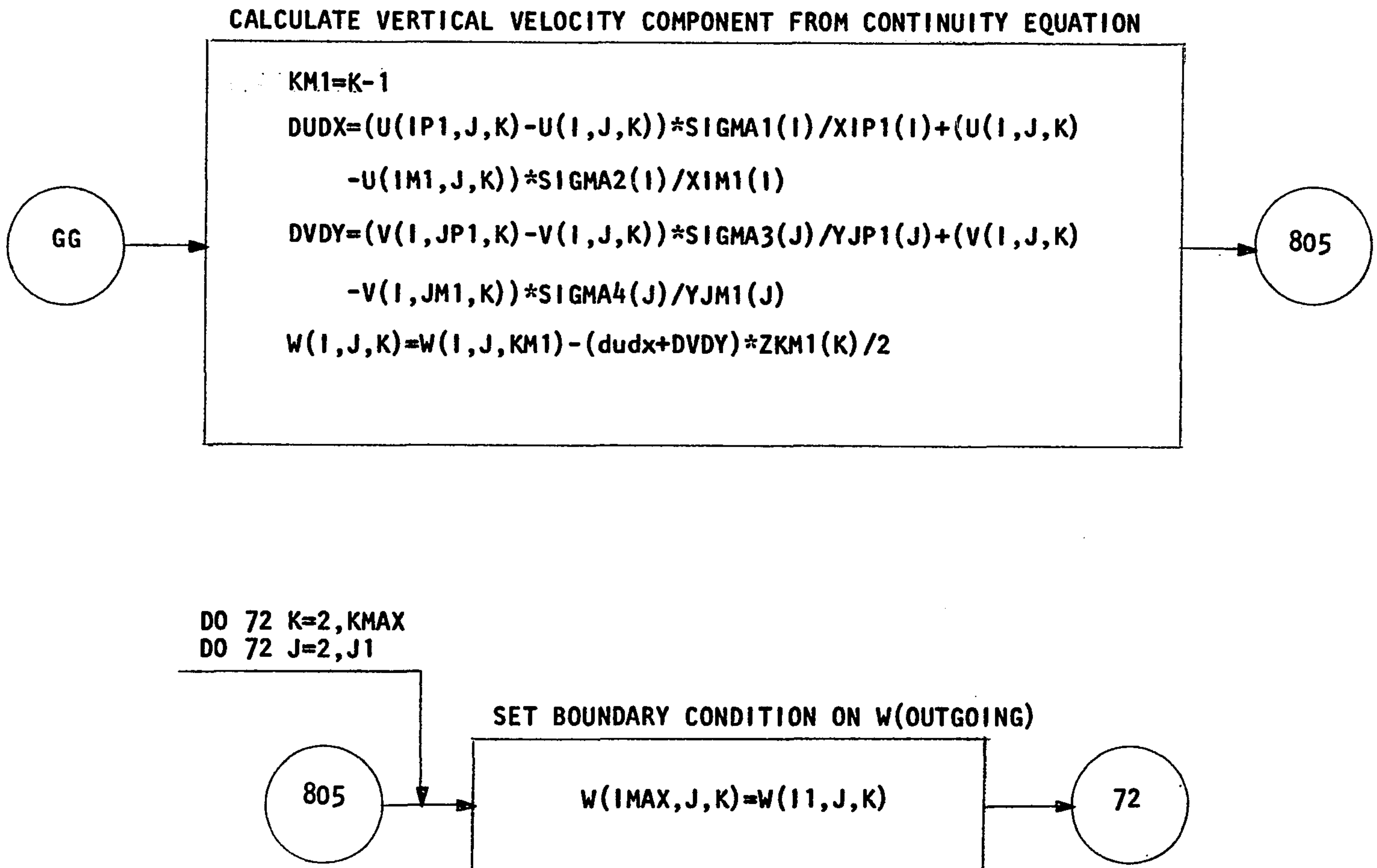


FIG 24ff FLOW CHART OF MAIN PROGRAM

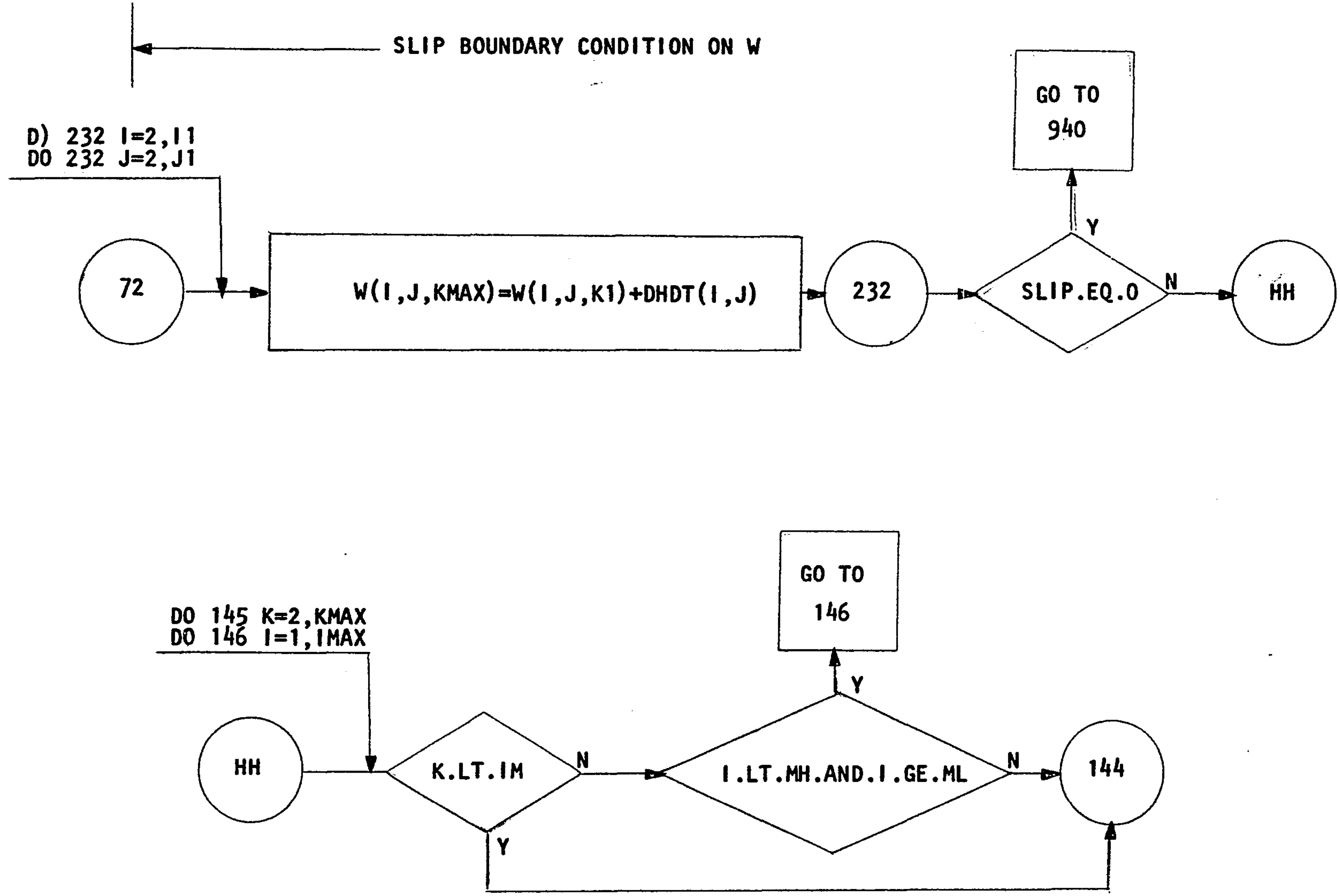


FIG 24gg FLOW CHART OF MAIN PROGRAM

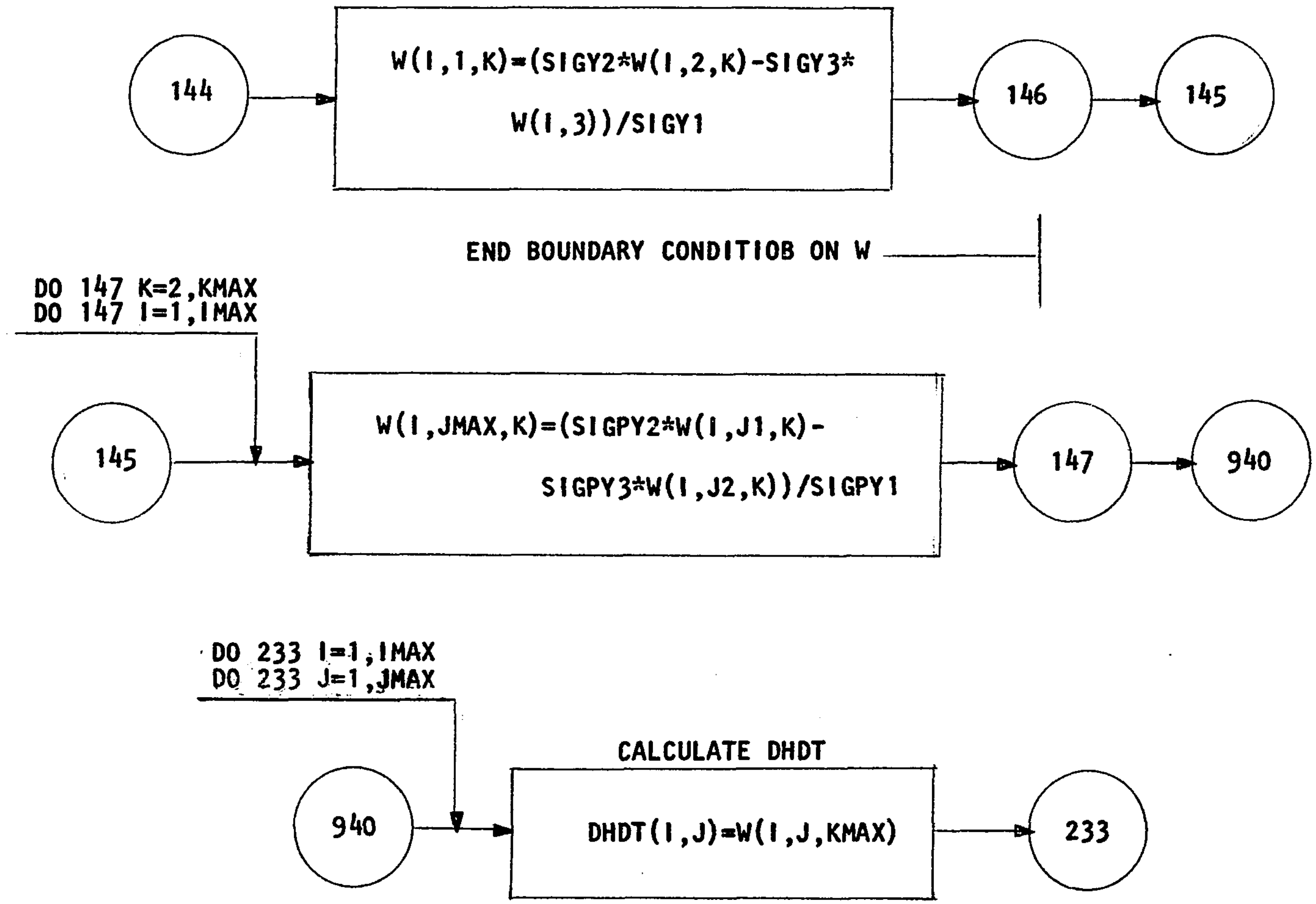


FIG 24hh FLOW CHART OF MAIN PROGRAM

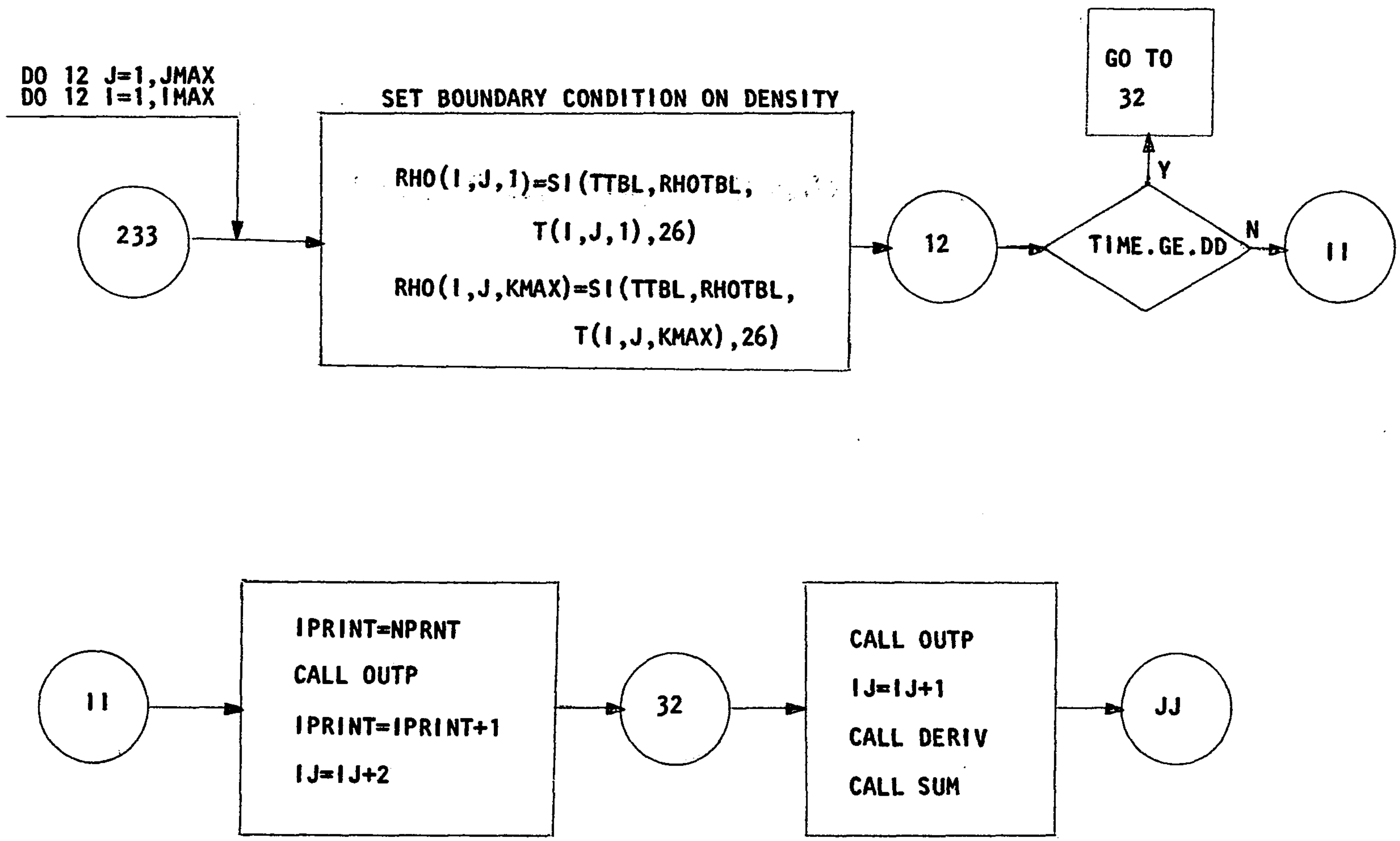


FIG 24ii FLOW CHART OF MAIN PROGRAM

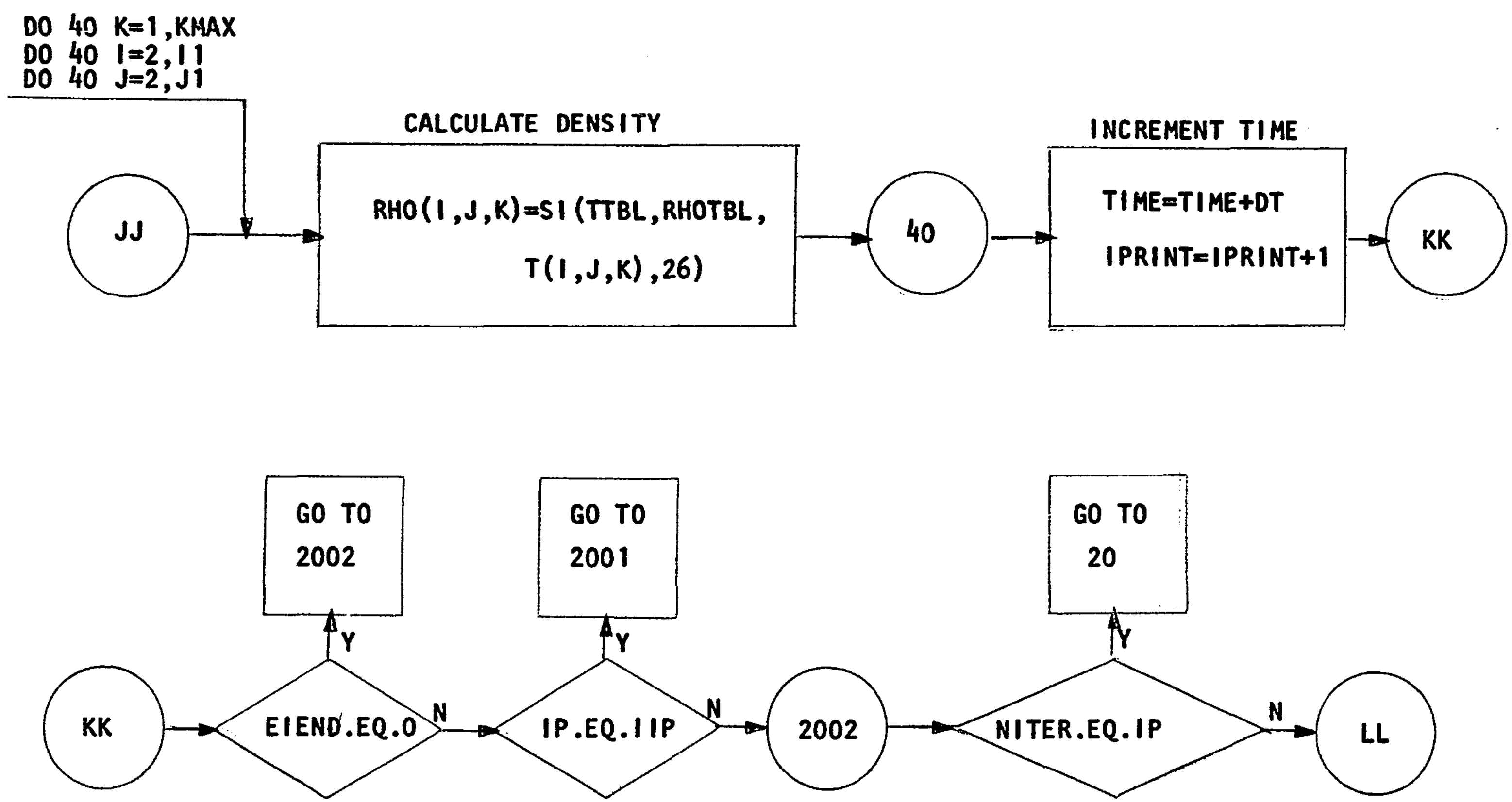


FIG 24jj FLOW CHART OF MAIN PROGRAM

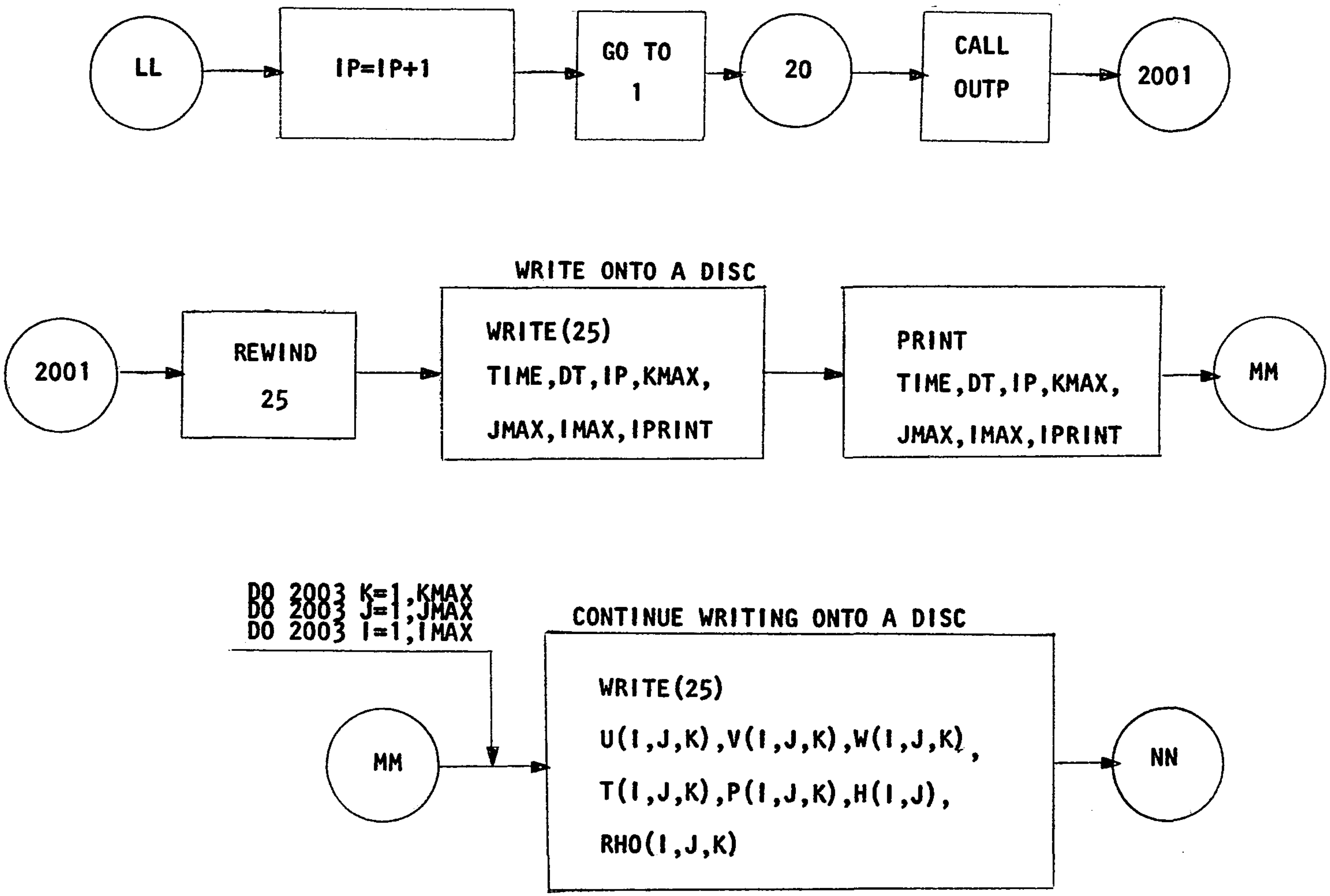


FIG 24kk FLOW CHART OF MAIN PROGRAM

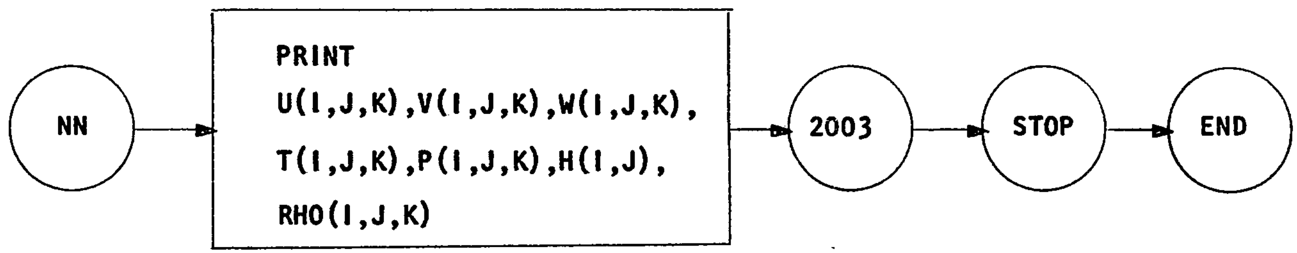


FIG2411 FLOW CHART OF MAIN PROGRAM

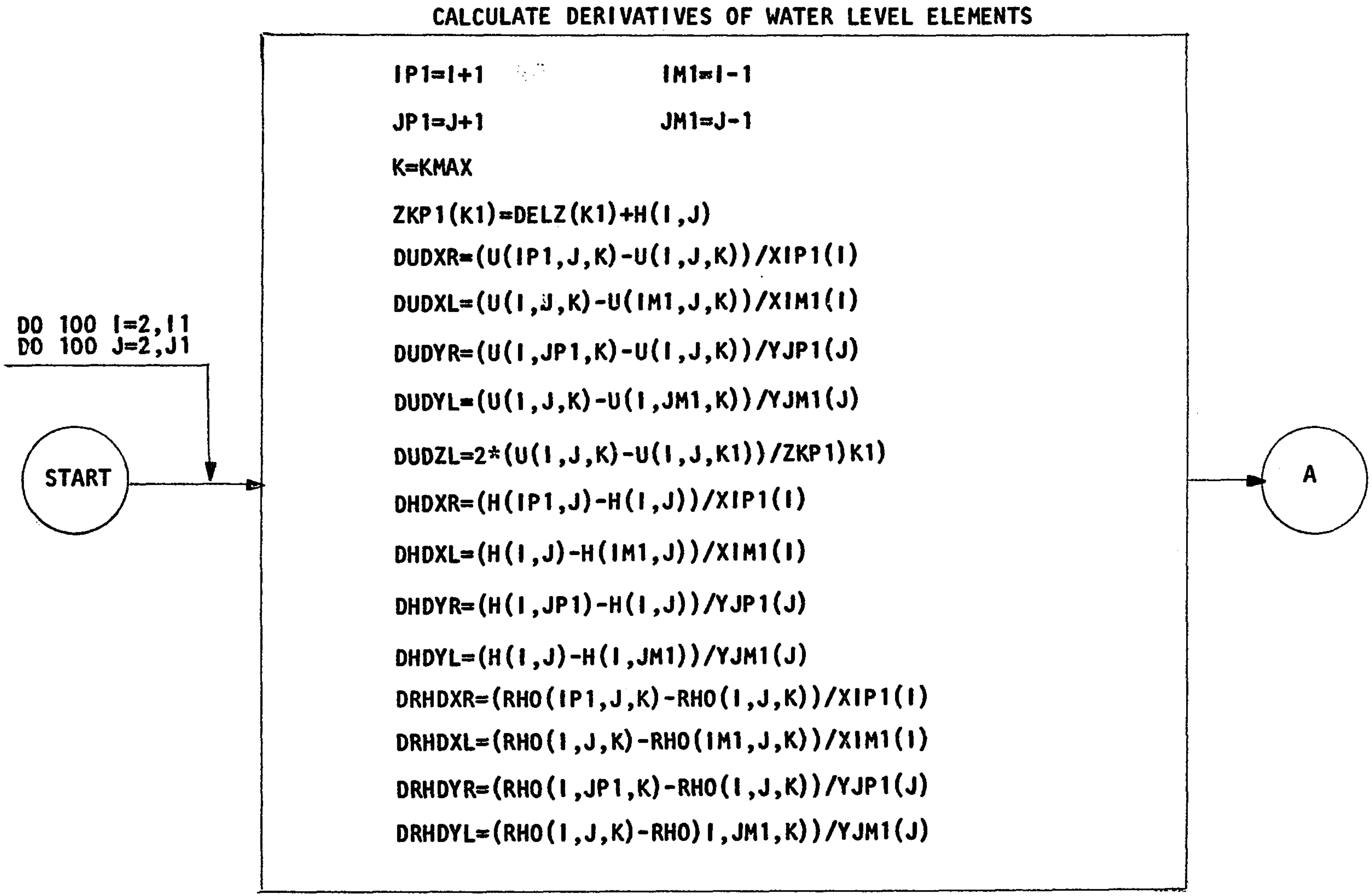


FIG 25a FLOW CHART OF SUBROUTINE DERIV

CALCULATE TIME DERIVATIVE OF U FOR WATER LEVEL ELEMENTS

(A) →

```
DUDT(I,J,K)=-U(I,J,K)
          *(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)
          -V(I,J,K)*(SIGMA3(J)*DUDYR+SIGMA4(J)*DUDYL)
          -H(I,J)*(SIGMA1(I)*DRHDXR+SIGMA2(I)*DRHDXL)/FO1
          +((SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)
          *(XNUR*(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)/H(I,J)
          -2*RHO(I,J,K)/FO1))
          +XNUR*8*((DUDXR-DUDXL)/XIPM1(I)
          +(DUDYR-DUDYL)/YJPM1(J))
          +XNUR*((SIGMA3(J)*DUDYR+SIGMA4(J)*DUDYL)
          *(SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL))/H(I,J)
          -ZNUR*DUDZL/H(I,J)
          +CFX*RHOA*WINDX*WINDX/(2*H(I,J))
```

→ (B)

FIG 25b FLOW CHART OF SUBROUTINE DERIV

CALCULATE TIME DERIVATIVE OF V FOR WATER LEVEL ELEMENTS

C →

```
DVDT(I,J,K)=-U(I,J,K)
        *(SIGMA1(I)*DVDXR+SIGMA2(I)*DVDXL)
        -V(I,J,K)*(SIGMA3(J)*DVDYR+SIGMA4(J)*DVDYL)
        -H(I,J)*(SIGMA3(J)*DRHDYR+SIGMA4(J)*DRHDYL)/FO1
        *XNUR*((SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)
        *(SIGMA1(I)*DVDXR+SIGMA2(I)*DVDXL)/H(I,J)
        *((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)
        *(XNUR*(SIGMA3(J)*DVDYR+SIGMA4(J)*DVDYL)/H(I,J)
        -2*RHO(I,J,K)/FO1
        +XNUR*8*((DVDXR-DVDXL)/XIPM1(I)
        +(DVDYR-DVDYL)/YJPM1(J))
        -ZNUR*DVDZL/H(I,J)
        +CFY*RHOA*WINDY*WINDY/(2*H(I,J))
```

→ D

FIG 25d FLOW CHART OF SUBROUTINE DERIV

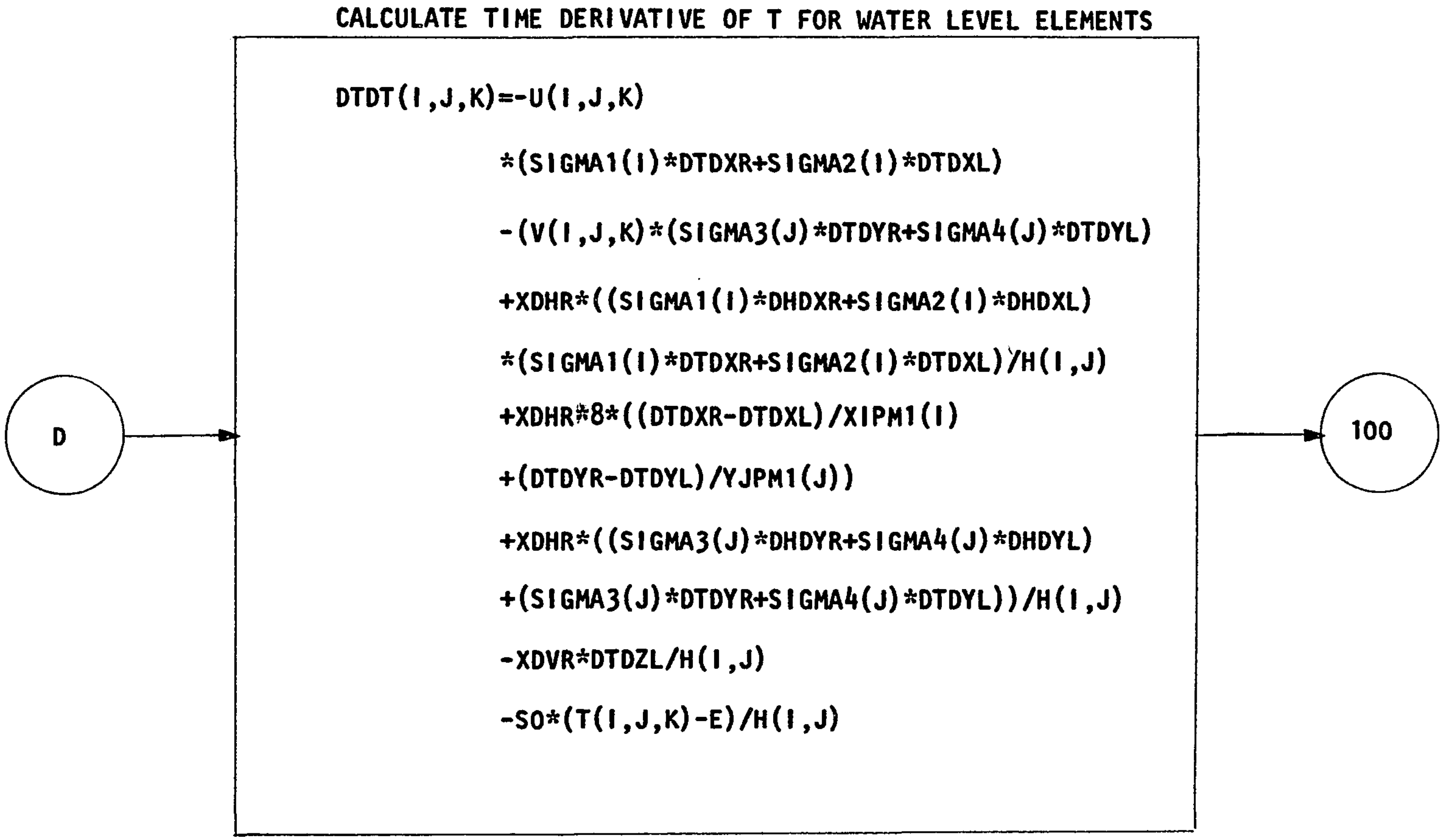


FIG 25e FLOW CHART OF SUBROUTINE DERIV

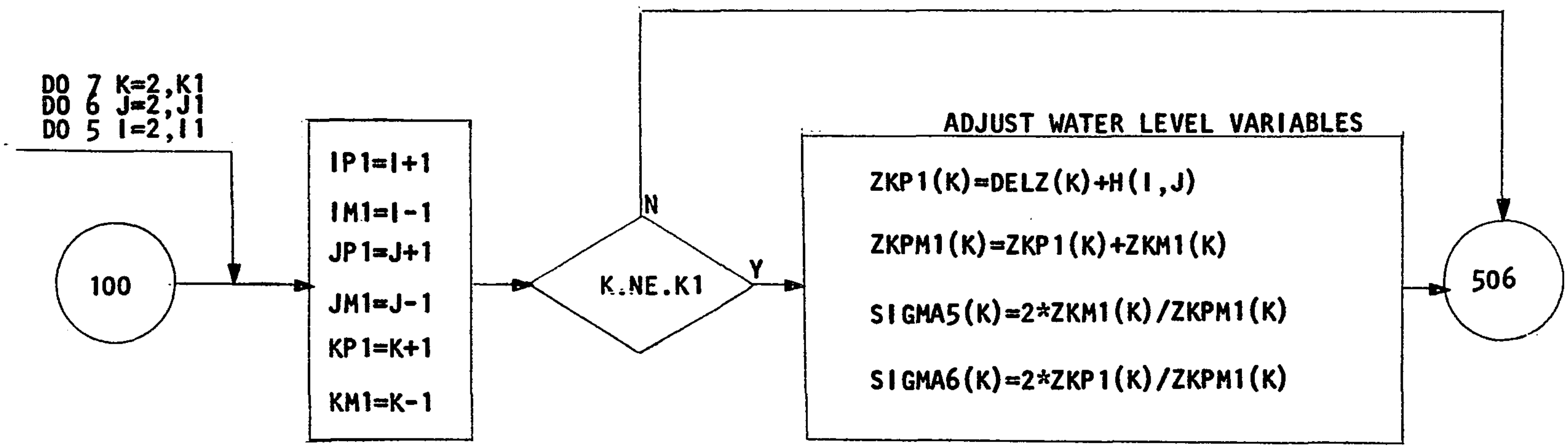


FIG 25f FLOW CHART OF SUBROUTINE DERIV

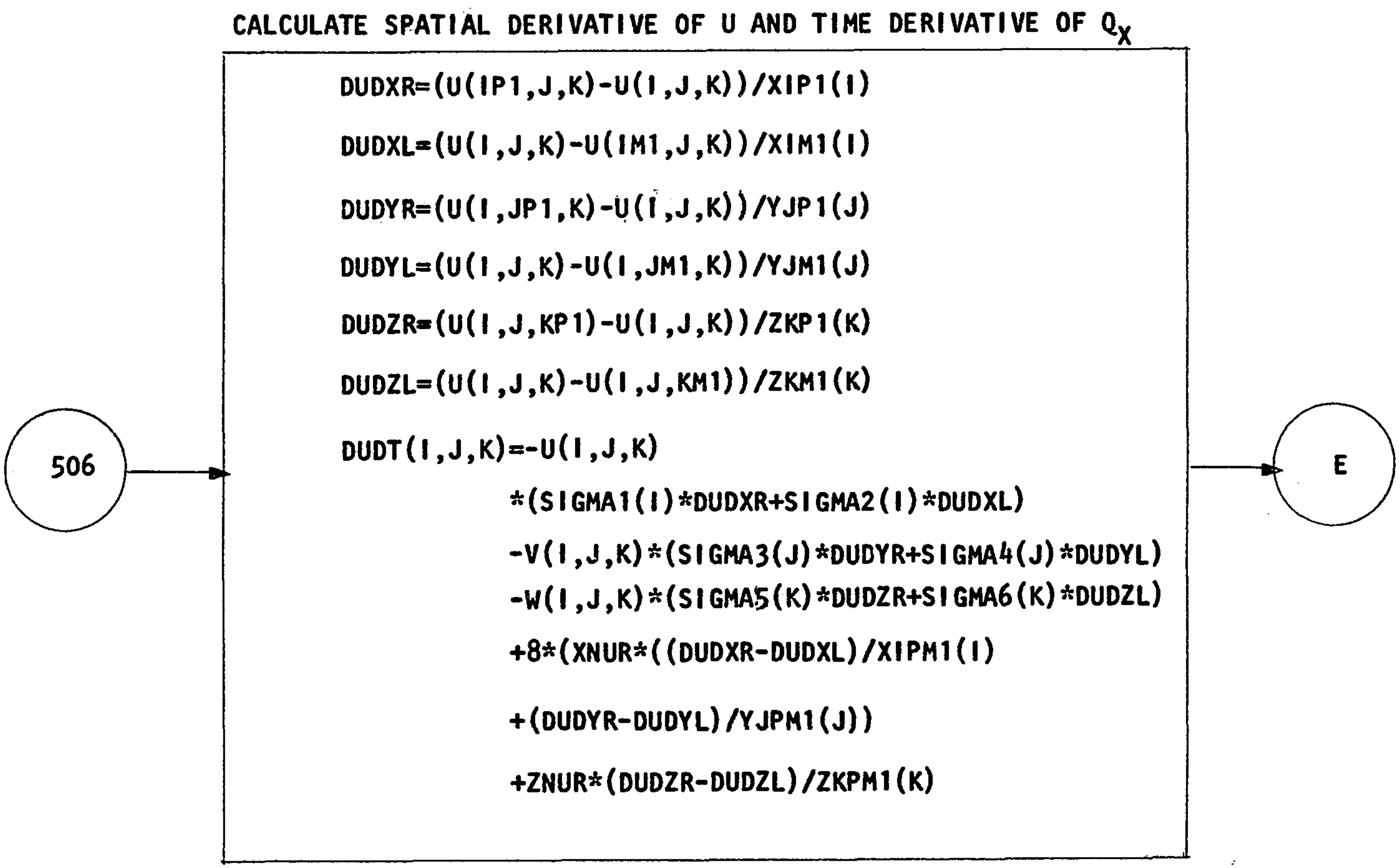


FIG 25g FLOW CHART OF SUBROUTINE DERIV

CALCULATE SPATIAL DERIVATIVE OF V AND TIME DERIVATIVE OF Q_Y

E →

```
DVDXR=(V(IP1,J,K)-V(I,J,K))/XIP1(I)
DVDXL=(V(I,J,K)-V(IM1,J,K))/XIM1(I)
DVDYR=(V(I,JP1,K)-V(I,J,K))/YJP1(J)
DVDYR=(V(I,JP1,K)-V(I,J,K))/YJP1(J)
DVDYL=(V(I,J,K)-V(I,JM1,K))/YJM1(J)
DVDZR=(V(I,J,KP1)-V(I,J,K))/ZKP1(K)
DVDZL=(V(I,J,K)-V(I,J,KM1))/ZKM1(K)
DVDT(I,J,K)=-U(I,J,K)
            *(SIGMA1(I)*DVDXR+SIGMA2(I)*DVDXL)
            -V(I,J,K)*(SIGMA3(J)*DVDYR+SIGMA4(J)*DVDYL)
            -W(I,J,K)*(SIGMA5(K)*DVDZR+SIGMA6(K)*DVDZL)
            +8*(XNUR*((DVDXR-DVDXL)/XIPM1(I)
            +(DVDYR-DVDYL)/YJPM1(J)
            +ZNUR*(DVDZR-DVDZL)/ZKPM1(K))
```

→ F

FIG 25h FLOW CHART OF SUBROUTINE DERIV

CALCULATE SPATIAL DERIVATIVE OF W AND TIME DERIVATIVE OF Q_Z

F →

```
DWDXR=(W(IP1,J,K)-W(I,J,K))/XIP1(I)

DWDXL=(W(I,J,K)-W(IM1,J,K))/XIM1(I)

DWDYR=(W(I,JP1,K)-W(I,J,K))/YJP1(J)

DWDYL=(W(I,J,K)-W(I,JM1,K))/YJM1(J)

DWDZR=(W(I,J,KP1)-W(I,J,K))/ZKP1(K)

DWDZL=(W(I,J,K)-W(I,J,KM1))/ZKM1(K)

DWDT(I,J,K)=-U(I,J,K)

         *(SIGMA1(I)*DWDXR+SIGMA2(I)*DWDXL)

         -V(I,J,K)*(SIGMA3(J)*DWDYR+SIGMA4(J)*DWDYL)

         -W(I,J,K)*2*DWDZL

         +8*(XNUR*((DWDXR-DWDXL)/XIPM1(I)

         +(DWDYR-DWDYL)/YJPM1(J))

         +ZNUR*(DWDZR-DWDZL)/ZKPM1(K))

         -RHO(I,J,K)/F01
```

→ G

FIG 25i FLOW CHART OF SUBROUTINE DERIV

CALCULATE SPATIAL DERIVATIVE OF T AND TIME DERIVATIVE OF T

```
(G) ---->

DTDXR=(T(IP1,J,K)-T(I,J,K))/XIP1(I)
DTDXL=(T(I,J,K)-T(IM1,J,K))/XIM1(I)
DTDYR=(T(I,JP1,K)-T(I,J,K))/YJP1(J)
DTDYL=(T(I,J,K)-T(I,JM1,K))/YJM1(J)
DTDZR=(T(I,J,KP1)-T(I,J,K))/ZKP1(K)
DTDZL=(T(I,J,K)-T(I,J,KM1))/ZKM1(K)
DTDT(I,J,K)=-U(I,J,K)
        *(SIGMA1(I)*DTDXR+SIGMA2(I)*DTDXL)
        -v(I,J,K)*(SIGMA3(J)*DTDYR+SIGMA4(J)*DTDYL)
        -W(I,J,K)*(SIGMA5(K)*DTDZR+SIGMA6(K)*DTDZL)
        +8*XDHR*((DTDXR-DTDXL)/XIPM1(I)
        +(DTDYR-DTDYL)/YJPM1(J))
        +XDVR*((DTDZR-DTDZL)/ZKPM1(K))

----> (H)
```

FIG 25j FLOW CHART OF SUBROUTINE DERIV

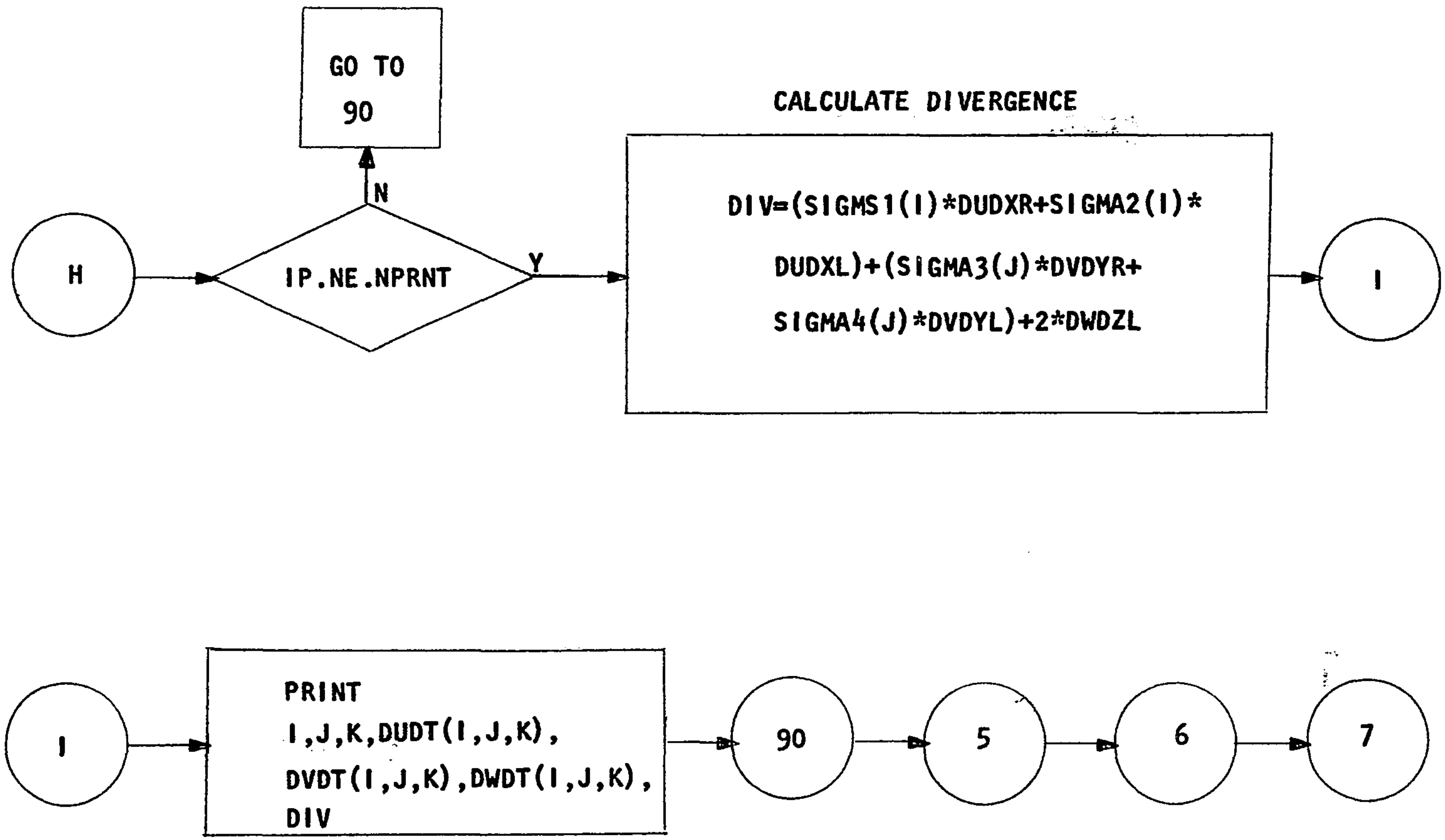


FIG 25k FLOW CHART OF SUBROUTINE DERIV

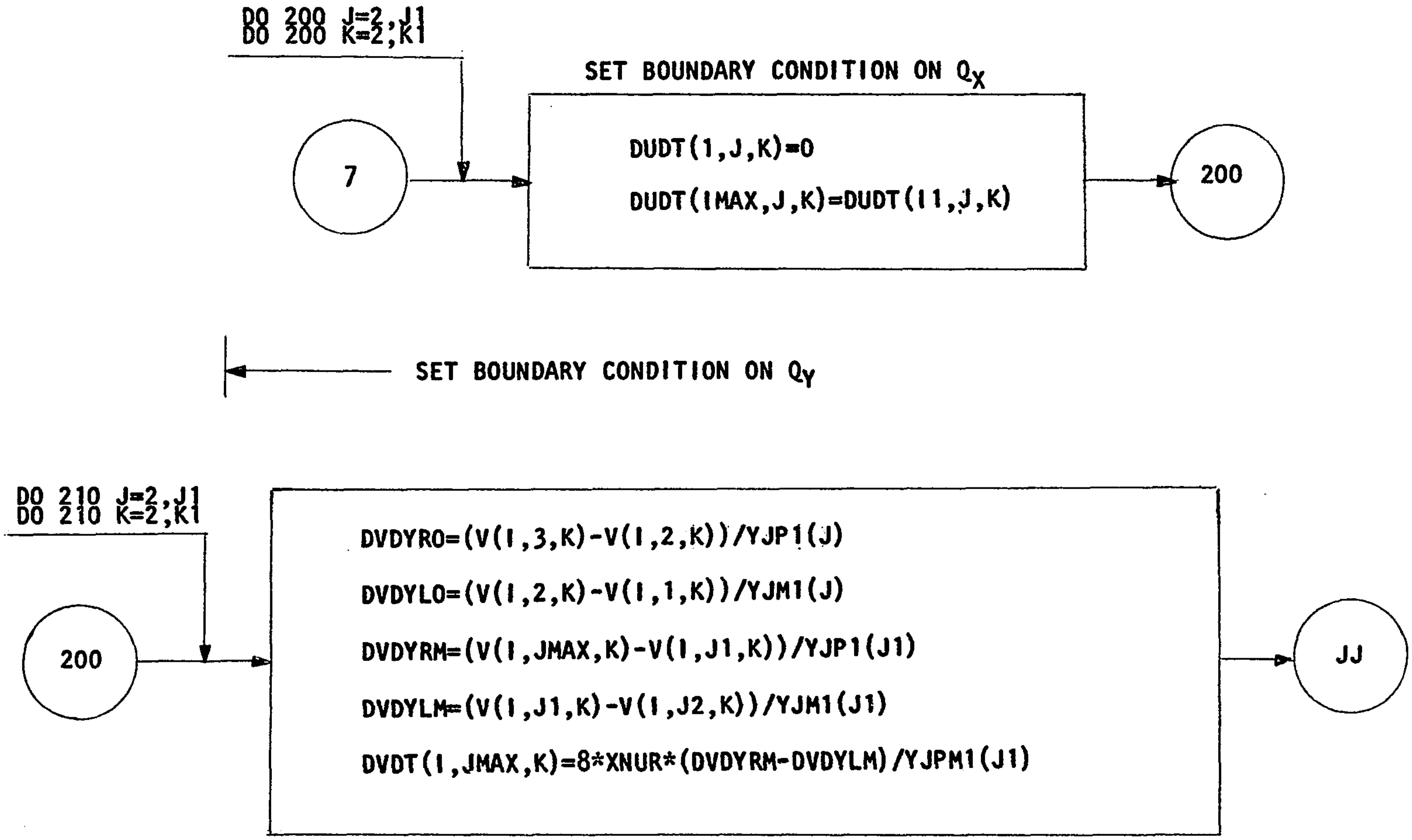


FIG 251 FLOW CHART OF SUBROUTINE DERIV

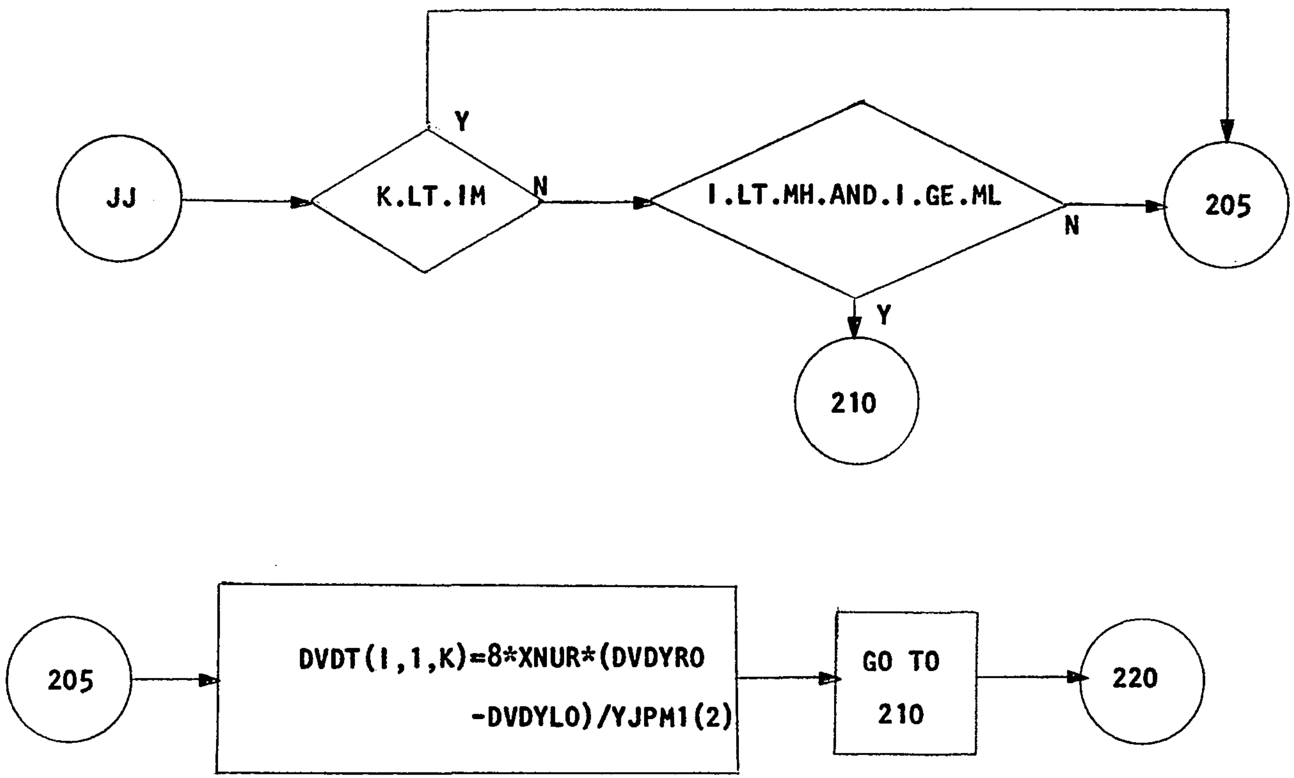


FIG 25m FLOW CHART OF SUBROUTINE DERIV

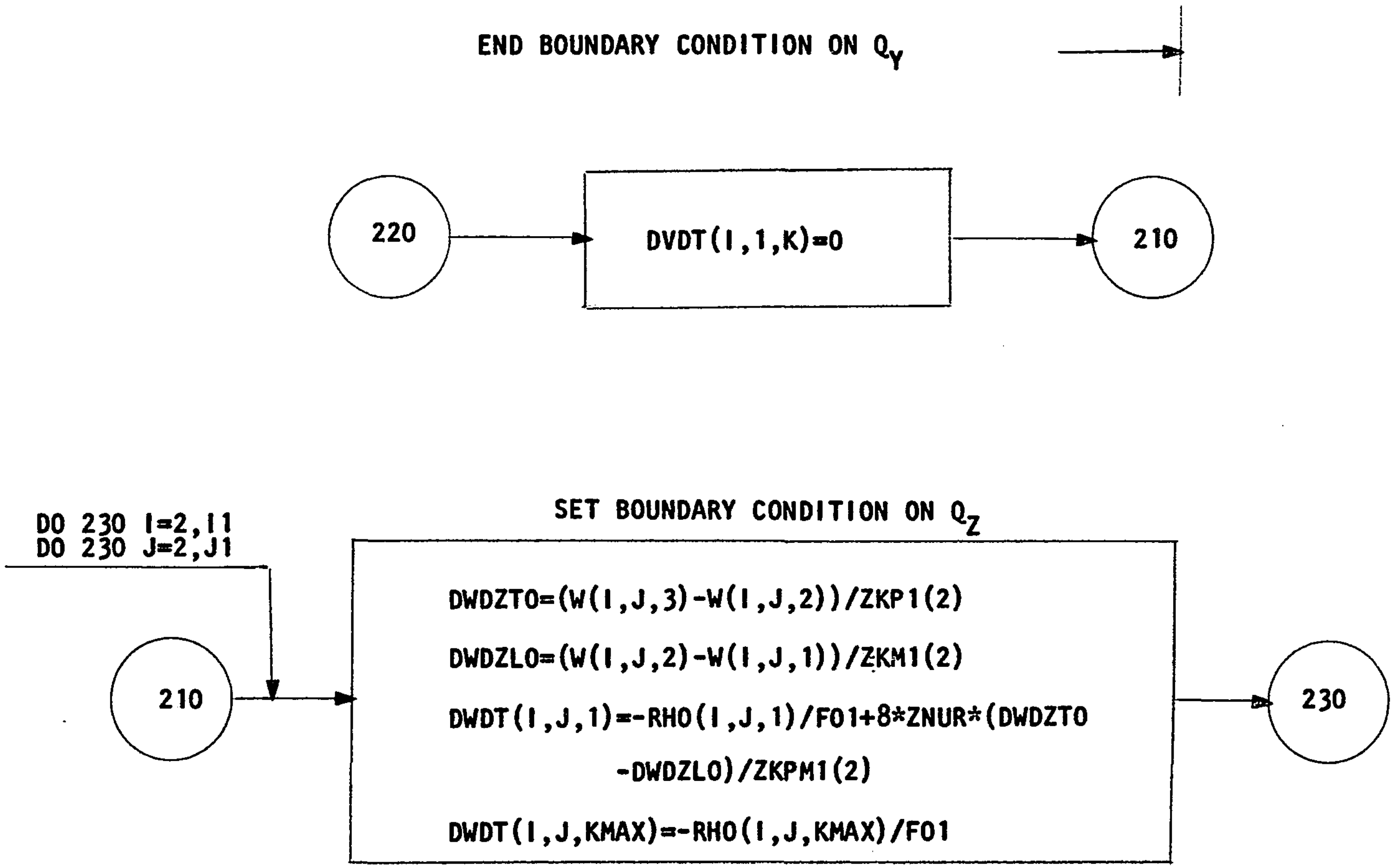


FIG 25n FLOW CHART OF SUBROUTINE DERIV

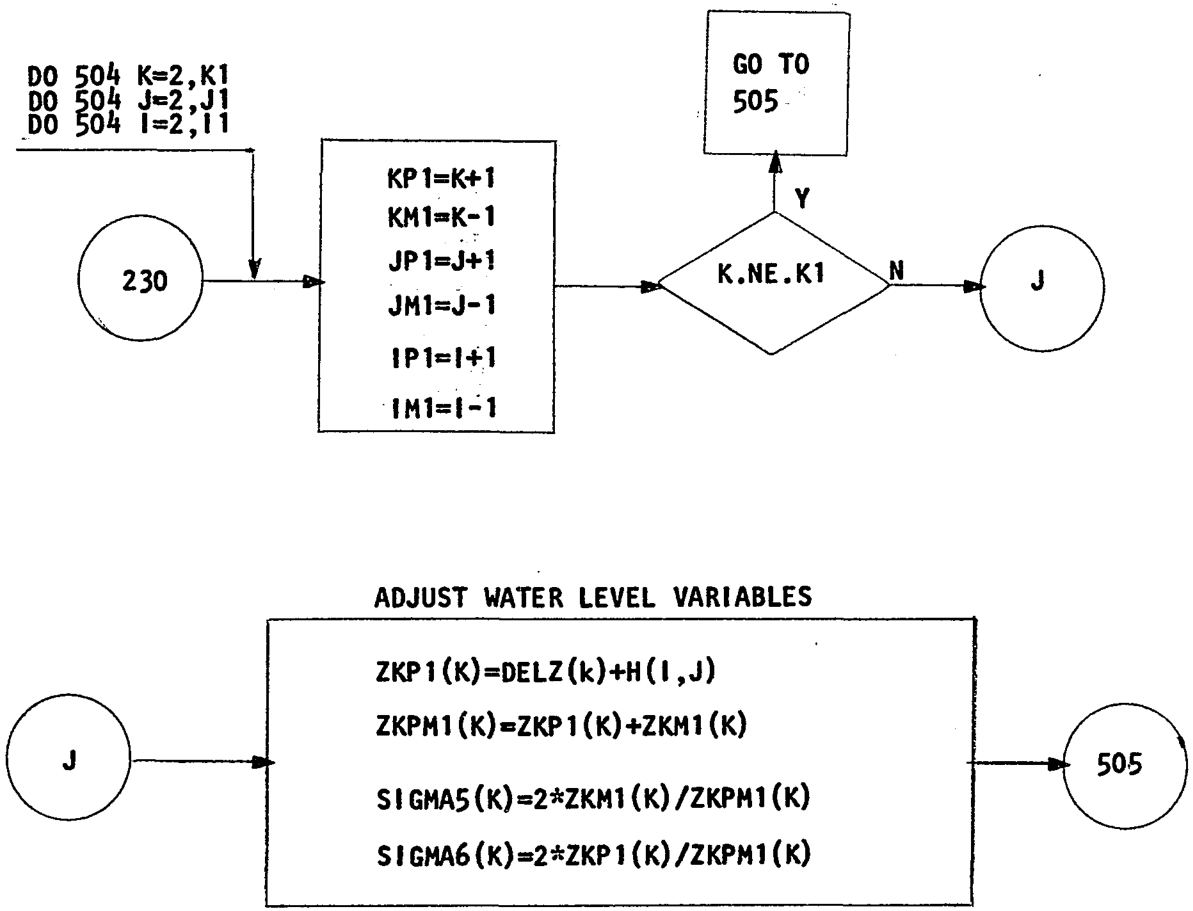


FIG 25o FLOW CHART OF SUBROUTINE DERIV

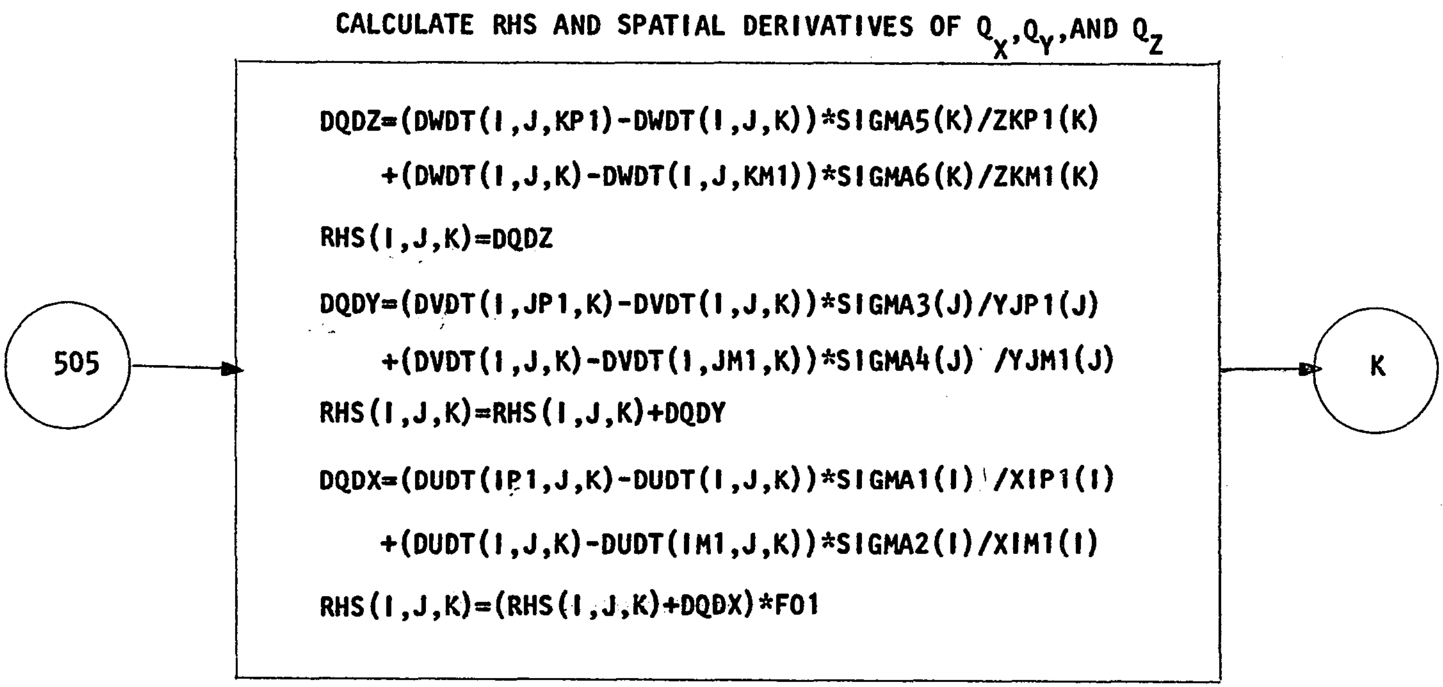


FIG 25p FLOW CHART OF SUBROUTINE DERIV

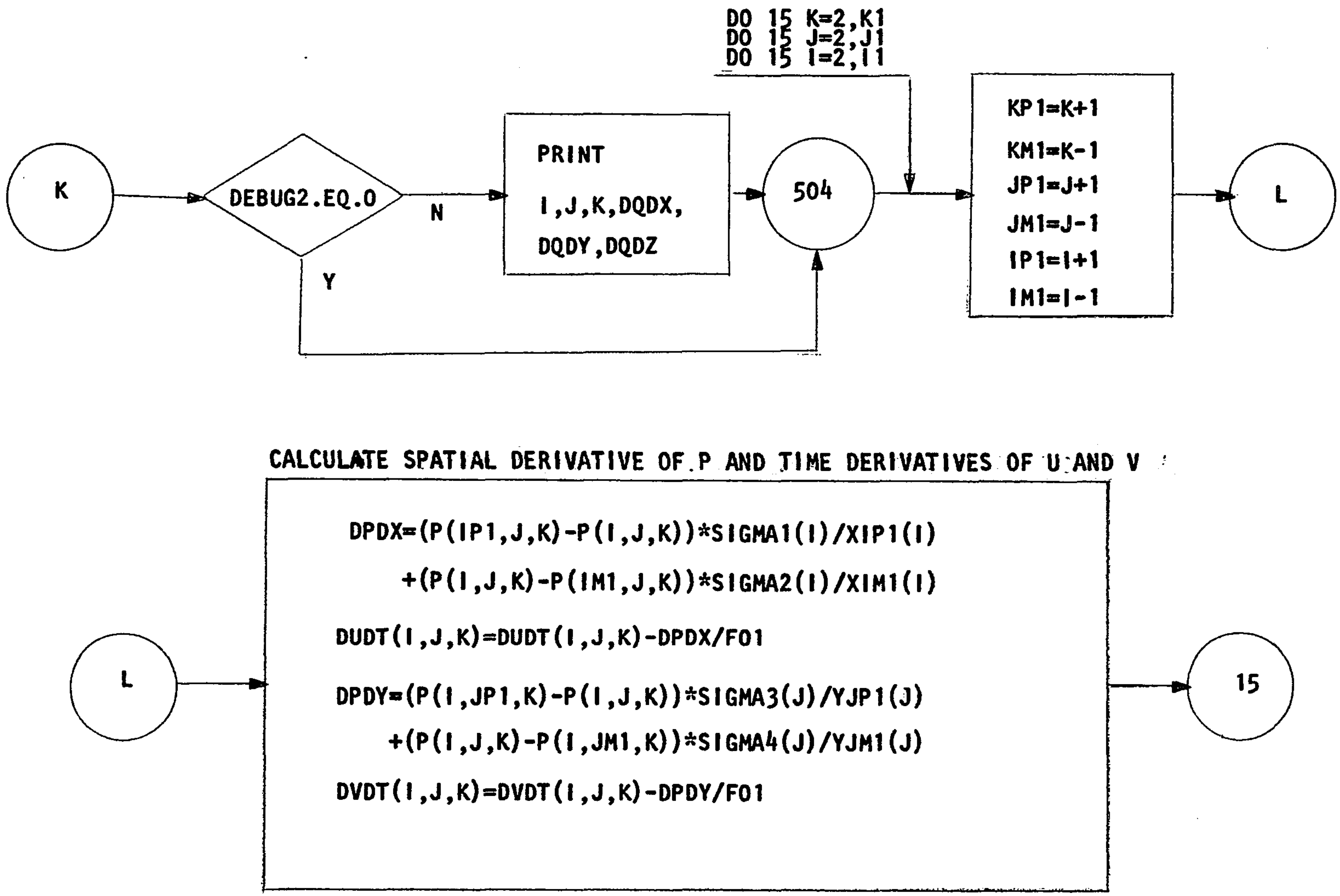


FIG 25q FLOW CHART OF SUBROUTINE DERIV

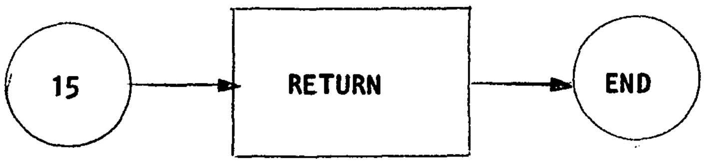


FIG 25r FLOW CHART OF SUBROUTINE DERIV

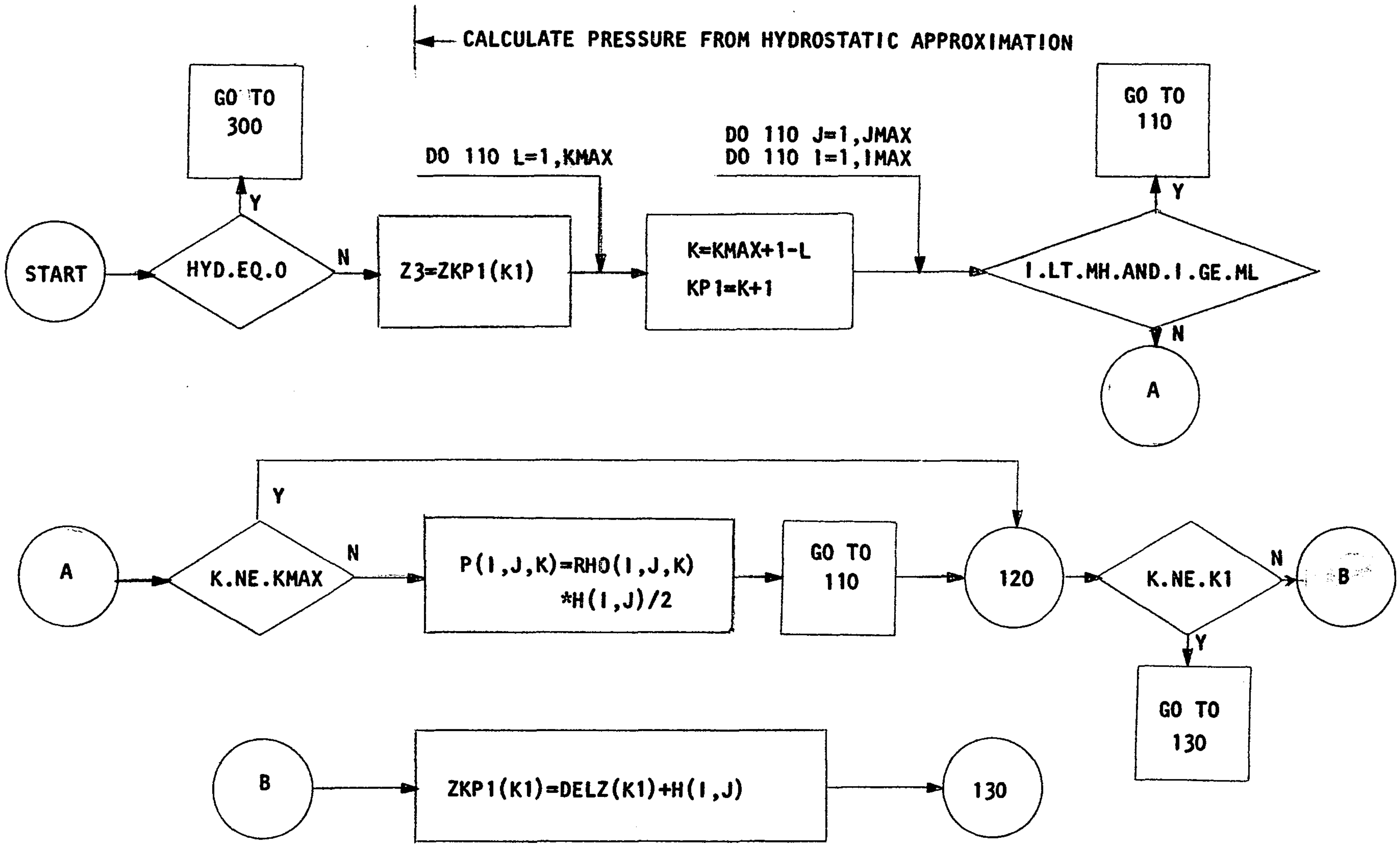


FIG 26a FLOW CHART OF SUBROUTINE PRESS

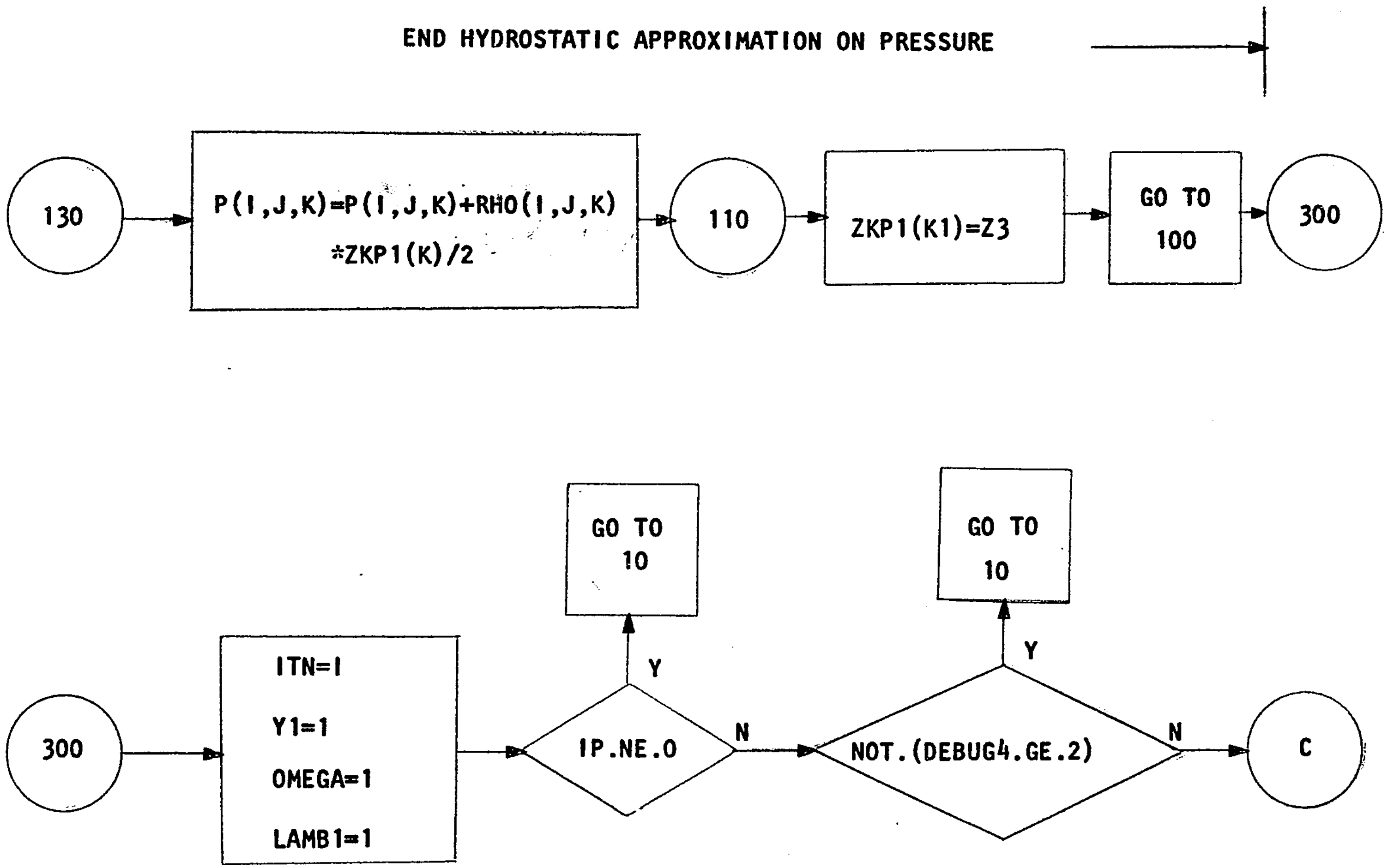


FIG 26b FLOW CHART OF SUBROUTINE PRESS

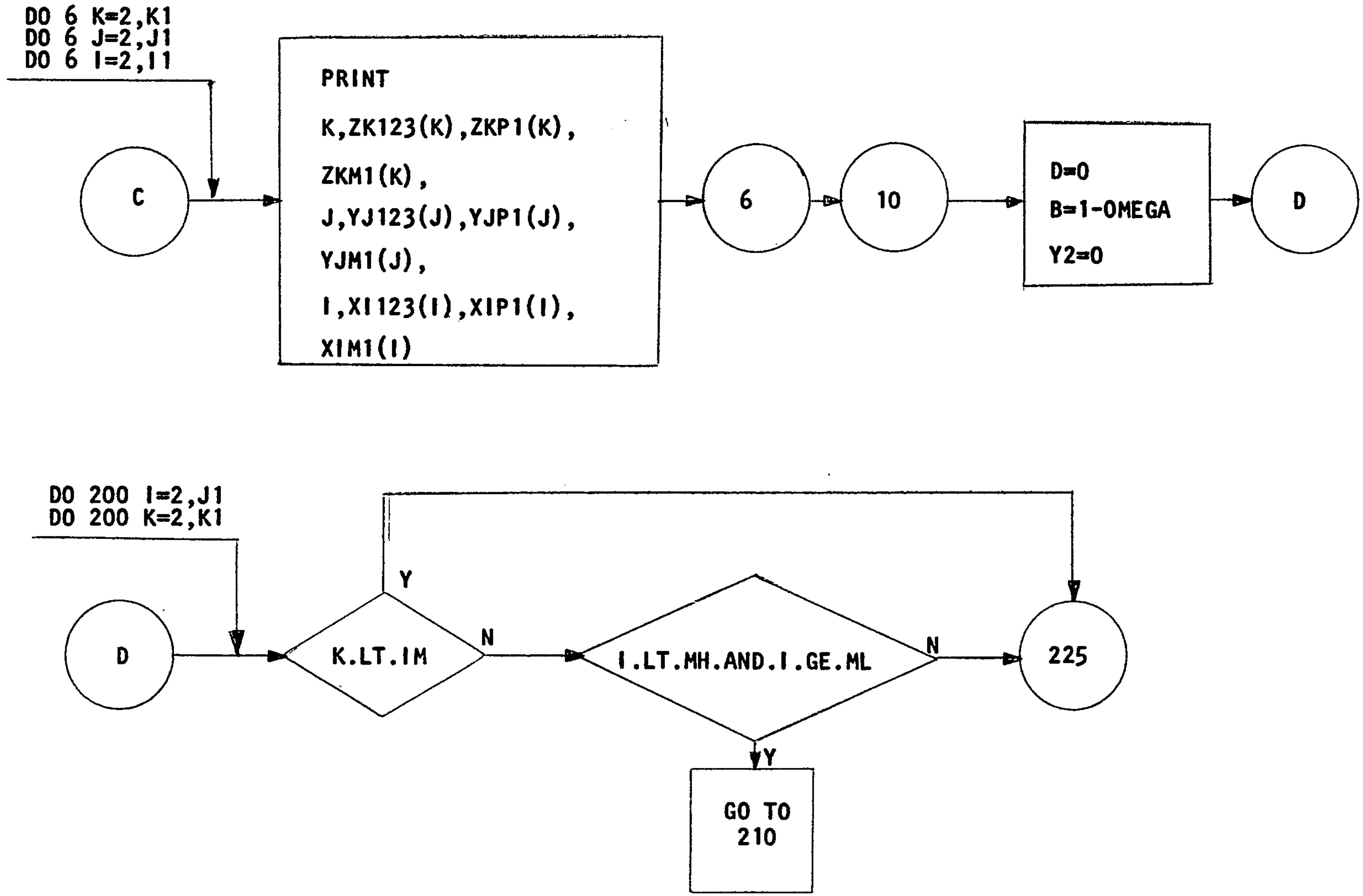


FIG 26c FLOW CHART OF SUBROUTINE PRESS

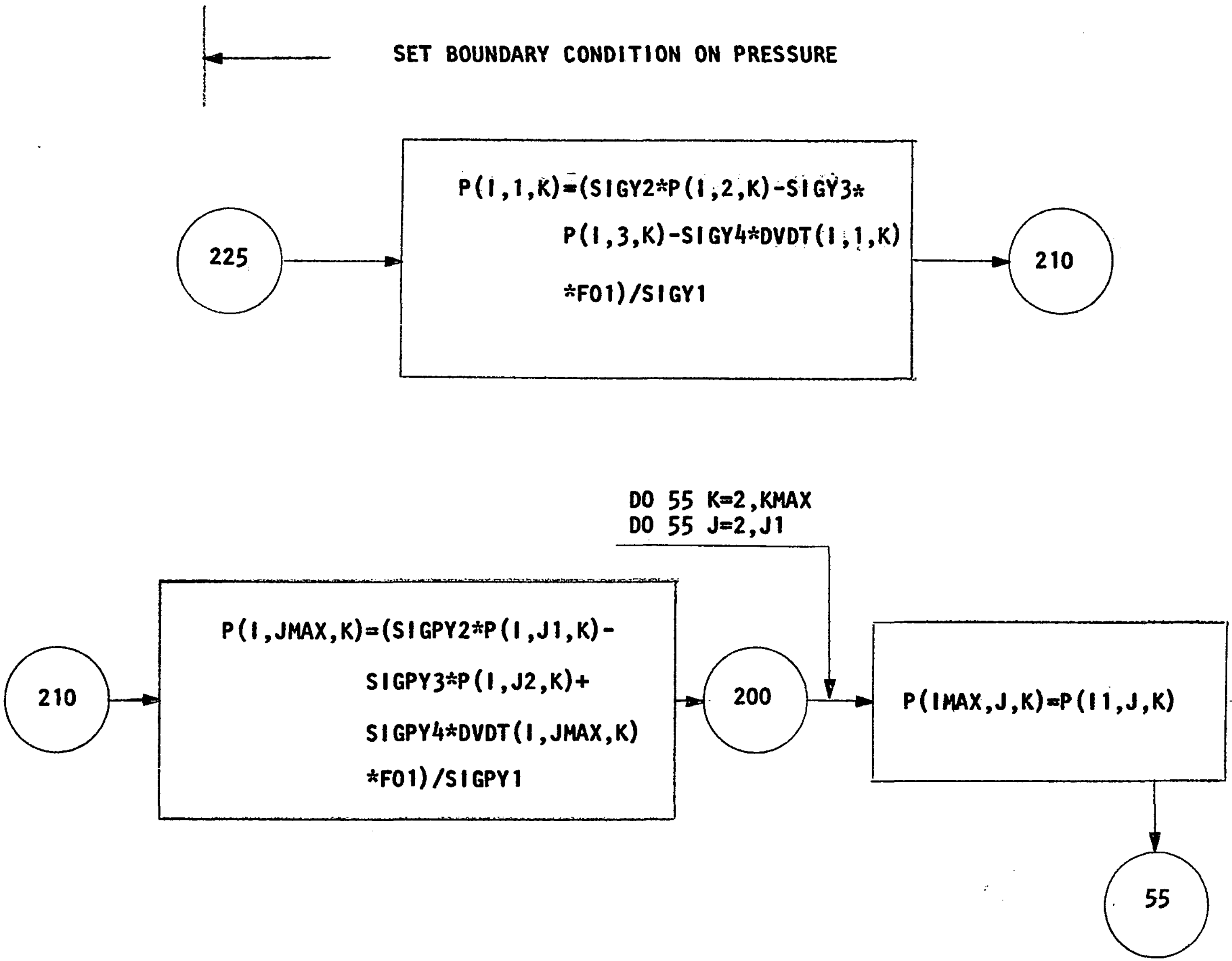


FIG 26d FLOW CHART OF SUBROUTINE PRESS

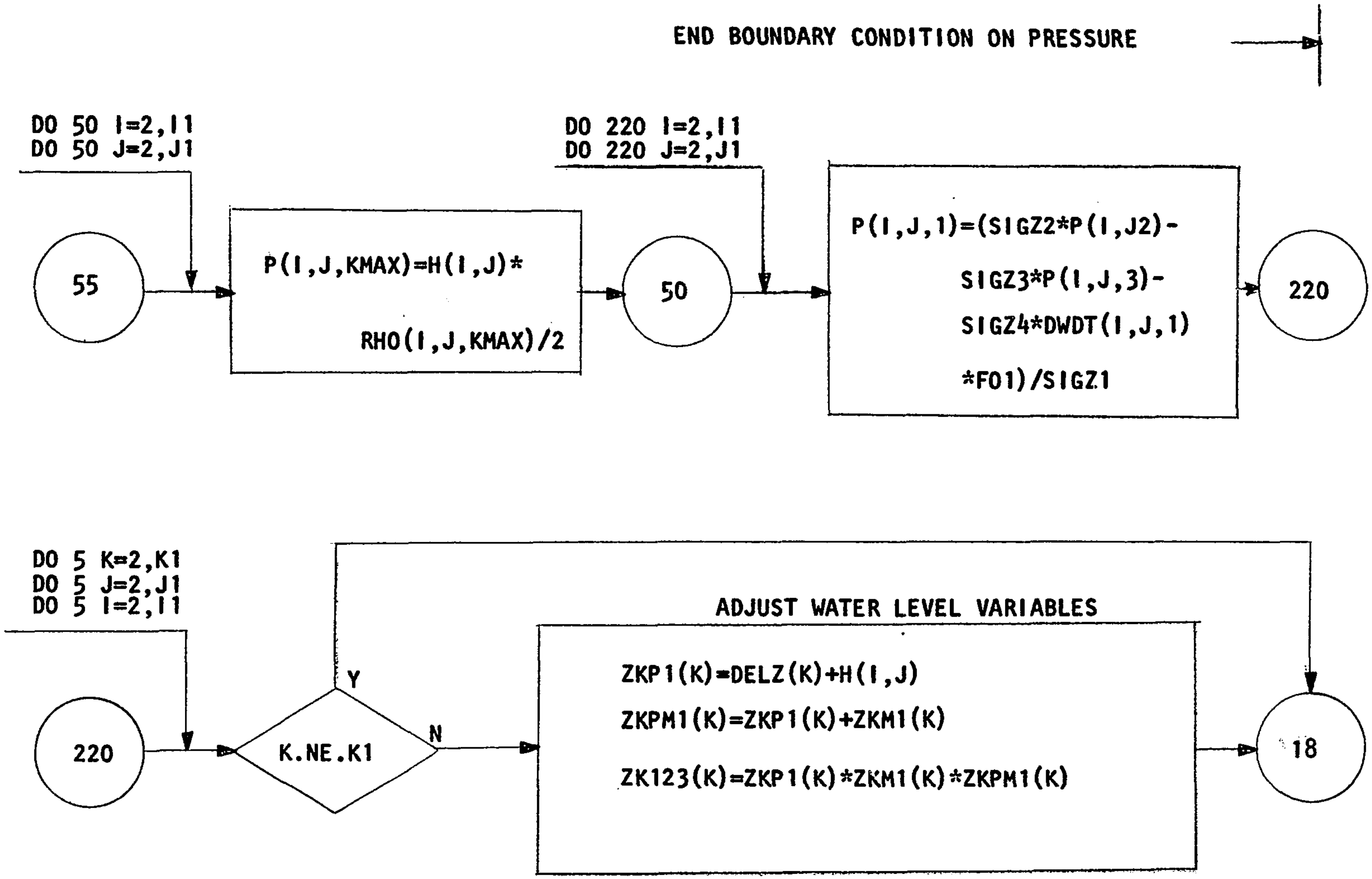


FIG 26e FLOW CHART OF SUBROUTINE PRESS

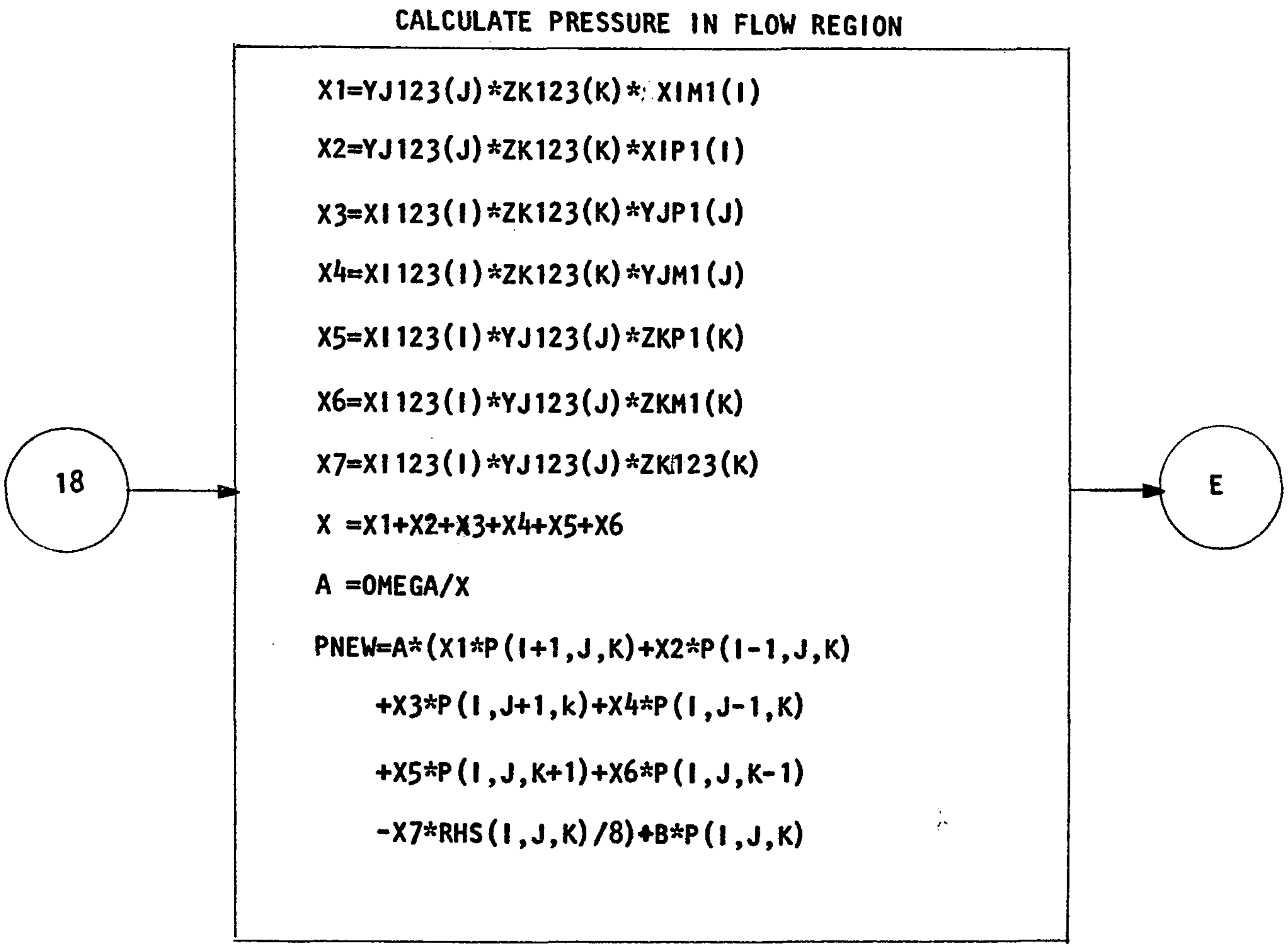


FIG 26f FLOW CHART OF SUBROUTINE PRESS

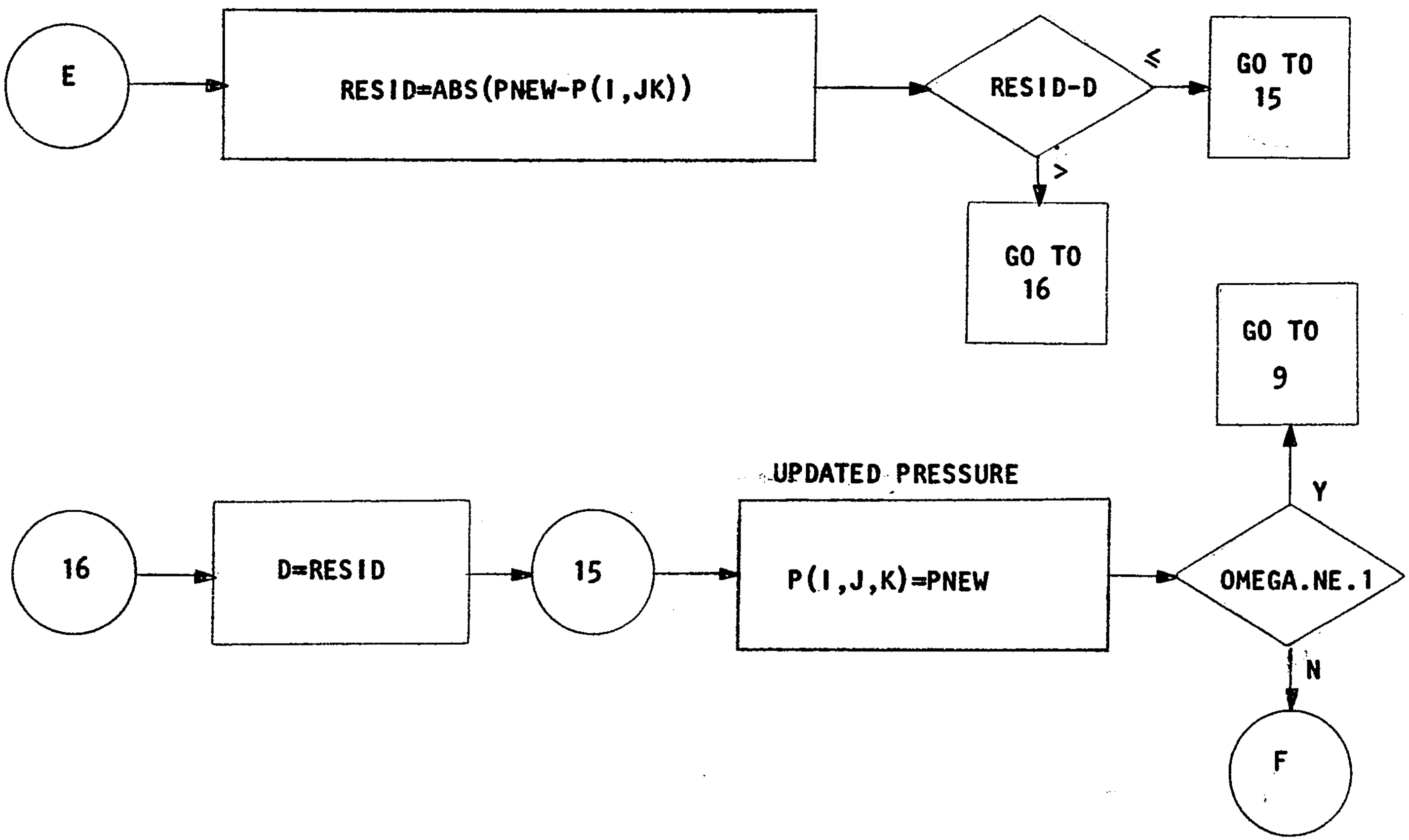


FIG 26g FLOW CHART OF SUBROUTINE PRESS

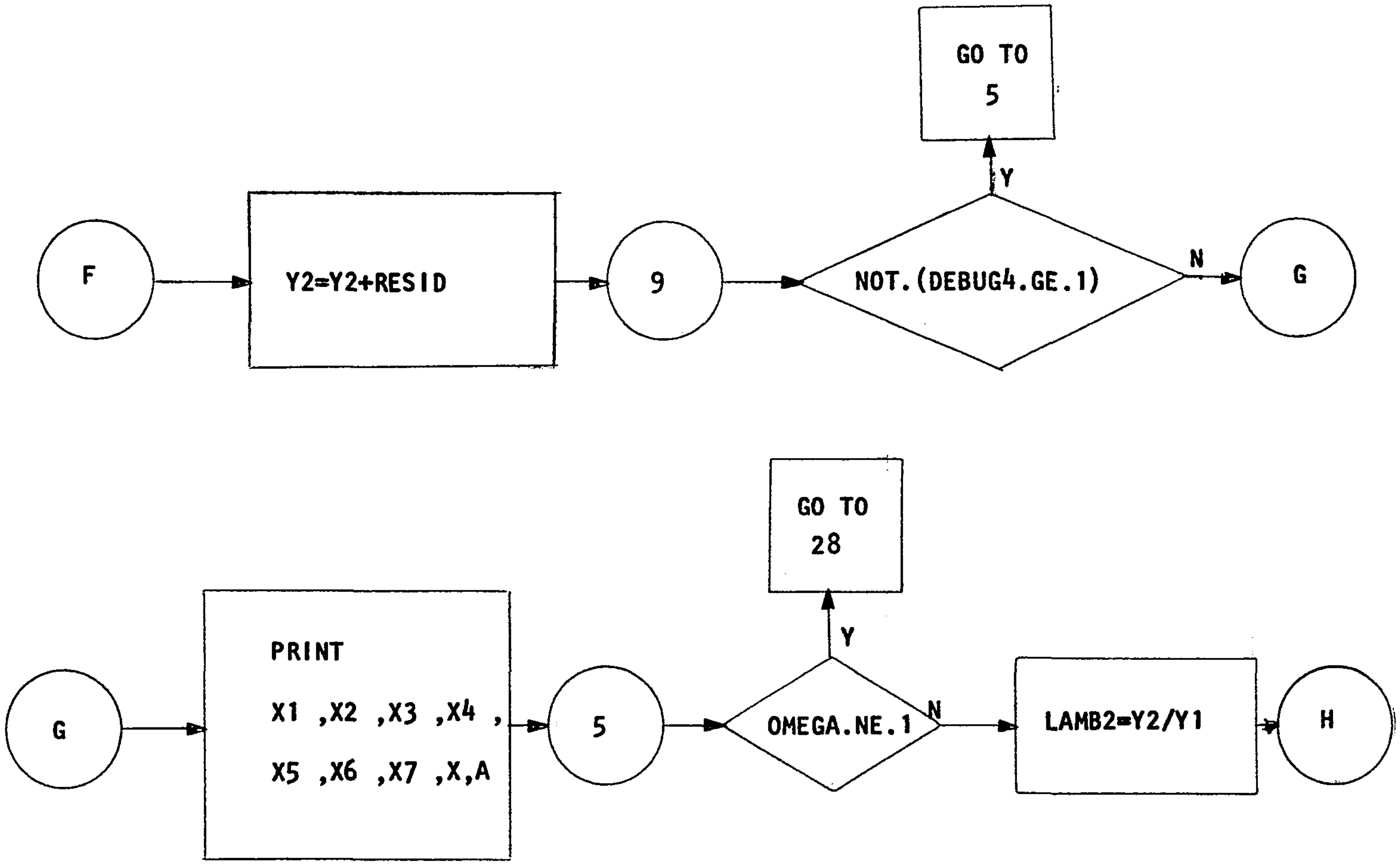


FIG 26h FLOW CHART OF SUBROUTINE PRESS

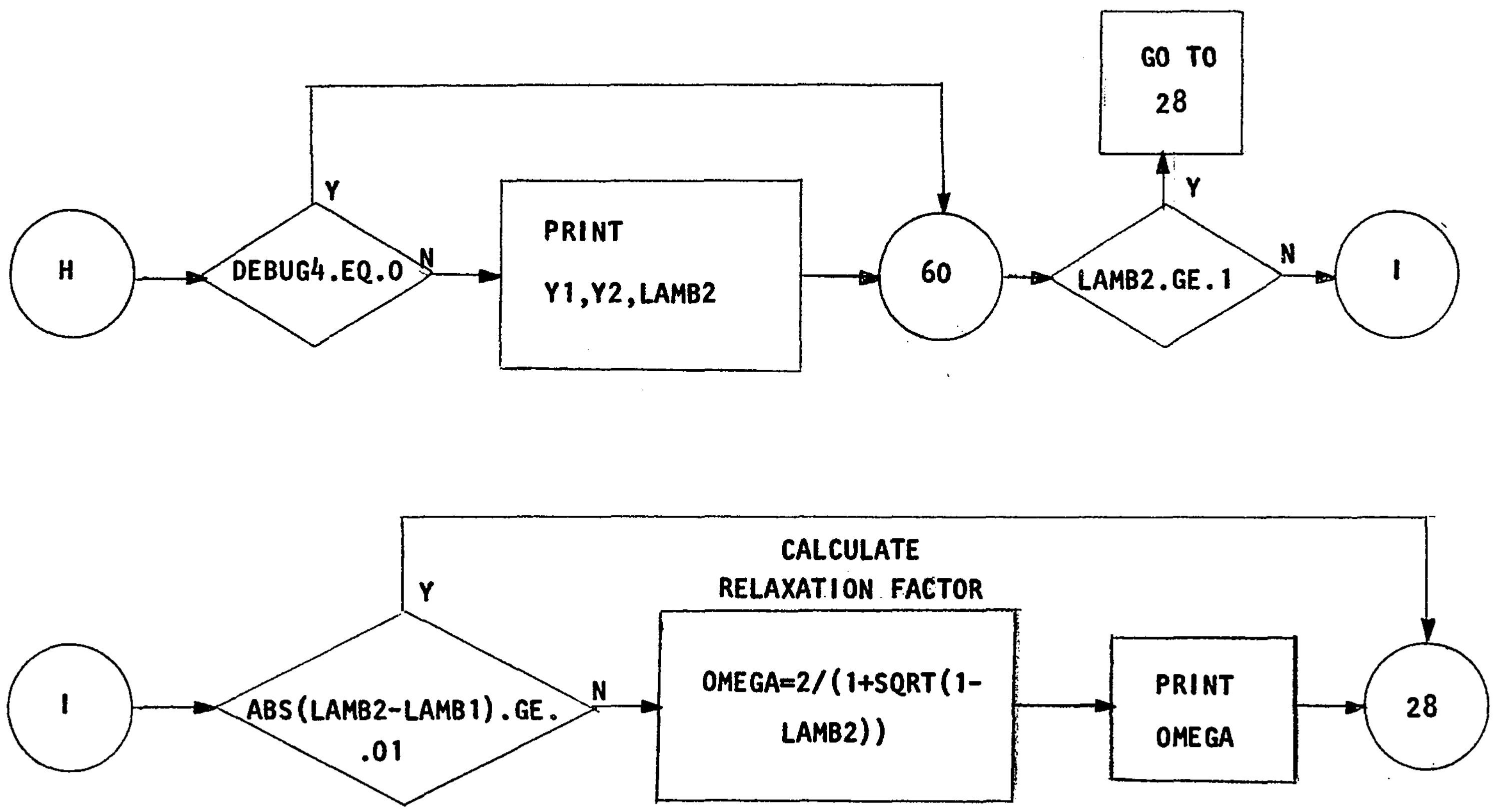


FIG 261 FLOW CHART OF SUBROUTINE PRESS

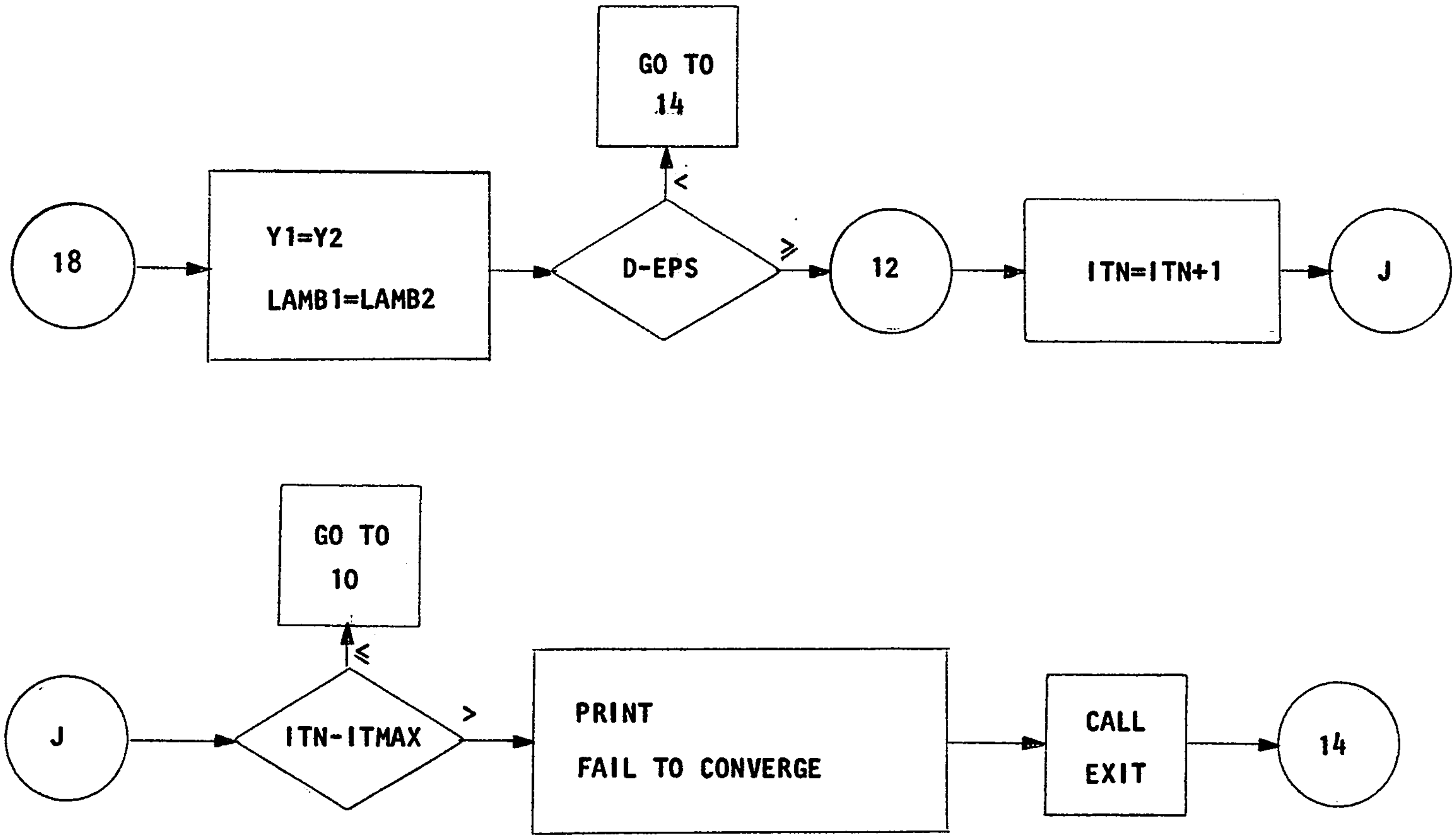


FIG 26j FLOW CHART OF SUBROUTINE PRESS

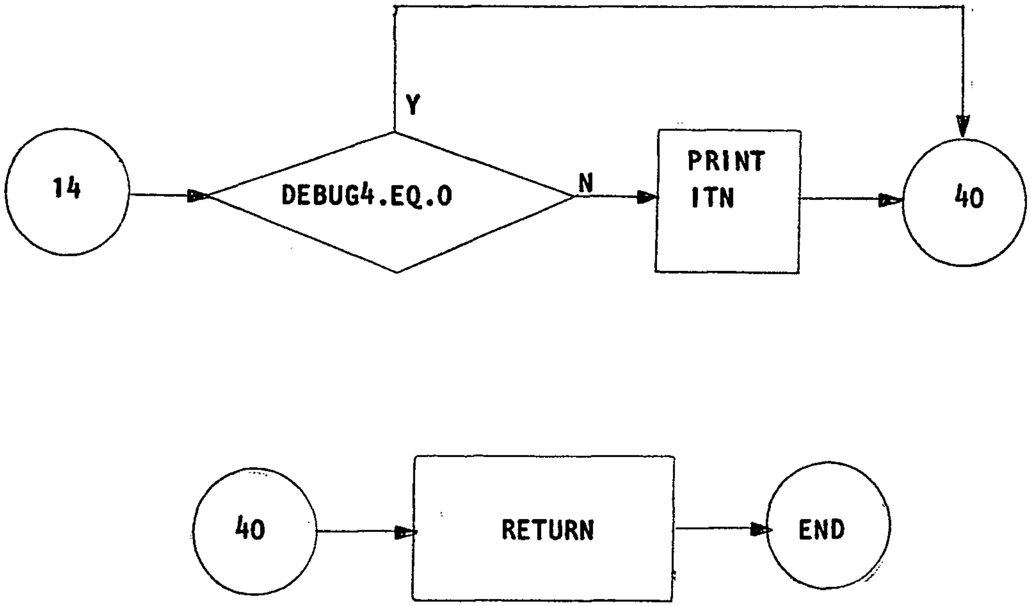


FIG 26k FLOW CHART OF SUBROUTINE PRESS

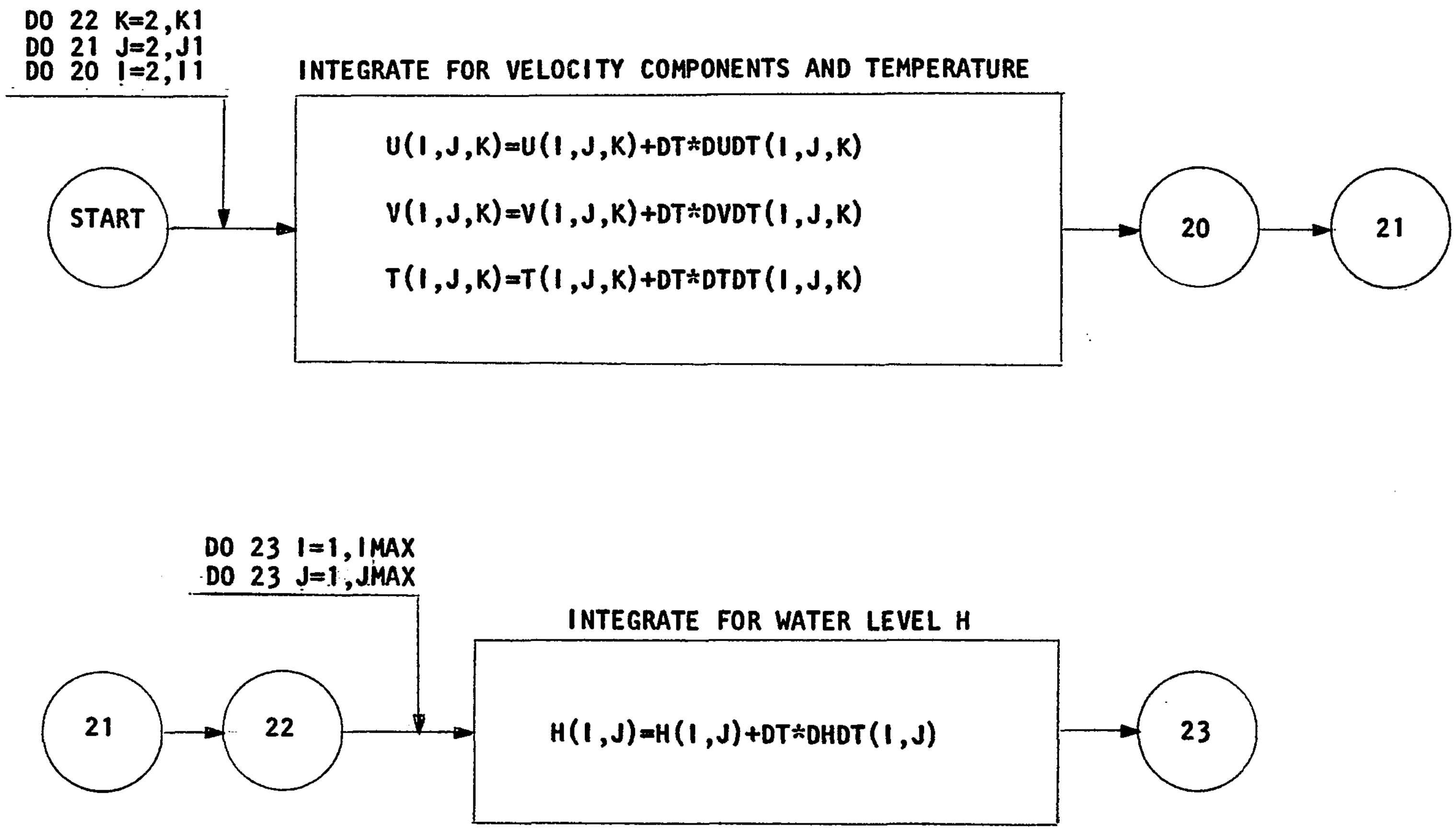


FIG 27a FLOW CHART OF SUBROUTINE SUM

FIG 27b FLOW CHART OF SUBROUTINE SUM

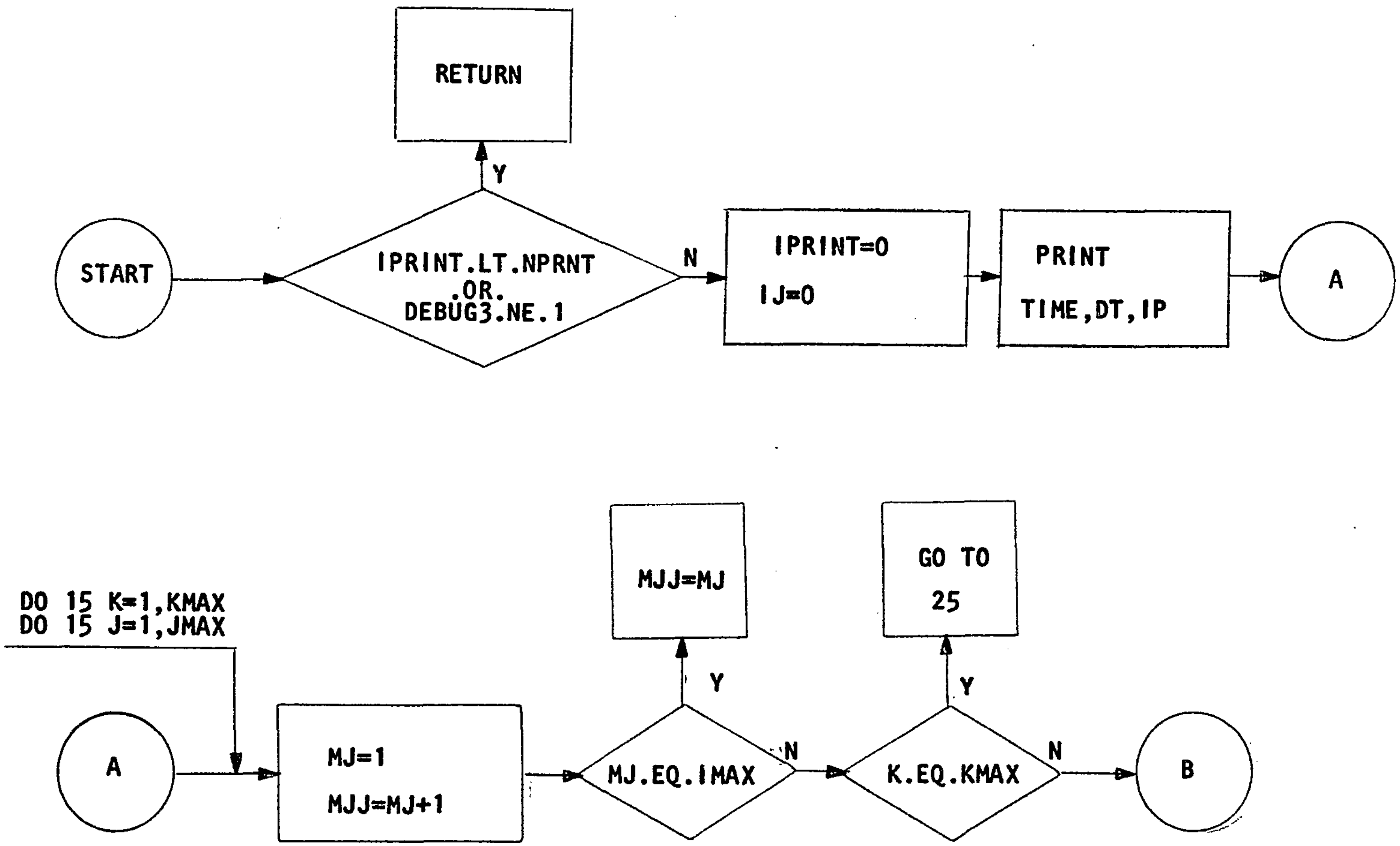


FIG 28a FLOW CHART OF SUBROUTINE OUTP

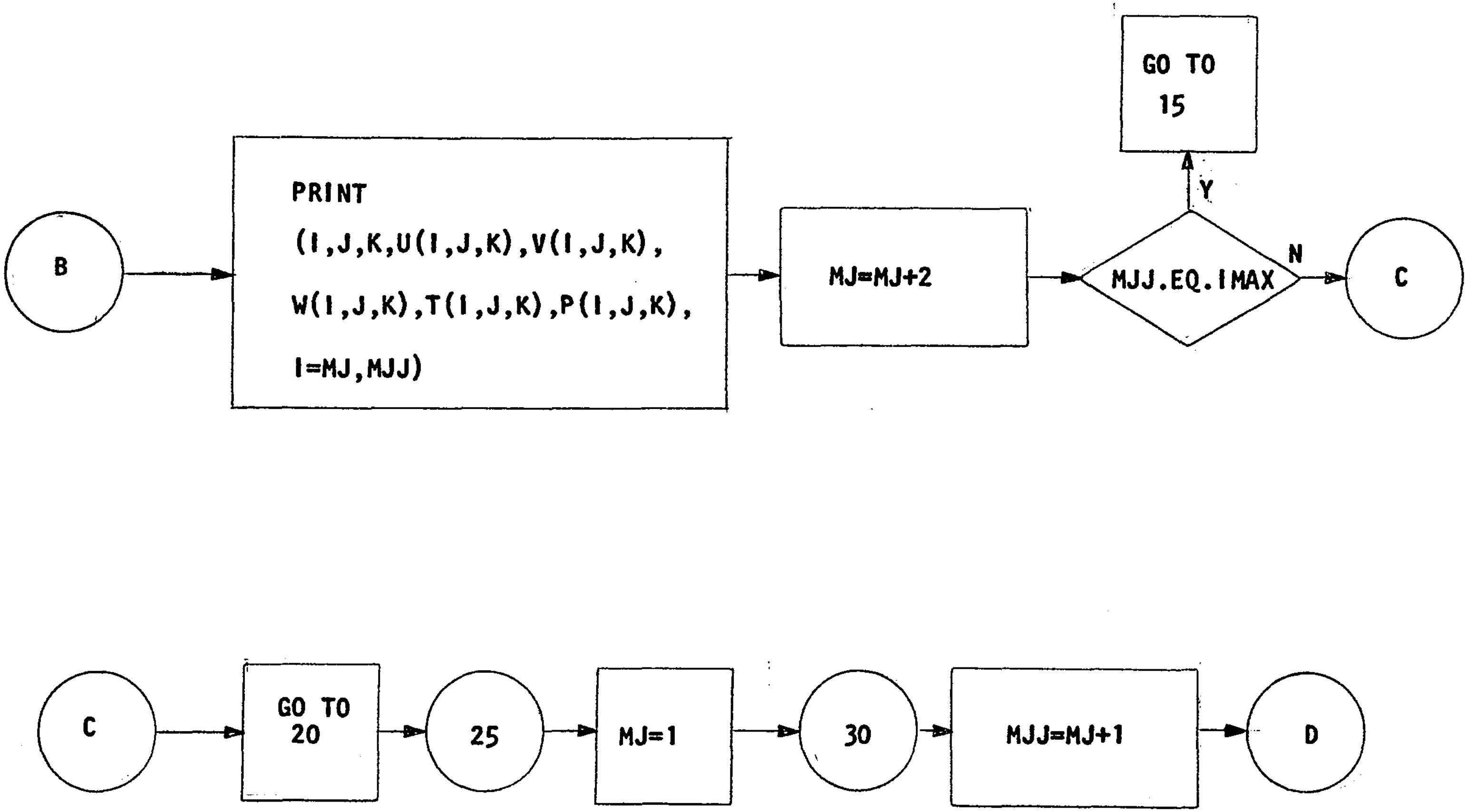


FIG 28b FLOW CHART OF SUBROUTINE OUTP

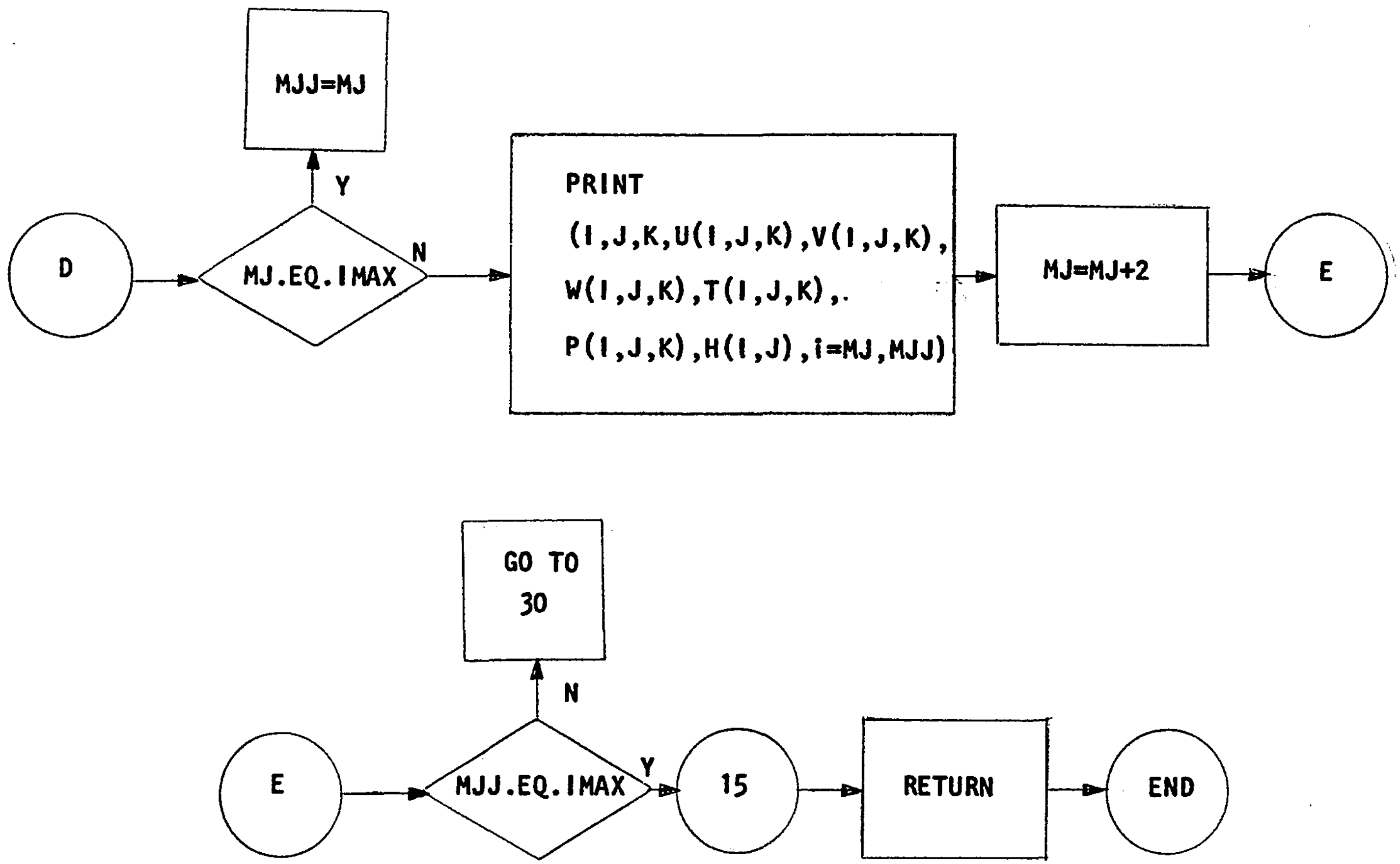


FIG 28c FLOW CHART OF SUBROUTINE OUTP

Subroutine	Function
SI Function	This subroutine performs linear interpolation.
OUTP	This subroutine prints out all the important variables.

1.1 MAIN Program

A detailed description of the MAIN program is presented in the flow charts in the Figs. 24a to 24ℓℓ. This program is employed to perform the following three main functions.

1) Reading in the input data. This part of the program reads in and prints out the input data to maintain a permanent record of each run.

2) Initialization of the system. This part of the program performs the following functions:

a) Converts input data from dimensional values to non dimensional values; and

b) Sets time independent boundary conditions.

3) Iterative operations performed after each time increment. This part of the program performs the following tasks:

a) Checks the stability conditions, if these conditions are not satisfied then OUTP subroutine is called and program terminated.

b) Sets time - dependent boundary conditions.

c) Calculates vertical velocity components by a spatial integration of equation (61) using updated values of horizontal velocity components.

d) Calls subroutine OUTP to print out updated values of the important variables.

e) Calls subroutine DERIV and SUM in the process of finding the updated values.

f) Compares the current time TIME with the final problem time TEND, in order to stop the problem if TEND is exceeded.

1.2 DERIV Subroutine

A detailed description of the subroutine DERIV is shown in Figs. 25a to 25r. This subroutine calculates the first and the second spatial derivatives of the variables. These spatial derivatives are used along with the pressure gradients calculated in subroutine PRESS to find the time derivatives of the horizontal velocity components, temperature, and level according to equation (78) through (84). Communication among DERIV, MAIN and other subroutine take place by means of a COMMON block. The calling sequence of this subroutine is: CALL DERIV.

1.3 PRESS Subroutine

A detailed description of the subroutine PRESS is shown in Figs. 26a to 26k. This subroutine solves for the pressure according to equations (73) and (74) by an iterative method, based on a modified Gauss-Seidel iteration technique called successive overrelaxation technique. The calculation of the pressure for the entire flow field is repeated until the values of pressure at every grid point converge within a prescribed error. This subroutine further sets the boundary condition on pressure and calculates the orverelaxation factor ω according to equations (75) through (77). This factor expedites the convergence of the pressure calculations. The variables are passed into and returned from PRESS by means of a COMMON block. The calling sequence of this subroutine is: CALL PRESS.

1.4 SUM Subroutine

A detailed description of the subroutine SUM is shown in Figs. 27a to 27b. This subroutine performs the time integration for the horizontal velocity components, temperature, and water-level. The communications among SUM, MAIN and other subroutines take place by means of a COMMON block. The calling sequence of this subroutine is: CALL SUM.

1.5 SI Function

This function is used to interpolate values from the table of temperature versus density. It is a standard subroutine:

SI(XTBL,YTBL,X,N)

XTBL=Tabular values of the independent variable.

YTBL=Tabular values of the dependent variable.

X=Value of independent variable.

N=Number of points in the table.

1.6 OUTP Subroutine

A detailed description of subroutine OUTP is shown in Figs. 28a to 28c. This subroutine prints out all variables encompassing the problem solution at any desired multiple of integration time step. These variables are passed into OUTP from the other subroutines, including the MAIN program, by means of a COMMON block. The calling sequence of this subroutine is: CALL OUTP.

2. DESCRIPTION OF INPUT DATA

The input data can be divided into two groups:

1) Non-subscripted variables

2) Subscripted variables.

A brief description of each input variable appears in the program listing to aid in entering the data. All input data are entered using FORMAT E10.3.

2.1 Non-Subscripted Variables

These variables are floating point numbers and are entered into the computer storage by means of cards as specified hereunder:

Card No. 1

1) ENRUN, Run designation number. This number is used to identify each run for record keeping purposes.

Card No. 2

1) TBEG, Starting time, sec. This quantity specifies the initial problem time at which the dynamic problem starts.

2) TEND, End of time, sec. This variable defines the final problem time at which the dynamic problem stops automatically. The dynamic analysis may be aborted, prior to TEND, due to the occurrence of computational errors.

3) ENITER, Number of integration steps. This quantity defines the number of integration steps to be used between starting time

TBEG, and end of time TEND. The program calculates the integration time increment by

$$\Delta t = (TEND-TBEG)/ENITER$$

4) ENPRNT, Frequency of printout. This variable sets the number of integration steps between two successive printouts.

Card No. 3

1) UI, Initial velocity component in x-direction, ft/sec. This is the initial value of the x-component of the fluid velocity assumed constant throughout the water body. It is used to obtain the dynamic response of the water body from a set of uniform velocity distributions as discussed in Part 1 Section 5.

2) VI, Initial velocity component in y-direction, ft/sec. This is the initial value of the y-component of the fluid velocity assumed constant throughout the water body. The purpose of the introduction of this variable is described under variable UI above.

3) TI, Initial temperature of water body and inflow, degrees Fahrenheit. This variable defines the initial value of the fluid temperature assumed constant throughout the water body. The purpose of the introduction of this variable is described under variable UI above. Furthermore, this variable defines the inflow temperature into the water body assumed constant during the dynamic analysis.

Card No. 4

1) UO, Thermal discharge velocity, ft/sec. This quantity specifies the water velocity in the thermal discharge channel from the power plant entering the water body. This variable is used to non-

dimensionalize all velocity components involved in the program.

2) ANGLE, Thermal discharge angle, radian. This variable defines the angle at which the thermal discharge enters the water body measured from the x-axis. The x and y velocities of the thermal discharge entering the water body are calculated by U_0Cos (ANGLE) and U_0Sin (ANGLE) respectively.

3) TO, Thermal discharge temperature, degrees Fahrenheit. This variable specifies the water temperature in the thermal channel from the power plant entering the water body. This variable is used to nondimensionalize all temperature quantities involved in the program and should be always entered regardless of the value of HEAT on Card No. 9 to be discussed later.

4) DO, The half width of thermal discharge channel, ft. This variable is the half width of the thermal discharge channel. This variable is used to nondimensionalize all geometric dimensions involved in the program.

5) XNUZ, Kinematic viscosity of thermal discharge, ft^2/sec. This variable defines the kinematic viscosity of the thermal discharge water. It is used to nondimensionalize the thermal eddy diffusivities and momentum eddy viscosities.

Card No. 5

1) WINDX, x-component of wind velocity, ft/sec. This quantity is the wind velocity component in x-direction assumed to be blowing horizontally over the water body.

2) WINDY, y-component of wind velocity, ft/sec. This quantity

is the wind velocity component in y-direction. This variable together with WINDX provides the means of analyzing wind effects blowing at an oblique angle with respect to the Cartesian coordinate system.

3) RHOA, Air density, lbm/ft^3. This variable specifies the air density above the water body. It affects the water body dynamics because the wind shear effect at the water surface is proportional to air density as described in Part 1, Section 2.3.

4) E, Water body equilibrium temperature, degrees Fahrenheit. This variable specifies the water equilibrium temperature. It is used in the calculation of heat dissipation from water body surface to atmosphere by employing equation (54).

5) XK, Heat exchange coefficient, Btu/(ft^2-day-^{o}F). This variable specifies the coefficient of heat transfer between water body surface and atmosphere. It is used in the calculation of heat dissipation from water body to atmosphere by employing equation (54).

6) CFX, The skin coefficient in x-direction. This variable specifies the skin coefficient in x-direction. It is used to calculate the wind shear effect in x-direction from equation (52).

7) CFY, The skin coefficient in y-direction. This variable specifies the skin coefficient in y-direction. It is used to calculate the wind shear effect in y-direction from equation (53).

Card No. 6

1) DH, Horizontal eddy diffusivity of heat, ft^2/sec. This variable is the eddy diffusivity of heat for the water body in the

horizontal direction. It is used to calculate the turbulent heat exchange in the water in both x and y directions.

2) Dv, Vertical eddy diffusivity of heat, ft^2/sec. This variable is the eddy diffusivity of heat for the water body in the vertical direction. It is used to calculate the turbulent heat exchange in the water in the z direction.

3) XNUH, Horizontal eddy viscosity, ft^2/sec. This variable defines the eddy diffusivity of momentum for the water body in the horizontal direction. It is used to calculate the turbulent momentum transfer in the water in both x and y directions.

4) XNUV, Vertical eddy viscosity, ft^2/sec. This variable defines the eddy diffusivity of momentum for the water body in the vertical direction. It is used to calculate the turbulent momentum transfer in the water in the z-direction.

5) CP, Specific heat of water, Btu/lbm-^oF. This quantity is the specific heat of water in the water body and is used in the calculation of heat transfer from the water to the atmosphere from equation (54).

Card No. 7

1) EIMAX, Number of cells in x-direction. This variable specifies the number of space increments, in the flow field, along x-axis.

2) EJMAX, Number of cells in y-direction. This variable represents the number of space increments, in the flow field, along the y-axis.

3) EKMAX, Number of cells in z-direction. This variable defines the number of space increments, in the flow field, along the z-axis.

Card No. 8

1) DEBUG1, Debugging printout controller in the MAIN program. This quantity controls the printout of the calculated variable in the MAIN program for debugging purposes. Calculated results are printed out when DEBUG1 = 1. No printout occurs when DEBUG1 = 0.

2) DEBUG2, Debugging printout controller in subroutine DERIV. This quantity controls the printout of the calculated variable in subroutine DERIV for debugging purposes. Calculated results are printed out when DEBUG2 = 1. No printout occurs when DEBUG2 = 0.

3) DEBUG3, Unused.

4) DEBUG4, Debugging printout controller in subroutine PRESS. This quantity controls the printout of the calculated variables in subroutine PRESS for debugging purposes. Calculated results are printed out when DEBUG4 = 1. No printout occurs when DEBUG4 = 0.

5) EIRUN, Restart switch. This quantity controls the initialization of the program. When EIRUN = 0, program will start from initial condition described earlier in this section. When EIRUN = 1, program will skip the initialization and will use previous output already stored on tape as initial values. The latter tape storage is achieved by setting EIEND = 1 in the previous run as discussed next.

6) EIEND, Restart controller. This quantity controls the output at the completion of a dynamic run. When EIEND = 1, the final program output will be stored on tape for restarting the run at a later date. When EIEND = 0, no storage of final program output will be made.

Card No. 9

1) EIM, Number of thermal discharge cells in vertical direction.

This parameter indicates the number of grid cells in the vertical direction in the thermal discharge channel.

2) EML, Starting location of thermal discharge. This parameter indicates the grid cell number in the x-direction where the thermal discharge channel begins.

3) EMK, End location of thermal discharge. This parameter indicates the grid cell number in the x-direction where the thermal discharge channel ends.

4) SLIP, Slip condition controller. This quantity indicates whether boundary conditions on velocity components are calculated using slip or non-slip conditions. When SLIP = 1., the boundary conditions are calculated using slip conditions. When SLIP = 0., the boundary conditions are calculated with non-slip conditions.

5) HYD, Hydrostatic pressure controller. This quantity indicates whether pressure is calculated by hydrostatic approximation or from equations of motion. When HYD = 1., the pressure is calculated by hydrostatic approximation. When HYD = 0., the pressure is not hydrostatic and is calculated from the equations of motion indicated by equations (73) and (74).

6) HEAT, Temperature controller. This quantity controls whether the thermal discharge is heated or not. When HEAT = 1., it indicates that the thermal discharge has a higher temperature than surrounding fluid. When HEAT = 0., it means that the thermal discharge is at same temperature as the surrounding fluid.

7) XIIP, Off line storage controller. This parameter controls the off-line storage of the output data for subsequent restart. At

the appropriate iteration, indicated by XIIP, the output are stored on a disc, and recalled in the restart mode.

2.2 Subscripted Variables

These variables involve fixed size arrays and are entered into the computer storage using FORMAT 7E10.3.

1) DELX(I), x-increments, ft. This array defines the spatial increments, in the flow field, along the x-axis. The number of cards needed to enter this array is equal to EIMAX/7.

2) DELY(J), y-increments. This array defines the spatial increments, in the flow field, along the y-axis. The number of cards needed to enter this array is equal to EJMAX/7.

3) DELZ(K), z-increments. This array defines the spatial increments, in the flow field, along the z-axis. The number of cards needed to enter this array is equal to EKMAX/7.

4) Density versus Temperature, lbm/ft^3 and degree Fahrenheit respectively. The densities and their corresponding temperatures, given in Table 1, are entered in a BLOCK DATA routine. The following format is used in entering the data:

```
BLOCK DATA
COMMON/TABLE/TTBL,RHOTBL
REAL * 4 TTBL(26)/26 values of temperature/
REAL * 4 RHOTBL(26)/26 values of density/
```

3. DESCRIPTION OF OUTPUT DATA

The output data can be conveniently divided into the following groups:

1) Input Data Printout

All input data are printed out and labeled using the variable names defined in the description of input data. The temperature and density relation, entered by BLOCK DATA, are printed in a tabular form with one column labeled "Temperature" and the other column labeled "Density", followed by the appropriate values.

2) Debugging Printout

At the user's option, under the control of DEBUG1, DEBUG2, DEBUG3, and DEBUG4 as described in the input data, the intermediate calculations may be printed out for debugging or checking purposes. Labels used for these printouts are described in Part 2, Section 5 in program nomenclature.

When DEBUG1 = 1., the following variables will be printed out: I,J,K,DT,DTT,DTMIN,DTTMIN,DTS,DTTS,DTSM,DTTSM,XIP1,XIM1,XIPM1,XI123, XIPM,YJP1,YJM1,YJPM1,YJ123,YJPM,ZKP1,ZKM1,ZKPM1,ZK123,SIGX,SIGX1,SIGX2, SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGZ,SIGZ1,SIGZ2,SIGZ3,SIGZ4, SIGMA1,SIGMA2,SIGMA3,SIGMA4,SIGMA5,SIGMA6,XN,XM,XL,YN,YL,ZN,ZL,DUDX, DUDY.

When DEBUG2 = 1., the following variables are printed out: I,J,K,IP1,IM1,JP1,JM1,KP1,KM1,DUDT,DVDT,DWDT,DTDT,DIV,DQDX,DQDY,DQDZ, RHS.

DEBUG3 is presently unused and is reserved for future applications.

For DEBUG4 = 1., the following variables are printed out :
I,J,K,ZK123,ZKP1,ZKM1,YJ123,YJP1,YJM1,XI123,XIP1,XIM1,X,X1,X2,X3,X4,
X5,X6,X7,Y1,Y2,LAMB2,OMEGA,ITN

3) Variables Defining the Problem Solution

At user's option, under the control of ENPRNT described in the input data, the solution variables can be printed out at any integration time increment or multiple thereof. These variables are printed in two columns, each column containing the variables:
I,J,K,U,V,W,T,P,and H.

4. OPERATING PROCEDURE

To employ the THERMA computer program effectively, the user should first familiarize himself with Part 1, Section 2, particularly with the mathematical formulation. The user should then draw a sketch of the receiving water similar to Fig. 7, and determine the space and time increments to be used in the analysis as indicated in Section 4. The input data is then prepared in accordance with the format and description given in Section 2 of this part. These input data are then punched on cards and assembled with the THERMA program deck and the necessary control cards. The control cards vary from one computer organization to the other. A sample of the deck assembly for the UNIVAC SERIES 70 is given below:

```
/LOGONbuserid,acct.no.,bTIME=in second
/PARAMb LIST=YES,MAP=YES,DEBUG=YES
/EXECbBGFOR
        PROGRAM Name
        (Fortran Source Statement)
        END
/FILE THERMA.RESTART1,LINK=DSFT24,FCBTYPE=SAM[1]
/FILE THERMA.RESTART2,LINK=DSFT25,FCBTYPE=SAM[2]
/EXECb *
        (Data)
/LOGOFF
```

---

[1] RESTART 1 designates the input offline storage

[2] RESTART 2 designates the output offline storage

The computer program is then run for several integration steps and the user should then verify the following points:

1) Correctness of input data. The input data printed out should be verified for possible error in punching.

2) Correctness of integration results. The integration results should be verified for consistency and the appropriateness of the initial conditions.

The program could then be run for maximum allowable computer time, at the end of which the results may be written on disc. This off-line storage may be used as input data in a later run to continue the computations. This procedure may be repeated until the results have reached steady state. After the program has been fully run, the user should verify the convergence of the solution by halving the time step Δt and the space increments Δx, Δy, and Δz. The program is then resubmitted and results are checked to verify the agreement between the two solutions. If the solutions agree, the convergence in time and space has been established. If the solutions do not agree, it would then be necessary to choose a smaller time step or further refine the mesh, and perform the above operations again until convergence is established.

5. PROGRAM NOMENCLATURE

To assist the user in the revision of the THERMA program, a list of some key variables with their corresponding analysis notations and definitions is presented in this section.

Program Notation	Analysis Notation	DESCRIPTION
CFX	C_{fx}	Skin friction in x-direction
CFY	C_{fy}	Skin friction in y-direction
CP	C_p	Specific heat of water
DH	D_h	Horizontal eddy diffusivity of heat
DV	D_v	Vertical eddy diffusivity of heat
DO	D_0	Half width of thermal discharge channel
DT	Δt	Time increment
DELX(I)	Δx_i	Space increment in x-direction
DELY(J)	Δy_j	Space increment in y-direction
DELZ(K)	Δz_k	Space increment in z-direction
DEBUG1	-	Control for debugging of MAIN program
DEBUG2	-	Control for debugging of subroutine DERIV
DEBUG3	-	Unused
DEBUG4	-	Control for debugging of subroutine PRESS

Program Notation	Analysis Notation	Description
DUDT	$\partial u/\partial t$	Time derivative of u component of velocity
DVDT	$\partial v/\partial t$	Time derivative of v component of velocity
DWDT	$\partial w/\partial t$	Time derivative of w component of velocity
DTDT	$\partial T/\partial t$	Time derivative of temperature
DHDT	$\partial H/\partial t$	Time derivative of water-level
E	E	Equilibrium temperature of water body
ENRUN	-	Run designation number
EPS	-	Pressure convergence error
FO	F_0	Froude number
G	g	Gravitational acceleration
H	H	Water level
IMAX	i_{max}	Maximum number of elements in x-direction
IP	-	Current number of iterations
IPRINT	-	Printout control; used in conjunction with NPRNT and IP to determine when printout occurs
ITMAX	-	Maximum number of iterations for the convergence of pressure
ITN	-	Current iteration number used to check against ITMAX

Program Notation	Analysis Notation	Description
JMAX	j_{max}	Maximum number of elements in y-direction
KMAX	k_{max}	Maximum number of elements in z-direction
LAMB2	λ^2	Norm ratio
NITER	-	Number of integration steps
NPRNT	-	Frequency of printout
OMEGA	ω	Relaxation factor
P	p	Pressure
RO	R_0	Reynolds number
RHO	ρ	Density of water
RHOA	ρ_a	Density of air
RHOZ	ρ_0	Density of thermal discharge
RHOTBL	-	Density table
RHS(I,J,K)	-	Defined by Eq. (66b)
SO	S_0	Stanton number
SIGMA1(I)	a_x	Weighting factor for first derivative in flow field defined by Eq. (67)
SIGMA2(I)	b_x	Weighting factor for first derivative in flow field defined by Eq. (67)
SIGMA3(J)	a_y	Weighting factor for first derivative in flow field defined by Eq.

Program Notation	Analysis Notation	Description
		(67)
SIGMA4(J)	b_y	Weighting factor for first derivative in flow field defined by Eq. (67)
SIGMA5(K)	a_z	Weighting factor for first derivative in flow field defined by Eq. (67)
SIGMA6(K)	b_z	Weighting factor for first derivative in flow field defined by Eq. (67)
SIGX	σ	Ratio of space increments in x-direction defined by Eq. (C-12)
SIGX1	$2\sigma + 1$	Coefficient of q_1 in Eq. (C-13)
SIGX2	$(1+\sigma)^2$	Coefficient of q_2 in Eq. (C-13)
SIGX3	σ^2	Coefficient of q_3 in Eq. (C-13)
SIGX4	$\frac{1}{2}\Delta n_{12}(1+\sigma)$	Coefficient of Eq. (C-13)
SIGY	-	Analogous to SIGX in y-direction
SIGY1	-	Analogous to SIGX1 in y-direction
SIGY2	-	Analogous to SIGX2 in y-direction
SIGY3	-	Analogous to SIGX3 in y-direction
SIGY4	-	Analogous to SIGX4 in y-direction
SIGPY	σ'	Analogous to $1/\sigma$, ratio of spatial increments in y-direction defined by Eq. (C-27)

Program Notation	Analysis Notation	Description
SIGPY1	$1+2\sigma'$	Coefficient of q_3 in Eq. (C-28)
SIGPY2	$(1+\sigma')^2$	Coefficient of q_2 in Eq. (C-28)
SIGPY3	σ'^2	Coefficient of q_1 in Eq. (C-28)
SIGPY4	$\frac{1}{2}\Delta n_{23}(1+\sigma')$	Coefficient of Eq. (C-28)
SIGZ	-	Analogous to SIGX in z-direction
SIGZ1	-	Analogous to SIGX1 in z-direction
SIGZ2	-	Analogous to SIGX2 in z-direction
SIGZ3	-	Analogous to SIGX3 in z-direction
SIGZ4	-	Analogous to SIGX4 in z-direction
T	T	Temperature
TO	T_0	Thermal discharge temperature
TIME	t	Time
TTBL	-	Temperature table
U	u	Velocity in x-direction
UO	u_0	Velocity of thermal discharge
V	v	Velocity in y-direction
W	w	Velocity in z-direction
WINDX	W_x	Wind velocity in x-direction
WINDY	W_y	Wind velocity in y-direction
XIMP1(I)	c_x	A quantity defined by Eq. (67)
XNUZ	ν_0	Kinematic viscosity of thermal discharge
XNUH	ν_h	Horizontal eddy viscosity

Program Notation	Analysis Notation	Description
XNUV	ν_v	Vertical eddy viscosity
YJPM1(J)	c_y	A quantity defined by Eq. (67)
ZKPM1(K)	c_z	A quantity defined by Eq. (67)

6. PROGRAM LISTING AND SAMPLE RUN

In this section, the FORTRAN program listing of the THERMA computer program is presented. This listing is followed by a sample run for non-buoyant jet discussed in Part 1, Section 5.3. To reduce the bulk of computer printout, the output data are supplied for iterations number 0, 1 and 50 corresponding to times 0, 0.2 and 10 seconds respectively.

```
      PROGRAM THERMA
C     CARD NO.  SYMBOL    DESCRIPTION
C      1-1     ENRUN     RUN DESIGNATION NUMBER
C      2-1     TBEG      STARTING TIME                                    SEC
C      2-2     TEND      END OF TIME                                      SEC
C      2-3     ENITER    NUMBER OF INTEGRATION STEPS
C      2-4     ENPRINT   FERQUENCY OF PRINTOUT
C      3-1     UI        INITIAL VELOCITY COMPONENT IN
C                        X-DIRECTION                                    FT/SEC
C      3-2     VI        INITIAL VELOCITY COMPONENT IN
C                        Y-DIRECTION                                    FT/SEC
C      3-3     TI        INITIAL TEMPERATURE OF LAKE AND
C      4-1     U0        THERMAL DISCHARGE VELOCITY                     FT/SEC
C      4-2     ANGLE     THERMAL DISCHARGE ANGLE                        RADIANS
C      4-3     T0        THERMA DISCHARGE TEMPERATURE                     DEGREE F
C      4-4     D0        HYDRAULIC DIAMETER OF THERMAL
C                          DISCHARGE CHANNEL
C      4-5     XNUZ      KINEMATIC VISCOSITY OF THERMAL
C                        DISCHARGE                                       FT2/SEC
C      5-1     WINDX     X-COMPONENT OF WIND VELOCITY                    FT/SEC
C      5-2     WINDY     Y-COMPONENT OF WIND VELOCITY                    FT/SEC
C      5-3     RHOA      AIR DENSITY                                   LBM/FT3
C      5-4     E         EQUILIBRIUM TEMEPRATURE OF WATER                 DEGREE F
C      5-5     XK        HEAT EXCHANGE COEFFICIENT                    BTU/FT2-DAY-F
C      5-6     CFX       SKIN COEFFICIENT IN X-DIRECTION
C      5-7     CFY       SKIN COEFFICIENT IN Y-DIRECTION
C      6-1     DH        HORIZONTAL EDDY DIFFUSIVITY OF HEAT            FT2/SEC
C      6-2     DV        VERTICAL   EDDY DIFFUSIVITY OF HEAT             FT2/SEC
C      6-3     XNUH      HORIZONTAL EDDY VISCOSITY                      FT2/SEC
C      6-4     XNUV      VERTICAL   EDDY VISCOSITY                       FT2/SEC
C      6-5     CP        SPECIFIC HEAT OF WATER                         BTU/LBM-F
C      7-1     EIMAX     NUMBER OF CELLS IN X-DIRECTION
C      7-2     EJMAX     NUMBER OFYCELLS IN Y-DIRECTION
C      7-3     EKMAX     NUMBER OFYCELLS IN Z-DIRECTION
C      8-1     DEBUG1    CONTROL FOR DEBUGGING OF MAIN PROGRAM
C      8-2     DEBUG2    CONTROL FOR DEBUGGING OF SUBROUTINE DERIV
C      8-3     DEBUG3    UNUSED
C      8-4     DEBUG4    CONTROL FOR DEBUGGING OF SUBROUTINE PRESS
C      8-5     EIRUN     RESTART SWITCH
C      8-6     EIEND     RESTART CONTROLLER
C      9-1     EIM       NUMBER OF THERMAL DISCHARGE CELLS IN
C                        VERTICAL-DIRECTION
C      9-2     EML       STARTING LOCATION OF THERMAL DISCHARGE
C      9-3     EMK       END LOCATION OF THERMAL DISCHARGE
C      9-4     SLIP      SLIP CONDITION CONTROLLER
C      9-5     HYD       HYDROSTATIC PRESSURE CONTROLLER
C      9-6     HEAT      TEMPERATURE CONTROLLER
C      9-7     XIIP      OFF LINE STORAGE CONTROLLER
C
C
```

```
C
C
C      SUBSCRIPTED VARIABLES
C       VARIABLE NAME      DESCRIPTION
C       DELX(I)            DIMENSION OF ELEMENT I IN X-DIRECTION       FT
C       DELY(J)            DIMENSION OF ELEMENT J IN Y-DIRECTION       FT
C       DELZ(K)            DIMENSION OF ELEMENT K IN Z-DIRECTION       FT
C      V(I,J,K)     =Y-COMPCNENT CF VELOCITY
C      W(I,J,K)     =Z-COMPONENT OF VELOCITY
C      T(I,J,K)     =TEMPERATURE
C      H(I,J)       = WATER LEVEL HEIGHT
C      P(I,J,K)     =PRESSURE
C      RHO(I,J,K)   =DENSITY
C      IMAX         =NUMBER OF CELLS IN X-DIRECTION
C      JMAX         =NUMBER OF CELLS IN Y-DIRECTION
C      KMAX         =NUMBER OF CELLS IN Z-DIRECTION
C      X            =COEFFICIENT OF P(I,J,K)
C      X1           =COEFFICIENT OF P(I+1,J,K)
C      X2           =COEFFICIENT OF P(I-1,J,K)
C      X3           =COEFFICIENT OF PI,J+1,K)
C      X4           =COEFFICIENT OF P(I,J-1,K)
C      X5           =COEFFICIENT OF P(I,J,K+1)
C      X6           =COEFFICIENT OF P(I,J,K-1)
C      X7           =COEFFICIENT OF RHS(I,J,K)
       COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
      1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
      2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
      3XNUH,XNUV,RO,FO,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,Z,
      4WINDX,WINDY,E,RHOA,IMAX,JMAX,KMAX,OMEGA,IM,MH,ML,MK,
      5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUR,ZNUR,XDHR,XDVR,G,DT,
      6NITER,NPRINT,I1,I2,J1,J2,K1,K2,TIME,IPRINT,RHS(20,14,10),
      7XIP1(20),XIM1(20),XIPM1(20),XI123(20),YJP1(14),XIP2(20),
      8YJM1(14),YJPM1(14),YJ123(14),H(20,14),DZ,YJP2(14),
      9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10)
      A,II,JJ,KK,I3,J3,K3,FO1,IIT,B,EPS,ITNMAX,SIGMA1(20),SIGMA2(20),
      BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHDT(20,14)
       COMMON SIGX1,SIGX2,SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY1,
      1SIGPY2,SIGPY3,SIGPY4,SIGZ1,SIGZ2,SIGZ3,SIGZ4
       COMMON/TABLE/TTBL,RHOTBL
       DIMENSION TTBL(26),RHOTBL(26)
       DIMENSION XM(20),YM(14),ZM(14)
       DIMENSION XN(20),YN(14),ZN(10),XL(20),YL(14),ZL(10),CLEVEL(20,14)
  1001 FORMAT(E10.3)
  1002 FORMAT(2E10.3)
  1003 FORMAT(3E10.3)
  1004 FORMAT(4E10.3)
  1005 FORMAT(5E10.3)
  1006 FORMAT(6E10.3)
  1007 FORMAT(7E10.3)
  1008 FORMAT(5E10.3)
```

```
 604  FORMAT(1H0,10X,'I',4X,'XN',15X,'XM')
 608  FORMAT(1H0,10X,3(I3,2X,2(E11.4)))
 609  FORMAT(1H0,10X,'J',4X,'YN(J)',15X,'YM(J)')
 610  FORMAT(1H0,10X,'K',4X,'ZN(K)',15X,'ZM(K)')
 614  FORMAT(1H0,10X,3(I2,1X),2(E11,4,2X))
 615  FORMAT(1H0,10X,'I',2X,'J',2X,'K',2X,'DT',5X,'DTT')
 616  FORMAT(1H ,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4,/)
 621  FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION  FAILS IN X-DIRECTION
     1')
 622  FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION FAILS IN Y-DIRECTION'
     1)
 623  FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION  FAILS IN Z-DIRECTION
     1')
 627  FORMAT(1H0,10X,6(I2,2X,E11,4,2X)/)
 629  FORMAT(1H0,10X,'I',4X,'XIMU(I)')
 630  FORMAT(1H0,10X,'J',4X,'YJM1(J)')
 631  FORMAT(1H0,10X,'K',4X,'ZKM1(K)')
 640  FORMAT(1H0,10X,'I',2X,'XL(I)')
 641  FORMAT(1H0,10X,5(I2,1X,E11.4))
 642  FORMAT(1H0,10X,'J',2X,'YL(J)')
 643  FORMAT(1H0,10X,'K',2X,'ZL(K)')
 644  FORMAT(1H0,10X,'I',5X,'SIGMA1',5X,'SIGMA2')
 645  FORMAT(1H0,10X,I2,5X,2(E11.4,2X))
 646  FORMAT(1H0,10X,'J',5X,'SIGMA3',5X,'SIGMA4')
 647  FORMAT(1H0,10X,'K',5X,'SIGMA5',5X,'SIGMA6')
 705  FORMAT(1H0,11X,'I',3X,'J',3X,'K',3X,'SUGGESTED DT',10X,
     X'I',3X,'J',3X,'K',3X,'SUGGESTED DTT',/)
 709  FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'DTS',5X,'DTTS')
 710  FORMAT(1H0,10X, 3(I2,1X),2(E11.4,2X))
 714  FORMAT(1H0,11X,'I',3X,'J',3X,'KMAX',3X,'SUGGESTED DTS',10X,
     X'I',3X,'J',3X,'KMAX',3X,'SUGGESTED DTTS',/)
 715  FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4,/)
 720  FORMAT(1H0,10X,'DT BECAME TOO SMALL AT',2X,'I=',I2,2X,'J=',I2,
     12X,'HLEVEL=',E14.7,2X,'CLEVEL(I,J)=',E14.7)
 1025 FORMAT(1H1)
 1026 FORMAT(1H0,10X,'ENRUN =',E11.4)
 1027 FORMAT(1H0,10X,'TBEG  =',E11.4,5X,'TEND  =',E11.4,5X,'ENITER='
     1,E11.4,5X,' NPRNT=',E11.4)
 1028 FORMAT(1H0,10X,'UI    =',E11.4,5X,'VI    =',E11.4,5X,
     1           'TI    =',E11.4)
 1029 FORMAT(1H0,10X,'WINDX =',E11.4,5X,'WINDY =',E11.4,5X,'RHOA  ='
     1,E11.4,5X,'E =',E11.4,9X,' XK =',E11.4,//,11X,'CFX  =',E11.4,5X,
     2'CFY  =',E11,4)
 1030 FORMAT(1H0,10X,'DH    =',E11.4,5X,'DV    =',E11.4,5X,'XNUH  ='
     1,E11.4,5X,'XNUV  =',E11.4,5X,'CP=',E11.4)
 1031 FORMAT(1H0,10X,'EIMAX =',E11.4,5X,'EJMAX =',E11.4,5X,'EKMAX ='
     1,E11,4)
 1032 FORMAT(1H0,10X,'RO    =',E11.4,5X,'FO    =',E11.4,5X,'SO    ='
     1,E11.4)
 1033 FORMAT(1H0,10X,'DEBUG1=',E11.4,5X,'DEBUG2=',E11.4,5X,'4EBU73=',
```

```
     1E11.4,5X,'DEBUG4=',E11.4,//,11X,'EIRUN=',E11.4,5X,'EIEND=',E11.4)
1034 FORMAT(1H0,10X,'IMAX  =',I5   ,5X,'JMAX  =',I5   ,5X,'KMAX  ='
     1,I5  )
1035 FORMAT(1H0,10X,'I1    =',I5   ,5X,'J1    =',I5   ,5X,'K1    ='
     1,I5 )
1036 FORMAT(1H0,10X,'I2    =',I5   ,5X,'J2    =',I5   ,5X,'K2    ='
     1,I5)
1037 FORMAT(1H0,10X,'G     =',E11.4,5X,'DT    =',E11.4,5X,'E=',E14.7)
1038 FORMAT(1H0,10X,'I',3X,'DELX(I)')
1039 FORMAT(2(1H0,10X,4(I2,2X,E11.4,5X)/))
1040 FORMAT(1H0,10X,'J',3X,'DELY(J)')
1041 FORMAT(1H0,10X,'K',3X,'DELZ(K)')
1042 FORMAT(1H0,10X,'XNUR  =',E11.4,5X,'ZNUR  =',E11.4,5X,'XDHR  ='
     1,E11.4,5X,'XDVR  =',E11,4)
1046 FORMAT(1H0,10X,'I3    =',I5   ,5X,'J3    =',I5   ,5X,'K3    ='
     1,I5)
1048 FORMAT(1H0,'K=',I2,1X,'ZKP1(K)=',E11,4,1X,
     1'ZKM1(K)=',E11.4,1X,'ZKPM1(K)=',E11,4,1X,
     2'ZK123(K)=',E11.4,/,1X,'J=',I2,1X,'YJP1(J)=',E11.4,5X,'YJM1(J)=',
     3E11.4,5X,'YJPM1(J)=',E11.4,5X,'YJ123(J)=',E11.4,/1X,'I=',I2,
     51X,'XIP1(I)=',E11,4,1X,'XIM1(I)=',E11,4,1X,'XIPM1(I)=',E11.4,1X,
     6'XI123(I)=',E11.4)
1054 FORMAT(1H ,2X,'I=',I3,2X,'J=',I3,2X,'K=',I3,'DUDX=',E14.7,2X,
     1'DVDY=',E14.7,2X,'CS(K)=',E14.7,2X,'DS(I,J,K)=',E14.7)
1058 FORMAT(1H0,10X,'U0=',E11.4,9X,'ANGLE=',E11.4,6X,'T0=',E11.4,
     19X,'D0=',E11.4,9X,'XNUZ=',E11.4)
1062 FORMAT(1H0,20X,20('*'),'NON-DIMENSIONAL INPUT VALUES ',20('*'))
1063 FORMAT(1H0,20X,20('*'),'END OF NON-DIMENSIONAL INPUT VALUES',
     120('*'))
1064 FORMAT(1H ,10X,2(E14.7,2X))
1065 FORMAT(1H0,15X,'TTBL',10X,'RHOTBL')
1066 FORMAT(1H0,10X,'RHCZ=',E15.7)
1067 FORMAT(1H0,10X,'TBEG  =',E11.4,5X,'TEND  =',E11.4,5X,' NITER='
     1,I6   ,5X,'NPRINT=',I6)
1068 FORMAT(1H0,10X,'DH    =',E11.4,5X,'DV    =',E11.4,5X,'XNUH  ='
     1,E11.4,5X,'XNUV  =',E11.4)
1069 FORMAT(1H0,10X,'WINDX =',E11.4,5X,'WINDY =',E11.4,5X,'RHOA  ='
     1,E11,4)
1071 FORMAT(1H0,10X,'EIM=',E11.4,5X,'EML=',E11.4,5X,'EMK=',E11.4,
     15X,'SLIP=',E11.4,5X,'HYD=',E11.4,//,11X,'HEAT=',E11.4,5X,'XIIP=',
     2E11.4)
1072 FORMAT(1H0,10X,'Z  =',E11.4)
1100 FORMAT(1H0,5(E14.7,4X),/,2X,5(E14.7,4X))
1110 FORMAT(1H0,5(E14.7,4X))
2005 FORMAT(1H ,3(I3,2X),7(E14.7,2X))
2006 FORMAT(1H0,5X,'TIME=',E14.7,5X,'DT=',E14.7,5X,'IP=',I5,5X,'KMAX=',
     1I5,5X,'JMAX=',I5,5X,'IMAX=',I5,5X,'IPRINT=',I5)
2007 FORMAT(1H0,1X,'I',2X,'J',2X,'K',9X,'U',12X,'V',12X,'W',12X,'T',
     112X,'P',12X,'RHO',12X,'H')
C     READ IN NON SUBSCRIPTED VARIABLES
```

```
      READ(5,1001)  ENRUN
      WRITE(6,1026) ENRUN
      READ(5,1004)  TBEG,TEND,ENITER,ENPRNT
      WRITE(6,1027) TBEG,TEND,ENITER,ENPRNT
      READ(5,1003)  UI,VI,TI
      WRITE(6,1028) UI,VI,TI
      READ(5,1005) UO,ANGLE,TO,DO,XNUZ
      WRITE(6,1058) UO,ANGLE,TO,DO,XNUZ
      READ(5,1007) WINDX,WINDY,RHOA,E,XK,CFX,CFY
      WRITE(6,1029) WINDX,WINDY,RHOA,E,XK,CFX,CFY
      READ(5,1005)  DH,DV,XNUH,XNUV,CP
      WRITE(6,1030) DH,DV,XNUH,XNUV,CP
      READ(5,1003)  EIMAX,EJMAX,EKMAX
      WRITE(6,1031) EIMAX,EJMAX,EKMAX
      READ (5,1006)  DEBUG1,DEBUG2,DEBUG3,DEBUG4,EIRUN,EIEND
      WRITE(6,1033) DEBUG1,DEBUG2,DEBUG3,DEBUG4,EIRUN,EIEND
       READ(5,1007) EIM,EML,EMK,SLIP,HYD,HEAT,XIIP
      WRITE(6,1071) EIM,EML,EMK,SLIP,HYD,HEAT,XIIP
      IMAX=EIMAX
      JMAX=EJMAX
      KMAX=EKMAX
      NITER=ENITER
      NPRINT=ENPRNT
      IIP=XIIP
C     READ IN SUBSCRIPTED VARIABLES
      READ(5,1007)  (DELX(I),I=1,IMAX)
      WRITE(6,1038)
      WRITE(6,1039) (I,DELX(I),I=1,IMAX)
      READ(5,1007)  (DELY(J),J=1,JMAX)
      WRITE(6,1040)
      WRITE(6,1039) (J,DELY(J),J=1,JMAX)
      READ(5,1007)  (DELZ(K),K=1,KMAX)
      WRITE(6,1041)
      WRITE(6,1039) (K,DELZ(K),K=1,KMAX)
      WRITE(6,1065)
      DO 505 L=1,26
      WRITE(6,1064) TTBL(L),RHOTBL(L)
  505 CONTINUE
      IM=EIM
      ML=EML
      MK=EMK
      MH=MK+1
      I1=IMAX-1
      I2=IMAX-2
      I3=IMAX-3
      J1=JMAX-1
      J2=JMAX-2
      J3=JMAX-3
      K1=KMAX-1
      K2=KMAX-2
```

```
      K3=KMAX-3
      ITNMAX=500
      EPS=.00001
      TIME       =0.
      IP=0
      IPRINT=0
      IJ=0
      G=32.2
      DTSM=1000.
      DTTSM=1000.
      DTMIN=1000.
      DTTMIN=1000.
      WRITE(6,1034) IMAX,JMAX,KMAX
      WRITE(6,1035) I1,J1,K1
      WRITE(6,1036) I2,J2,K2
      WRITE(6,1046) I3,J3,K3
      DO 624 I=2,I1
      XIM1(I)=DELX(I-1)+DELX(I)
      XN(I)=4.*XNUH/XIM1(I)
      XM(I)=4.*DH/XIM1(I)
  624 CONTINUE
      DO 625 J=2,J1
      YJM1(J)=DELY(J-1)+DELY(J)
      YN(J)=4.*XNUH/YJM1(J)
      YM(J)=4.*DH/YJM1(J)
  625 CONTINUE
      DO 626  K=2,K1
      ZKM1(K)=DELZ(K-1)+DELZ(K)
      ZN(K)=4.*XNUV/ZKM1(K)
      ZM(K)=4.*DV/ZKM1(K)
  626 CONTINUE
C     CALCULATE DENSITY OF THERMAL DISCHARGE
      RHOZ=SI(TTBL,RHOTBL,T0,26)
      WRITE(6,1066) RHOZ
      IF(DEBUG1.EQ.0.) GO TO 51
      WRITE(6,629)
      WRITE(6,627) (I,XIM1(I),I=2,I1)
      WRITE(6,630)
      WRITE(6,627) (J,YJM1(J),J=2,J1)
      WRITE(6,631)
      WRITE(6,627) (K,ZKM1(K),K=2,K1)
      WRITE(6,604)
      WRITE(6,608)(I,XN(I),XM(I),I=2,I1)
      WRITE(6,609)
      WRITE(6,608)(J,YN(J),YM(J),J=2,J1)
      WRITE(6,610)
      WRITE(6,608)(K,ZN(K),ZM(K),K=2,K1)
   51 CONTINUE
C     INITIALIZE WATER LEVEL HEIGHT
      DO 706 I=1,IMAX
```

```
      DO 706 J=1,JMAX
      H(I,J)=DELZ(KMAX)/2.
  706 CONTINUE
C     CALCULATE DT
      IF(DEBUG1.EQ.0.) GO TO 655
      WRITE(6,615)
  655 CONTINUE
      DO 613 K=2,K1
      DO 613 J=2,J1
      DO 613 I=2,I1
      DT=1/(2.*(XN(I)/XIM1(I)+YN(J)/YJM1(J)+ZN(K)/ZKM1(K)))
      DTT=1/(2.*(XM(I)/XIM1(I)+YM(J)/YJM1(J)+ZM(K)/ZKM1(K)))
      IF(DEBUG1.EQ.0.) GO TO 656
      WRITE(6,614) I,J,K,DT,DTT
  656 CONTINUE
      IF(DT.LT.DTMIN) GO TO 701
      GO TO 702
  701 DTMIN=DT
      IIM=I
      JJM=J
      KKM=K
  702 IF(DTT.LT.DTTMIN) GO TO 703
      GO TO 613
  703 DTTMIN=DTT
      IIT=I
      JJT=J
      KKT=K
  613 CONTINUE
      WRITE(6,705)
C************************
      WRITE(6,616) IIM,JJM,KKM,DTMIN,IIT,JJT,KKT,DTTMIN
      IF(DEBUG1.EQ.0.) GO TO 717
      WRITE(6,709)
  717 CONTINUE
      DO 707 I=2,I1
      DO 707 J=2,J1
      DTS=1/(2.*(XN(I)/XIM1(I)+YN(J)/YJM1(J)+XNUV/(H(I,J)*(H(I,J)+
     1DELZ(K1)))))
      DTTS=1/(2.*(XM(I)/XIM1(I)+YM(J)/YJM1(J)+DV/(H(I,J)*(H(I,J)+
     1DELZ(K1)))))
      IF(DEBUG1.EQ.0.) GO TO 708
      WRITE(6,710) I,J,KMAX,DTS,DTTS
  708 CONTINUE
      IF(DTS.LT.DTSM) GO TO 711
      GO TO 712
  711 DTSM=DTS
      IIM=I
      JJM=J
  712 IF(DTTS.LT.DTTSM) GO TO 713
      GO TO 707
```

```
  713 DTTSM=DTTS
      IIT=I
      JJT=J
  707 CONTINUE
      WRITE(6,714)
      WRITE(6,715) IIM,JJM,KMAX,DTSM,IIT,JJT,KMAX,DTTSM
      DT=DTMIN
C
C*****BEGIN NONDIMENSIONALIZATION
C
      WRITE(6,1062)
      TBEG=TBEG*U0/D0
      TEND=TEND*U0/D0
      DT=(TEND-TBEG)/ENITER
      UI=UI/U0
      VI=VI/U0
      TI=TI/T0
      WINDX=WINDX/U0
      WINDY=WINDY/U0
      RHOA=RHOA/RHOZ
      TD=TD/T0
      DH=DH/XNUZ
      DV=DV/XNUZ
      XNUH=XNUH/XNUZ
      XNUV=XNUV/XNUZ
      DO 13 I=1,IMAX
      DO 13 J=1,JMAX
      H(I,J)=H(I,J)/D0
   13 CONTINUE
      DO 504 LL=1,26
      TTBL(LL)=TTBL(LL)/T0
      RHOTBL(LL)=RHOTBL(LL)/RHOZ
  504 CONTINUE
      DO 601 I=1,IMAX
      DELX(I)=DELX(I)/D0
  601 CONTINUE
      DO 602 J=1,JMAX
      DELY(J)=DELY(J)/D0
  602 CONTINUE
      DO 603 K=1,KMAX
      DELZ(K)=DELZ(K)/D0
  603 CONTINUE
      RO=D0*U0/XNUZ
      FO=U0/((G*D0)**.5)
      E=E/T0
      XK=XK/(3600*24)
      SD=XK /(RHOZ*CP*U0)
      U0=U0/U0
      T0=T0/T0
      XNUZ=XNUZ/XNUZ
```

```
      WRITE(6,1026) ENRUN
      WRITE(6,1067) TBEG,TEND,NITER,NPRINT
      WRITE(6,1028) UI,VI,TI
      WRITE(6,1058) U0,ANGLE,T0,D0,XNUZ
      WRITE(6,1069) WINDX,WINDY,RHOA
      WRITE(6,1068) DH,DV,XNUH,XNUV
      WRITE(6,1037) G,DT,E
      WRITE(6,1038)
      WRITE(6,1039) (I,DELX(I),I=1,IMAX)
      WRITE(6,1040)
      WRITE(6,1039) (J,DELY(J),J=1,JMAX)
      WRITE(6,1041)
      WRITE(6,1039) (K,DELZ(K),K=1,KMAX)
      WRITE(6,1032) RO,FO,SO
      WRITE(6,1065)
      DO 507 L=1,26
      WRITE(6,1064) TTBL(L),RHOTBL(L)
  507 CONTINUE
      WRITE(6,1063)
C
C*****END NONDIMENSIONALIZATION *************************
C
      DO 33 K=2,K1
      ZKP1(1)=DELZ(1)+DELZ(2)
      ZKP1(K)  =DELZ(K+1)+DELZ(K)
      ZKM1(K)  =DELZ(K-1)+DELZ(K)
      ZKPM1(K) =ZKP1(K)+ZKM1(K)
      ZK123(K)=ZKP1(K)*ZKM1(K)*ZKPM1(K)
   33 CONTINUE
      DO 34 J=2,J1
      YJP1(J)  =DELY(J+1)+DELY(J)
      YJM1(J)  =DELY(J-1)+DELY(J)
      YJPM1(J) =YJP1(J)+YJM1(J)
      YJ123(J) =YJP1(J)*YJM1(J)*YJPM1(J)
   34 CONTINUE
      DO 35 I=2,I1
      XIP1(I)  =DELX(I+1)+DELX(I)
      XIM1(I)  =DELX(I-1)+DELX(I)
      XIPM1(I) =XIP1(I)+XIM1(I)
      XI123(I) =XIP1(I)*XIM1(I)*XIPM1(I)
   35 CONTINUE
      IF(DEBUG1.EQ.0.) GO TO 18
      DO 17 J=2,J1
      DO 17 K=2,K1
      DO 17 I=2,I1
      WRITE(6,1048) K,ZKP1(K),ZKM1(K),ZKPM1(K),ZK123(K),
     1J,YJP1(J),YJM1(J),YJPM1(J),YJ123(J),
     2I,XIP1(I),XIM1(I),XIPM1(I),XI123(I)
   17 CONTINUE
   18 CONTINUE
```

```
C     COEFFICIENT OF VISCOUS TERMS
      XNUR       =XNUH/RC
      ZNUR       =XNUV/RC
      XDHR       =  DH/RC
      XDVR       =  DV/RC
      WRITE(6,1042) XNUR,ZNUR,XDHR,XDVR
      DD=2.*DT
   26 FORMAT(1H0,3(1X,I2),9(1X,E11.4))
      F01=F0**2
C
C     Z=TOTAL HEIGHT OF LAKE
      Z=0.
      DO 110 L=1,K1
      Z=Z-ZKP1(L)/2.
      IF(DEBUG1.EQ.0.) GO TO 58
      WRITE(6,1072) Z
   58 CONTINUE
  110 CONTINUE
   55 FORMAT(1H 5X,'Z=',F5.2)
C     FIRST DERIVATIVE CORRECTION
      DO 311 K=2,K1
      SIGMA5(K)=2.*ZKM1(K)/ZKPM1(K)
      SIGMA6(K)=2.*ZKP1(K)/ZKPM1(K)
  311 CONTINUE
      DO 312 J=2,J1
      SIGMA3(J)=2.*YJM1(J)/YJPM1(J)
      SIGMA4(J)=2.*YJP1(J)/YJPM1(J)
  312 CONTINUE
      DO 313 I=2,I1
      SIGMA1(I)=2.*XIM1(I)/XIPM1(I)
      SIGMA2(I)=2.*XIP1(I)/XIPM1(I)
  313 CONTINUE
C
      IF(DEBUG1.EQ.0.) GO TO 52
      WRITE(6,644)
C**************
      DO 657 I=2,I1
      WRITE(6,645) I,SIGMA1(I),SIGMA2(I)
  657 CONTINUE
C**************
      WRITE(6,646)
      DO 658 J=2,J1
      WRITE(6,645) J,SIGMA3(J),SIGMA4(J)
  658 CONTINUE
C**************
      WRITE(6,647)
      DO 659 K=2,K1
      WRITE(6,645) K,SIGMA5(K),SIGMA6(K)
  659 CONTINUE
C**************
```

```
  648 CONTINUE
   52 CONTINUE
C*************
C     CALCULATE COEFFICIENT OF FIRST DERIVATIVE AT BOUNDARY
C*************
      SIGY=(DELY(1)+DELY(2))/(DELY(2)+DELY(3))
      SIGY1=(1.+2.*SIGY)
      SIGY2=(1.+SIGY)*(1.+SIGY)
      SIGY3=SIGY*SIGY
      SIGY4=(DELY(1)+DELY(2))*(1.+SIGY)/2.
      SIGPY=(DELY(JMAX)+DELY(J1))/(DELY(J1)+DELY(J2))
      SIGPY1=(1.+2.*SIGPY)
      SIGPY2=(1.+SIGPY)*(1.+SIGPY)
      SIGPY3=SIGPY*SIGPY
      SIGPY4=(DELY(JMAX)+DELY(J1))*(1.+SIGPY)/2.
      IF(DEBUG1.EQ.0.) GC TO 53
      WRITE(6,1100) SIGY,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY,SIGPY1,SIGPY2,
     2SIGPY3,SIGPY4
   53 CONTINUE
      SIGX=(DELX(1)+DELX(2))/(DELX(2)+DELX(3))
      SIGX2=(1.+SIGX)*(1.+SIGX)
      SIGX1=(1.+2.*SIGX)
      SIGX3=SIGX*SIGX
      SIGX4=(DELX(1)+DELX(2))*(1.+SIGX)/2.
      IF(DEBUG1.EQ.0.) GC TO 54
      WRITE(6,1110) SIGX,SIGX1,SIGX2,SIGX3,SIGX4
   54 CONTINUE
      SIGZ=(DELZ(1)+DELZ(2))/(DELZ(2)+DELZ(3))
      SIGZ1=(1.+2.*SIGZ)
      SIGZ2=(1.+SIGZ)*(1.+SIGZ)
      SIGZ3=SIGZ*SIGZ
      SIGZ4=(DELZ(1)+DELZ(2))*(1.+SIGZ)/2.
      IF(DEBUG1.EQ.0.) GC TO 56
      WRITE(6,1110) SIGZ,SIGZ1,SIGZ2,SIGZ3,SIGZ4
   56 CONTINUE
C     END CALCULATION OF COEFFICIENT OF FIRST DERIVATIVE
      DMH=AMAX1(XDHR,XNUR)
      DMV=AMAX1(XDVR,ZNUR)
      DO 718 I=2,I1
      DO 718 J=2,J1
      CLEVEL(I,J)=(1./DT)-8.*DMH/(XIM1(I)*XIM1(I))-8.*DMH/(YJM1(J)*
     1YJM1(J))
  718 CONTINUE
C**************
C     CHECK FOR RESTART ENTRY FOR DYNAMIC ANALYSIS
C**********************
C     BEGIN INITIALIZATION
      IF(EIRUN.EQ.1.) GO TO 75
C**********************
      DO 5  K=1,KMAX
```

```
      DO 5  J=1,JMAX
      DO 5  I=1,IMAX
      U(I,J,K)=0.
      V(I,J,K)=0.
      W(I,J,K)=0.
      T(I,J,K)=0.
      RHS(I,J,K)=0.
      DUDT(I,J,K)=0.
      DVDT(I,J,K)=0.
      DWDT(I,J,K)=0.
      DTDT(I,J,K)=0.
      DHDT(I,J)=0.
    5 CONTINUE
C******INITIALIZATION OF HORIZONTAL VELOCITY COMPONENTS U AND V
      DO 25 K=2,KMAX
      DO 25 J=2,J1
      DO 25 I=1,IMAX
      U(I,J,K)  =UI
      V(I,J,K)  =VI
   25 CONTINUE
      Z1=Z
C*****INITIALIZATION OF TEMPERATURE
C*****FLUID-FLUID BOUNDARY ON PRESSURE
      DO 502 K=1,KMAX
      DO 506 J=1,JMAX
      DO 506 I=1,IMAX
      T(I,J,K)  =TI
      RHO(I,J,K)=SI(TTBL,RHOTBL,T(I,J,K),26)
      P(I,J,K)=-RHO(I,J,K)*Z
  506 CONTINUE
      IF(K.EQ.KMAX) GO TO 502
      Z=Z+ZKP1(K)/2.
      IF(K.EQ.K1) Z=-DELZ(KMAX)/4.
      IF(DEBUG1.EQ.0.) GO TO 57
      WRITE(6,1072) Z
   57 CONTINUE
  502 CONTINUE
      Z=Z1
C     END INITIALIZATION
C     SET THE BOUNDARY CONDITIONS
C     FIXED BOUNDARY CONDITIONS
C*****FIXED BOUNDARY CONDITION ON TEMPERATURE IS BEEN SET DURING INITIALIZATION*
C     FIXED VELOCITY BOUNDARY CONDITION
C
      DO 4 K=2,KMAX
      DO 4 J=2,J1
      U(1,J,K)=UI
    4 CONTINUE
C********
C     SET VELOCITY OF THERMAL DISCHARGE
```

```
      DO 63 I=ML,MK
      DO 63 K=IM,KMAX
      U(I,1,K)=U0*COS(ANGLE)
      V(I,1,K)=U0*SIN(ANGLE)
 63   CONTINUE
      GO TO 79
 75   CONTINUE
C*****READING FROM DISC*******************************
      REWIND 24
      READ(24) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
C**********DISC OUTPUT************
      WRITE(6,2006) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
      WRITE(6,2007)
      DO 76 K=1,KMAX
      DO 76 J=1,JMAX
      DO 76 I=1,IMAX
      READ(24) U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K),H(I,J),
     1RHO(I,J,K)
      WRITE(6,2005) I,J,K,U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K),
     1RHO(I,J,K),H(I,J)
 76   CONTINUE
C******END OF DISC OUTPUT****************
      IPRINT=0
 79   CONTINUE
C
      DO 635 I=2,I1
      XN(I)=4.*XNUR/XIM1(I)
      XM(I)=4.*XDHR/XIM1(I)
      XL(I)=AMIN1(XN(I),XM(I))
 635  CONTINUE
      DO 636 J=2,J1
      YN(J)=4.*XNUR/YJM1(J)
      YM(J)=4.*XDHR/YJM1(J)
      YL(J)=AMIN1(YN(J),YM(J))
 636  CONTINUE
      DO 637 K=2,K1
      ZN(K)=4.*ZNUR/ZKM1(K)
      ZM(K)=4.*XDVR/ZKM1(K)
      ZL(K)=AMIN1(ZN(K),ZM(K))
 637  CONTINUE
      IF(DEBUG1.EQ.0.) GO TO 654
      WRITE(6,604)
      WRITE(6,608)(I,XN(I),XM(I),I=2,I1)
      WRITE(6,609)
      WRITE(6,608)(J,YN(J),YM(J),J=2,J1)
      WRITE(6,610)
      WRITE(6,608)(K,ZN(K),ZM(K),K=2,K1)
      WRITE(6,640)
      WRITE(6,641) (I,XL(I),I=2,I1)
      WRITE(6,642)
```

```
            WRITE(6,641) (J,YL(J),J=2,J1)
            WRITE(6,643)
            WRITE(6,641) (K,ZL(K),K=2,K1)
      654   CONTINUE
            WRITE(6,1025)
            IF(HEAT.EQ.0.) GO TO 1
C           SET BOUNDARY AT THERMAL DISCHARGE INLET
C
C           TEMPERATURE
            DO 61 K=IM,KMAX
            DO 61 I=ML,MK
            T(I,1,K)=1.
       61   CONTINUE
C
C           PRESSURE
C
            Z1=Z
            DO 43 K=1,KMAX
            DO 42 I=ML,MK
            P(I,1,K)=-Z
       42   CONTINUE
            Z=Z+ZKP1(K)/2.
            IF(K.EQ.K1) Z=-DELZ(KMAX)/4.
       43   CONTINUE
            Z=Z1
C
C ********* BEGINNING OF THE DYNAMIC LOOP *************
C           VARIABLE BOUNDARY CONDITIONS
C
        1   CONTINUE
C           CHECK STABILITY FOR U,V, AND W
            DO 617 K=2,KMAX
            DO 617 I=2,I1
            DO 617 J=2,J1
            IF(ABS(U(I,J,K)).GT.XL(I)) GO TO 618
            IF(ABS(V(I,J,K)).GT.YL(J)) GO TO 619
            IF(K.EQ.KMAX) GO TO 716
            IF(ABS(W(I,J,K)).GT.ZL(K)) GO TO 620
      716   CONTINUE
            GO TO 617
      618   WRITE(6,621) I,J,K
            IPRINT=NPRINT
            CALL OUTP
            CALL EXIT
      619   WRITE(6,622) I,J,K
            IPRINT=NPRINT
            CALL OUTP
            CALL EXIT
      620   WRITE(6,623) I,J,K
            IPRINT=NPRINT
```

```
      CALL OUTP
      CALL EXIT
  617 CONTINUE
      DO 719 I=2,I1
      DO 719 J=2,J1
      HLEVEL=2.*DMV/(H(I,J)*(H(I,J)+DELZ(K1)))
      IF(HLEVEL.LT.CLEVEL(I,J)) GO TO 719
      WRITE(6,720) I,J,HLEVEL,CLEVEL(I,J)
      IPRINT=NPRINT
      CALL OUTP
      CALL EXIT
  719 CONTINUE
C     END STABILITY ANALYSIS
C
C     OUTGOING BOUNDARY ON VELOCITY
      DO 70 J=1,JMAX
      DO 70 K=2,KMAX
      U(IMAX,J,K)=U(I1,J,K)
      V(IMAX,J,K)=V(I1,J,K)
   70 CONTINUE
C*****BOTTOM BOUNDARY ON HORIZONTAL VELOCITY COMPONENTS
      DO 236 I=1,IMAX
      DO 236 J=1,JMAX
      V(I,J,1)=0.
      U(I,J,1)=0.
  236 CONTINUE
C*****SOLID-FLUID BOUNDARY ON HORIZONTAL VELOCITY COMPONENT U
      IF(SLIP.EQ.0.) GO TO 950
      DO 6 K=2,KMAX
      DO 6 I=1,IMAX
      U(I,JMAX,K)=(SIGPY2*U(I,J1,K)-SIGPY3*U(I,J2,K))/SIGPY1
    6 CONTINUE
      DO 135 K=2,KMAX
      DO 137 I=1,IMAX
      IF(K.LT.IM) GO TO 134
      IF( I .LT. MH .AND. I .GE. ML ) GO TO 137
  134 CONTINUE
      U(I,1,K)=(SIGY2*U(I,2,K)-SIGY3*U(I,3,K))/SIGY1
  137 CONTINUE
  135 CONTINUE
  950 CONTINUE
C*****SOLID-FLUID BOUNDARY ON TEMPERATURE
      DO 130 K=2,KMAX
      DO 130 I=1,IMAX
      T(I,JMAX,K)=(SIGPY2*T(I,J1,K)-SIGPY3*T(I,J2,K))/SIGPY1
  130 CONTINUE
      DO 125 K=2,KMAX
      DO 126 I=1,IMAX
      IF(K.LT.IM) GO TO 124
      IF( I .LT. MH .AND. I .GE. ML ) GO TO 126
```

```
 124  CONTINUE
      T(I,1,K)=(SIGY2*T(I,2,K)-SIGY3*T(I,3,K))/SIGY1
 126  CONTINUE
 125  CONTINUE
C*****SURFACE BOUNDARY ON VERTICAL VELOCITY COMPONENT IS ZEROED DURING
C     INITIALIZATION
C*****SURFACE BOUNDARY ON HORIZONTAL VELOCITY COMPONENTS U AND V
C
      DO 7 I=1,IMAX
      DO 7 J=1,JMAX
      T(I,J,1)=(SIGZ2*T(I,J,2)-SIGZ3*T(I,J,3))/SIGZ1
   7  CONTINUE
C
C
      DO 73 I=1,IMAX
      DO 73 J=1,JMAX
      T(IMAX,J,K)=T(I1,J,K)
 73   CONTINUE
C*****CALCULATION OF DHDT(I,J) FOR THE TOP ELEMENTS ***************************
 800  DO 661 I=2,I1
      IM1=I-1
 1120 DO 661 J=2,J1
      JM1=J-1
      DHDT(I,J)=-2.*(H(I,J)*(U(I,J,KMAX)-U(IM1,J,KMAX))*
     1U(I,J,KMAX)*(H(I,J)-H(IM1,J)))/XIM1(I)
     2-2.*(H(I,J)*(V(I,J,KMAX)-V(I,JM1,KMAX))+
     3V(I,J,KMAX)*(H(I,J)-H(I,JM1)))/YJM1(J)
 661  CONTINUE
C
C     CALCULATE VERTICAL VELOCITY COMPONENT FROM CONTINUITY
      DO 805 I=2,I1
      IP1=I+1
      IM1=I-1
      DO 805 J=2,J1
      JP1=J+1
      JM1=J-1
      DO 805 K=2,K1
      KM1=K-1                                                              3/8
      DUDX=(U(IP1,J,K)-U(I,J,K))*SIGMA1(I)/XIP1(I)+(U(I,J,K)-U(IM1,J,K))
     1*SIGMA2(I)/XIM1(I)
      DVDY=(V(I,JP1,K)-V(I,J,K))*SIGMA3(J)/YJP1(J)+(V(I,J,K)-V(I,JM1,K))
     1*SIGMA4(J)/YJM1(J)
      W(I,J,K)=W(I,J,KM1)-(DUDX+DVDY)*ZKM1(K)/2.
 805  CONTINUE
      DO 72 K=2,KMAX
      DO 72 J=2,J1
      W(IMAX,J,K)=W(I1,J,K)
 72   CONTINUE
      DO 232 I=2,I1
      DO 232 J=2,J1
```

```
            W(I,J,KMAX)=W(I,J,K1)+DHDT(I,J)
      232   CONTINUE
C           END OF W CALCULATION
C           SET THE BOUNDARY ON W
C*****SOLID-FLUID BOUNDARY ON VERTICAL VELOCITY COMPONENT W
            IF(SLIP.EQ.0.) GO TO 940
            DO 145 K=2,KMAX
            DO 146 I=1,IMAX
            IF(K.LT.IM) GO TO 144
            IF( I .LT. MH .AND. I .GE. ML ) GO TO 146
      144   CONTINUE
            W(I,1,K)=(SIGY2*W(I,2,K)-SIGY3*W(I,3,K))/SIGY1
      146   CONTINUE
      145   CONTINUE
            DO 147 K=2,KMAX
            DO 147 I=1,IMAX
            W(I,JMAX,K)=(SIGPY2*W(I,J1,K)-SIGPY3*W(I,J2,K))/SIGPY1
      147   CONTINUE
      940   CONTINUE
C
C*****SET DHDT(I,J)=W(I,J,KMAX)**********************************************
C
            DO 233 I=1,IMAX
            DO 233 J=1,JMAX
            DHDT(I,J)=W(I,J,KMAX)
      233   CONTINUE
C
C*****BOTTOM BOUNDARY ON DENSITY
C
            DO 12 J=1,JMAX
            DO 12 I=1,IMAX
            RHO(I,J,1)=SI(TTBL,RHOTBL,T(I,J,1),26)
            RHO(I,J,KMAX)=SI(TTBL,RHOTBL,T(I,J,KMAX),26)
       12   CONTINUE
C
C           END OF BOUNDARY CONDITIONS
C
            IF(TIME.GT.DD) GO TO 32
            IPRINT=NPRINT
            CALL OUTP
            IPRINT=IPRINT+2
            IJ=IJ+2
       32   CONTINUE
            CALL OUTP
            IJ=IJ+1
            CALL DERIV
            CALL SUM
C           DENSITY CALCULATION
            DO 40 K=1,KMAX
            DO 40 I=2,I1
```

```
      DO 40 J=2,J1
      RHO(I,J,K)=SI(TTBL,RHOTBL,T(I,J,K),26)
   40 CONTINUE
      TIME=TIME+DT
      IPRINT=IPRINT+1
      IF(EIEND.EQ.0.) GO TO 2002
      IF(IP.EQ.IIP) GO TO 2001
 2002 CONTINUE
      IF(NITER.EQ.IP) GO TO 20
      IP=IP+1
      GO TO 1
   20 CALL OUTP
 2001 CONTINUE
C*****WRITING OUTPUT ON THE DISC*************
      WRITE(6,1025)
      REWIND 25
      WRITE(25) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
      WRITE(6,2006) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
      WRITE(6,2007)
      DO 2003 K=1,KMAX
      DO 2003 J=1,JMAX
      DO 2003 I=1,IMAX
      WRITE(25) U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K),H(I,J),
     1RHO(I,J,K)
      WRITE(6,2005) I,J,K,U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K),
     1RHO(I,J,K),H(I,J)
 2003 CONTINUE
      STOP
      END
```

```
      SUBROUTINE OUTP
      COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
     1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
     2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
     3XNUH,XNUV,RO,FO,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,Z,
     4WINDX,WINDY,E,RHOA,IMAX,JMAX,KMAX,OMEGA,IM,MH,ML,MK,
     5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUR,ZNUR,XDHR,XDVR,G,DT,
     6NITER,NPRINT,I1,I2,J1,J2,K1,K2,TIME,IPRINT,RHS(20,14,10),
     7XIP1(20),XIM1(20),XIPM1(20),XI123(20),YJP1(14),XIP2(20),
     8YJM1(14),YJPM1(14),YJ123(14),H(20,14),DZ,YJP2(14),
     9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10)
     A,II,JJ,KK,I3,J3,K3,FO1,IIT,B,EPS,ITNMAX,SIGMA1(20),SIGMA2(20),
     BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHDT(20,14)
      COMMON SIGX1,SIGX2,SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY1,
     1SIGPY2,SIGPY3,SIGPY4,SIGZ1,SIGZ2,SIGZ3,SIGZ4
      COMMON/TABLE/TTBL,RHOTBL
      DIMENSION TTBL(26),RHOTBL(26)
      IF(IPRINT.LT.NPRINT.OR.DEBUG3.NE.0.) RETURN
      IPRINT=0
      IJ=0
   10 CONTINUE
      WRITE(6,1022) TIME,DT ,IP
 1022 FORMAT(1H0,40X,'TIME=',E11.4,5X,'DT  =',E11.4,5X,'ITERATION NUMBER
     1=',I4/)
      WRITE(6,1020)
      DO 15 K=1,KMAX
      DO 15 J=1,JMAX
      MJ=1
   20 MJJ=MJ+1
      IF(MJ.EQ.IMAX) MJJ=MJ
      IF(K.EQ.KMAX) GO TO 25
      WRITE(6,1021) (I,J,K,U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K)
     1,I=MJ,MJJ)
      MJ=MJ+2
      IF(MJJ.EQ.IMAX) GO TO 15
      GO TO 20
   25 CONTINUE
      MJ=1
   30 CONTINUE
      MJJ=MJ+1
      IF(MJ.EQ.IMAX) MJJ=MJ
      WRITE(6,1023) (I,J,K,U(I,J,K),V(I,J,K),W(I,J,K),T(I,J,K),P(I,J,K)
     1,H(I,J),I=MJ,MJJ)
      MJ=MJ+2
      IF(MJJ.EQ.IMAX) GO TO 15
      GO TO 30
   15 CONTINUE
 1021 FORMAT(1H ,2(3(I2,1X),3(E11.4),2(F6.4),11X))
 1023 FORMAT(1H ,2(3(I2,1X),3(E11.4),2(F6.4),E11.4))
 1020 FORMAT(1H ,2(1X,'I',2X,'J',2X,'K',7X,'U',9X,'V',9X,'W',7X,'T',7X,
```

```
     1'P',7X,'H',5X))
      RETURN
      END
```

```
      SUBROUTINE SUM
       COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
     1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
     2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
     3XNUH,XNUV,RO,FO,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,Z,
     4WINDX,WINDY,E,RHOA,IMAX,JMAX,KMAX,OMEGA,IM,MH,ML,MK,
     5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUR,ZNUR,XDHR,XDVR,G,DT,
     6NITER,NPRINT,I1,I2,J1,J2,K1,K2,TIME,IPRINT,RHS(20,14,10),
     7XIP1(20),XIM1(20),XIPM1(20),XI123(20),YJP1(14),XIP2(20),
     8YJM1(14),YJPM1(14),YJ123(14),H(20,14),DZ,YJP2(14),
     9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10)
     A,II,JJ,KK,I3,J3,K3,FO1,IIT,B,EPS,ITNMAX,SIGMA1(20),SIGMA2(20),
     BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHDT(20,14)
       COMMON SIGX1,SIGX2,SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY1,
     1SIGPY2,SIGPY3,SIGPY4,SIGZ1,SIGZ2,SIGZ3,SIGZ4
       COMMON/TABLE/TTBL,RHOTBL
       DIMENSION TTBL(26),RHOTBL(26)
C      INTEGRATE FOR HORIZONTAL VELOCITY COMPONENTS AND TEMPERATURE
       DO 22 K=2,KMAX
       DO 21 J=2,J1
       DO 20 I=2,I1
       U(I,J,K)=U(I,J,K)+DT*DUDT(I,J,K)
       V(I,J,K)=V(I,J,K)+DT*DVDT(I,J,K)
       T(I,J,K)=T(I,J,K)+DT*DTDT(I,J,K)
  20   CONTINUE
  21   CONTINUE
 22    CONTINUE
C      INTEGRATE FOR WATER LEVEL HEIGHT H
       DO 23 I=1,IMAX
       DO 23 J=1,JMAX
        H(I,J)=H(I,J)+DT*DHDT(I,J)
 23    CONTINUE
       RETURN
       END
```

```
      SUBROUTINE DERIV
      COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
     1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
     2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
     3XNUH,XNUV,RO,FO,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,Z,
     4WINDX,WINDY,E,RHOA,IMAX,JMAX,KMAX,OMEGA,IM,MH,ML,MK,
     5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUR,ZNUR,XDHR,XDVR,G,DT,
     6NITER,NPRINT,I1,I2,J1,J2,K1,K2,TIME,IPRINT,RHS(20,14,10),
     7XIP1(20),XIM1(20),XIPM1(20),XI123(20),YJP1(14),XIP2(20),
     8YJM1(14),YJPM1(14),YJ123(14),H(20,14),DZ,YJP2(14),
     9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10)
     A,II,JJ,KK,I3,J3,K3,FO1,IIT,B,EPS,ITNMAX,SIGMA1(20),SIGMA2(20),
     BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHDT(20,14)
      COMMON SIGX1,SIGX2,SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY1,
     1SIGPY2,SIGPY3,SIGPY4,SIGZ1,SIGZ2,SIGZ3,SIGZ4
      COMMON/TABLE/TTBL,RHOTBL
      DIMENSION TTBL(26),RHOTBL(26)
C
C     CALCULATE DERIVATIVES FOR WATER LEVEL ELEMENTS
C
      DO 100 I=2,I1
      DO 100 J=2,J1
      IP1=I+1
      IM1=I-1
      JP1=J+1
      JM1=J-1
C
C     ADJUST WATER LEVEL VARIABLE
C
      ZKP1(K1)=DELZ(K1)+H(I,J)
      DUDXR=(U(IP1,J,KMAX)-U(I,J,KMAX))/XIP1(I)
      DUDXL=(U(I,J,KMAX)-U(IM1,J,KMAX))/XIM1(I)
      DUDYR=(U(I,JP1,KMAX)-U(I,J,KMAX))/YJP1(J)
      DUDYL=(U(I,J,KMAX)-U(I,JM1,KMAX))/YJM1(J)
      DUDZL=2.*(U(I,J,KMAX)-U(I,J,K1))/ZKP1(K1)
      DHDXR=(H(IP1,J)-H(I,J))/XIP1(I)
      DHDXL=(H(I,J)-H(IM1,J))/XIM1(I)
      DHDYL=(H(I,J)-H(I,JM1))/YJM1(J)
      DHDYR=(H(I,JP1)-H(I,J))/YJP1(J)
      DRHDXR=(RHO(IP1,J,KMAX)-RHO(I,J,KMAX))/XIP1(I)
      DRHDXL=(RHO(I,J,KMAX)-RHO(IM1,J,KMAX))/XIM1(I)
      DRHDYR=(RHO(I,JP1,KMAX)-RHO(I,J,KMAX))/YJP1(J)
      DRHDYL=(RHO(I,J,KMAX)-RHO(I,JM1,KMAX))/YJM1(J)
C
C     CALCULATE HORIZONTAL MOMENTUM IN X-DIRECTION FOR WATER LEVEL ELEMENTS
C
      DUDT(I,J,KMAX)=-U(I,J,KMAX)*(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)
     1-V(I,J,KMAX)*(SIGMA3(J)*DUDYR+SIGMA4(J)*DUDYL)
     2-H(I,J)*(SIGMA1(I)*DRHDXR+SIGMA2(I)*DRHDXL)/FO1
     3+((SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)
```

```
     4*(XNUR*(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)/H(I,J)
     5-2.*RHO(I,J,KMAX)/F01))
     6+XNUR*8.*((DUDXR-DUDXL)/XIPM1(I)+(DUDYR-DUDYL)/YJPM1(J))
     7+XNUR*((SIGMA3(J)*DUDYR+SIGMA4(J)*DUDYL)
     8*(SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL))/H(I,J)
     A-ZNUR*DUDZL/H(I,J)
     B+CFX*RHOA*WINDX*WINDX/(2.*H(I,J))
C
C     CALCULATE VELOCITY GRADIENT FOR Y-COMPONENT OF VELOCITY FOR WATER LEVEL
C
      DVDXR=(V(IP1,J,KMAX)-V(I,J,KMAX))/XIP1(I)
      DVDXL=(V(I,J,KMAX)-V(IM1,J,KMAX))/XIM1(I)
      DVDYR=(V(I,JP1,KMAX)-V(I,J,KMAX))/YJP1(J)
      DVDYL=(V(I,J,KMAX)-V(I,JM1,KMAX))/YJM1(J)
      DVDZL=2.*(V(I,J,KMAX)-V(I,J,K1))/ZKP1(K1)
C
C     CALCULATE HORIZONTAL MOMENTUM IN Y-DIRECTION FOR WATER LEVEL ELEMENTS
C
      DVDT(I,J,KMAX)=-U(I,J,KMAX)*(SIGMA1(I)*DVDXR+SIGMA2(I)*DVDXL)
     1-V(I,J,KMAX)*(SIGMA3(J)*DVDYR+SIGMA4(J)*DVDYL)
     2-H(I,J)*(SIGMA3(J)*DRHDYR+SIGMA4(J)*DRHDYL)/F01
     3+XNUR*((SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)
     4*(SIGMA1(I)*DVDXR+SIGMA2(I)*DVDXL))/H(I,J)
     5+((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)
     6*(XNUR*(SIGMA3(J)*DVDYR+SIGMA4(J)*DVDYL)/H(I,J)
     7-2.*RHO(I,J,KMAX)/F01))
     8+8.*XNUR*((DVDXR-DVDXL)/XIPM1(I)+(DVDYR-DVDYL)/YJPM1(J))
     A-ZNUR*DVDZL/H(I,J)
     B+CFY*RHOA*WINDY*WINDY/(2.*H(I,J))
C
C     CALCULATE TEMPERATURE GRADIENT FOR WATER LEVEL ELEMENTS
C
      DTDXR=(T(IP1,J,KMAX)-T(I,J,KMAX))/XIP1(I)
      DTDXL=(T(I,J,KMAX)-T(IM1,J,KMAX))/XIM1(I)
      DTDYR=(T(I,JP1,KMAX)-T(I,J,KMAX))/YJP1(J)
      DTDYL=(T(I,J,KMAX)-T(I,JM1,KMAX))/YJM1(J)
      DTDZL=2.*(T(I,J,KMAX)-T(I,J,K1))/ZKP1(K1)
C
C     CALCULATE ENERGY EGUATION FOR WATER LEVEL ELEMENTS
C
      DTDT(I,J,KMAX)=-U(I,J,KMAX)*(SIGMA1(I)*DTDXR+SIGMA2(I)*DTDXL)
     1-V(I,J,KMAX)*(SIGMA3(J)*DTDYR+SIGMA4(J)*DTDYL)
     2+XDHR*((SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)
     3*(SIGMA1(I)*DTDXR+SIGMA2(I)*DTDXL))/H(I,J)
     4+XDHR*8.*((DTDXR-DTDXL)/XIPM1(I)+(DTDYR-DTDYL)/YJPM1(J))
     5+XDHR*((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)
     6*(SIGMA3(J)*DTDYR+SIGMA4(J)*DTDYL))/H(I,J)
     8-XDVR*DTDZL/H(I,J)
     9-SO*(T(I,J,KMAX)-E)/H(I,J)
  100 CONTINUE
```

```
      DO 7 K=2,K1
      DO 6 J=2,J1
      DO 5 I=2,I1
      IP1=I+1
      IM1=I-1
      JP1=J+1
      JM1=J-1
      KP1=K+1
      KM1=K-1
C
C     ADJUST WATER LEVEL VARIABLE
C
      IF(K.NE.K1) GO TO 506
      ZKP1(K1)=DELZ(K1)+H(I,J)
      ZKPM1(K1)=ZKP1(K1)+ZKM1(K1)
      SIGMA5(K)=2.*ZKM1(K)/ZKPM1(K)
      SIGMA6(K)=2.*ZKP1(K)/ZKPM1(K)
 506  CONTINUE
      IF(DEBUG2.EQ.0.) GO TO 11
      WRITE(6,12) I,J,K
  12  FORMAT(1H0,2X,'I=',I3,2X,'J=',I3,2X,'K=',I3,/)
      WRITE(6,10) IP1,IM1,JP1,JM1,KP1,KM1
  10  FORMAT(1H ,2X,'IP1=',I3,2X,'IM1=',I3,2X,'JP1=',I3,2X,'JM1=',I3,2X,
     1'KP1=',I3,2X,'KM1=',I3)
  11  CONTINUE
C
C*****CALCULATION OF HORIZONTAL MOMENTUM IN X-DIRECTION
C
      DUDXR=(U(IP1,J,K)-U(I,J,K))/XIP1(I)
      DUDXL=(U(I,J,K)-U(IM1,J,K))/XIM1(I)
      DUDYR=(U(I,JP1,K)-U(I,J,K))/YJP1(J)
      DUDYL=(U(I,J,K)-U(I,JM1,K))/YJM1(J)
      DUDZR=(U(I,J,KP1)-U(I,J,K))/ZKP1(K)
      DUDZL=(U(I,J,K)-U(I,J,KM1))/ZKM1(K)
      DUDT(I,J,K)=-U(I,J,K)*(DUDXR*SIGMA1(I)+DUDXL*SIGMA2(I))
     1-V(I,J,K)*(DUDYR*SIGMA3(J)+DUDYL*SIGMA4(J))
     2-W(I,J,K)*(DUDZR*SIGMA5(K)+DUDZL*SIGMA6(K))
     3+8.*(XNUR*((DUDXR-DUDXL)/XIPM1(I)+(DUDYR-DUDYL)/YJPM1(J))
     4+ZNUR*(DUDZR-DUDZL)/ZKPM1(K))
C
C*****CALCULATION OF HORIZONTAL MOMENTUM IN Y-DIRECTION
C
      DVDXR=(V(IP1,J,K)-V(I,J,K))/XIP1(I)
      DVDXL=(V(I,J,K)-V(IM1,J,K))/XIM1(I)
      DVDYR=(V(I,JP1,K)-V(I,J,K))/YJP1(J)
      DVDYL=(V(I,J,K)-V(I,JM1,K))/YJM1(J)
      DVDZR=(V(I,J,KP1)-V(I,J,K))/ZKP1(K)
      DVDZL=(V(I,J,K)-V(I,J,KM1))/ZKM1(K)
      DVDT(I,J,K)=-U(I,J,K)*(DVDXR*SIGMA1(I)+DVDXL*SIGMA2(I))
     1-V(I,J,K)*(DVDYR*SIGMA3(J)+DVDYL*SIGMA4(J))
```

```
     2-W(I,J,K)*(DVDZR*SIGMA5(K)+DVDZL*SIGMA6(K))
     3+8.*(XNUR*((DVDXR-DVDXL)/XIPM1(I)
     4+(DVDYR-DVDYL)/YJPM1(J))
     5+ZNUR*(DVDZR-DVDZL)/ZKPM1(K))
C
C*****CALCULATION OF VERTICAL MOMENTUM IN Z-DIRECTION
C
      DWDXR=(W(IP1,J,K)-W(I,J,K))/XIP1(I)
      DWDXL=(W(I,J,K)-W(IM1,J,K))/XIM1(I)
      DWDYR=(W(I,JP1,K)-W(I,J,K))/YJP1(J)
      DWDYL=(W(I,J,K)-W(I,JM1,K))/YJM1(J)
      DWDZR=(W(I,J,KP1)-W(I,J,K))/ZKP1(K)
      DWDZL=(W(I,J,K)-W(I,J,KM1))/ZKM1(K)
      DWDT(I,J,K)=-U(I,J,K)*(DWDXR*SIGMA1(I)+DWDXL*SIGMA2(I))
     1-V(I,J,K)*(DWDYR*SIGMA3(J)+DWDYL*SIGMA4(J))
     2-W(I,J,K)*DWDZL*2.
     3+8.*(XNUR*((DWDXR-DWDXL)/XIPM1(I)
     4+(DWDYR-DWDYL)/YJPM1(J))
     5+ZNUR*(DWDZR-DWDZL)/ZKPM1(K))
     6-RHO(I,J,K)/F01
C
C*****CALCULATION OF ENERGY EQUATION
C
      DTDXR=(T(IP1,J,K)-T(I,J,K))/XIP1(I)
      DTDXL=(T(I,J,K)-T(IM1,J,K))/XIM1(I)
      DTDYR=(T(I,JP1,K)-T(I,J,K))/YJP1(J)
      DTDYL=(T(I,J,K)-T(I,JM1,K))/YJM1(J)
      DTDZR=(T(I,J,KP1)-T(I,J,K))/ZKP1(K)
      DTDZL=(T(I,J,K)-T(I,J,KM1))/ZKM1(K)
      DTDT(I,J,K)=-U(I,J,K)*(DTDXR*SIGMA1(I)+DTDXL*SIGMA2(I))
     1-V(I,J,K)*(DTDYR*SIGMA3(J)+DTDYL*SIGMA4(J))
     2-W(I,J,K)*(DTDZR*SIGMA5(K)+DTDZL*SIGMA6(K))
     3+8.*(XDHR*((DTDXR-DTDXL)/XIPM1(I)
     4+(DTDYR-DTDYL)/YJPM1(J))
     5+XDVR*((DTDZR-DTDZL)/ZKPM1(K)))
C
C*****CALCULATION OF DIVERGENCE
C
      IF(DEBUG2.EQ.0.) GO TO 90
      IF(IJ.NE.NPRINT) GO TO 90
      DIV=(DUDXR*SIGMA1(I)+DUDXL*SIGMA2(I))+(DVDYR*SIGMA3(J)+
     1DVDYL*SIGMA4(J))+DWDZL*2.
      WRITE(6,80) I,J,K,DUDT(I,J,K),DVDT(I,J,K),DWDT(I,J,K),DIV
   90 CONTINUE
   80 FORMAT(1H ,3(I2,2X),2X,'DUDT(I,J,K)=',E15.7,2X,'DVDT(I,J,K)=',
     1E15.7,2X,'DWDT(I,J,K)=',E15.7,2X,'DIV=',E15.7)
    5 CONTINUE
    6 CONTINUE
    7 CONTINUE
C     SET QX AT THE BOUNDARIES
```

```
      DO 200 J=2,J1
      DO 200 K=2,K1
      DUDT(1,J,K)=0.
      DUDT(IMAX,J,K)=DUDT(I1,J,K)
  200 CONTINUE
C     SET QY AT THE BOUNDARIES
      DO 210 I=2,I1
      DO 210 K=2,K1
      DVDYRO=(V(I,3,K)-V(I,2,K))/YJP1(2)
      DVDYLO=(V(I,2,K)-V(I,1,K))/YJM1(2)
      DVDYRM=(V(I,JMAX,K)-V(I,J1,K))/YJP1(J1)
      DVDYLM=(V(I,J1,K)-V(I,J2,K))/YJM1(J1)
      DVDT(I,JMAX,K)=8.*XNUR*(DVDYRM-DVDYLM)/YJPM1(J1)
      IF(K.LT.IM) GO TO 205
      IF( I .LT. MH .AND. I .GE. ML ) GO TO  220
  205 CONTINUE
      DVDT(I,1,K)=8.*XNUR*(DVDYRO-DVDYLO)/YJPM1(2)
      GO TO 210
  220 CONTINUE
      DVDT(I,1,K)=0.
  210 CONTINUE
C     SET QZ AT THE BOUNDARIES
      DO 230 I=2,I1
      DO 230 J=2,J1
      DWDZTO=(W(I,J,3)-W(I,J,2))/ZKP1(2)
      DWDZLO=(W(I,J,2)-W(I,J,1))/ZKM1(2)
      DWDT(I,J,1)=-RHO(I,J,1)/FO1+8.*ZNUR*(DWDZTO-DWDZLO)/ZKPM1(2)
      DWDT(I,J,KMAX)=-RHO(I,J,KMAX)/FO1
  230 CONTINUE
      DO 504 K=2,K1
      KP1=K+1
      KM1=K-1
      DO 504 J=2,J1
      JP1=J+1
      JM1=J-1
      DO 504 I=2,I1
      IP1=I+1
      IM1=I-1
C
C     ADJUST WATER LEVEL VARIABLE
C
      IF(K.NE.K1) GO TO 505
      ZKP1(K1)=DELZ(K1)+H(I,J)
      ZKPM1(K1)=ZKP1(K1)+ZKM1(K1)
      SIGMA5(K)=2.*ZKM1(K)/ZKPM1(K)
      SIGMA6(K)=2.*ZKP1(K)/ZKPM1(K)
  505 CONTINUE
      DQDZ=(DWDT(I,J,KP1)-DWDT(I,J,K))*SIGMA5(K)/ZKP1(K)+(DWDT(I,J,K)-
     1DWDT(I,J,KM1))*SIGMA6(K)/ZKM1(K)
      RHS(I,J,K)=DQDZ
```

```
      DQDY=(DVDT(I,JP1,K)-DVDT(I,J,K))*SIGMA3(J)/YJP1(J)+(DVDT(I,J,K)-
     1DVDT(I,JM1,K))*SIGMA4(J)/YJM1(J)
      RHS(I,J,K)=RHS(I,J,K)+DQDY
      DQDX=(DUDT(IP1,J,K)-DUDT(I,J,K))*SIGMA1(I)/XIP1(I)+(DUDT(I,J,K)-
     1DUDT(IM1,J,K))*SIGMA2(I)/XIM1(I)
      RHS(I,J,K)=(RHS(I,J,K)+DQDX)*FO1
      IF(DEBUG2.EQ.0.) GO TO 504
      WRITE(6,1000) I,J,K,DQDX,DQDY,DQDZ
 1000 FORMAT(1H ,2X,'I=',I2,2X,'J=',I2,2X,'K=',I2,2X,'DQDX=',E14.7,2X,
     1'DQDY=',E14.7,2X,'DQDZ=',E14.7)
  504 CONTINUE
  600 CONTINUE
      CALL PRESS
      DO 15 K=2,K1
      DO 15 J=2,J1
      DO 15 I=2,I1
      IP1=I+1
      IM1=I-1
      JP1=J+1
      JM1=J-1
      KP1=K+1
      KM1=K-1
C
C     CALCULATE PRESSURE GRADIENT IN X-DIRECTION
C
      DPDX=(P(IP1,J,K)-P(I,J,K))*SIGMA1(I)/XIP1(I)+(P(I,J,K)-P(IM1,J,K))
     1*SIGMA2(I)/XIM1(I)
      DUDT(I,J,K)=DUDT(I,J,K)-DPDX/FO1
C
C     CALCULATE PRESSURE GRADIENT IN Y-DIRECTION
C
      DPDY=(P(I,JP1,K)-P(I,J,K))*SIGMA3(J)/YJP1(J)+(P(I,J,K)-P(I,JM1,K))
     1*SIGMA4(J)/YJM1(J)
      DVDT(I,J,K)=DVDT(I,J,K)-DPDY/FO1
   15 CONTINUE
      RETURN
      END
```

```
       SUBROUTINE PRESS
       REAL LAMB1,LAMB2
       COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
      1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
      2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
      3XNUH,XNUV,RO,FO,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,Z,
      4WINDX,WINDY,E,RHOA,IMAX,JMAX,KMAX,OMEGA,IM,MH,ML,MK,
      5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUR,ZNUR,XDHR,XDVR,G,DT,
      6NITER,NPRINT,I1,I2,J1,J2,K1,K2,TIME,IPRINT,RHS(20,14,10),
      7XIP1(20),XIM1(20),XIPM1(20),XI123(20),YJP1(14),XIP2(20),
      8YJM1(14),YJPM1(14),YJ123(14),H(20,14),DZ,YJP2(14),
      9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10)
      A,II,JJ,KK,I3,J3,K3,FO1,IIT,B,EPS,ITNMAX,SIGMA1(20),SIGMA2(20),
      BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHDT(20,14)
       COMMON SIGX1,SIGX2,SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY1,
      1SIGPY2,SIGPY3,SIGPY4,SIGZ1,SIGZ2,SIGZ3,SIGZ4
       COMMON/TABLE/TTBL,RHOTBL
       DIMENSION TTBL(26),RHOTBL(26)
       IF(HYD.EQ.0.) GO TO 300
C      CALCULATE PRESSURE FROM HYDROSTATIC APPROXIMATION
       DO 110 L=1,KMAX
       K=KMAX+1-L
       KP1=K+1
       DO 110 J=1,JMAX
       DO 110 I=1,IMAX
       IF( I .LT. MH .AND. I .GE. ML ) GO TO 110
       IF(K.NE.KMAX) GO TO 120
       P(I,J,K)=RHO(I,J,K)*H(I,J)/2.
       GO TO 110
  120  CONTINUE
       IF(K.NE.K1) GO TO 130
       ZKP1(K1)=DELZ(K1)+H(I,J)
  130  CONTINUE
       P(I,J,K)=P(I,J,KP1)+RHO(I,J,K)*ZKP1(K)/2.
  110  CONTINUE
       GO TO 100
 300    CONTINUE
C      END PRESSURE CALCULATION FROM HYDROSTATIC APPROXIMATION
   33  FORMAT(1H0,2X,'OMEGA=',E14.6,/)
C      CALCULATE FROM PRESSURE EQUATION
       ITN        =1
       Y1=1.
       LAMB1=1.
       OMEGA=1.
       IF(IP.NE.0) GO TO 10
       IF(.NOT.(DEBUG4.GE.2.)) GO TO 10
       WRITE(6,33) OMEGA
       DO 6 K=2,K1
       DO 6 J=2,J1
       DO 6 I=2,I1
```

```
      WRITE(6,23) K,ZK123(K),ZKP1(K),ZKP2(K),ZKM1(K),J,YJ123(J),
     1YJP1(J),YJM1(J),I,XI123(I),XIP1(I),XIM1(I)
    6 CONTINUE
   10 D             =0.
      B=1.-OMEGA
      Y2=0.
C
C     PRESSURE BOUNDARY CONDITIONS
C ****SOLID-FLUID BOUNDARY ON PRESSURE
C
      DO 200 I=2,I1
      DO 200 K=2,K1
      IF(K.LT.IM) GO TO 225
      IF( I .LT. MH .AND. I .GE. ML ) GO TO  210
  225 CONTINUE
      P(I,1,K)=(SIGY2*P(I,2,K)-SIGY3*P(I,3,K)-SIGY4*DVDT(I,1,K)*F01)/SIG
     AY1
  210 CONTINUE
      P(I,JMAX,K)=(SIGPY2*P(I,J1,K)-SIGPY3*P(I,J2,K)+SIGPY4*DVDT(I,JMAX,
     AK)*F01)/SIGPY1
  200 CONTINUE
C
C     CALCULATION OF PRESSURE IN THE WATER LEVEL ELEMENTS
C
      DO 50 I=2,I1
      DO 50 J=2,J1
      P(I,J,KMAX)=H(I,J)*RHO(I,J,KMAX)/2.
   50 CONTINUE
C
C     FLUID-FLUID BOUNDARY CONDITION IS SET DURING INITIALIZATION
C*****BOTTOM BOUNDARY ON PRESSURE
C
      DO 220 I=2,I1
      DO 220 J=2,J1
      P(I,J,1)=(SIGZ2*P(I,J,2)-SIGZ3*P(I,J,3)-SIGZ4*DWDT(I,J,1)*F01)/SIG
     AZ1
  220 CONTINUE
C
C     OUTGOIONG BOUNDARY ON PRESSURE
C
      DO 55 K=2,KMAX
      DO 55 J=2,J1
      P(IMAX,J,K)=P(I1,J,K)
   55 CONTINUE
C
C     CALCULATE PRESSURE IN FLOW REGION
C
      DO 5 K=2,K1
      DO 5 J=2,J1
      DO 5 I=2,I1
```

```
C
C       ADJUST WATER LEVEL VARIABLE
C
        IF(K.NE.K1) GO TO 18
        ZKP1(K1)=DELZ(K1)+H(I,J)
        ZKPM1(K1)=ZKP1(K1)+ZKM1(K1)
        ZK123(K1)=ZKP1(K1)*ZKM1(K1)*ZKPM1(K1)
  18    CONTINUE
        X1       =YJ123(J)*ZK123(K)*XIM1(I)
        X2       =YJ123(J)*ZK123(K)*XIP1(I)
        X3       =XI123(I)*ZK123(K)*YJM1(J)
        X4       =XI123(I)*ZK123(K)*YJP1(J)
        X5       =XI123(I)*YJ123(J)*ZKM1(K)
        X6       =XI123(I)*YJ123(J)*ZKP1(K)
        X7       =XI123(I)*YJ123(J)*ZK123(K)
        X        =X1+X2+X3+X4+X5+X6
        A        =OMEGA/X
        PNEW=A*(X1*P(I+1,J,K)+X2*P(I-1,J,K)+X3*P(I,J+1,K)+X4*P(I,J-1,K)+
       1X5*P(I,J,K+1)+X6*P(I,J,K-1)-X7*RHS(I,J,K)/8.)+B*P(I,J,K)
        RESID      =ABS(PNEW-P(I,J,K))
        IF(RESID-D) 15,15,16
  16    D          =RESID
  15    P(I,J,K)   =PNEW
        IF(OMEGA.NE.1.) GO TO 9
        Y2=Y2+RESID
   9    CONTINUE
        IF(IP.NE.0) GO TO 5
        IF(.NOT.(DEBUG4.GE.1.)) GO TO 5
        WRITE(6,26) X,X1,X2,X3,X4,X5,X6,X7,A
   5    CONTINUE
C
C       CALCULATE RELAXATION FACTOR OMEGA
C
        IF(OMEGA.NE.1.) GO TO 28
        LAMB2=Y2/Y1
        IF(DEBUG4.EQ.0.) GO TO 60
        WRITE(6,42)Y1,Y2,LAMB2
  60    CONTINUE
        IF(LAMB2.GE.1.) GO TO 28
        IF(ABS(LAMB2-LAMB1).GT. .01) GO TO 28
        OMEGA=2./(1+SQRT(1-LAMB2))
        IF(DEBUG4.EQ.0.) GO TO 28
        WRITE(6,33) OMEGA
  28    CONTINUE
        Y1=Y2
        LAMB1=LAMB2
C
  42    FORMAT(1H ,'Y1=',E14.7,2X,'Y2=',E14.7,2X,'OMEGAS=',E14.7)
        IF(D-EPS) 14,12,12
  12    ITN        =ITN+1
```

```
      IF(ITN-ITNMAX) 10,10,21
   21 WRITE(6,22)
      CALL EXIT
   22 FORMAT(1H0,4X,' FAILS TO CONVERGE IN GIVEN ITNMAX ',/)
   26 FORMAT(1H0,2X,'I',2X,'J',3X,'K',6X,'X',6X,'X1',6X,'X2',6X,'X3',
     16X,'X4',6X,'X5',6X,'X6',6X,'X7',6X,'A')
   14 CONTINUE
      IF(DEBUG4.EQ.0.) GO TO 40
      WRITE(6,20) ITN
   20 FORMAT(1H0,10X,'ITN=',I3)
   23 FORMAT(1H0,'K=',I2,2X,'ZK123(K)=',E11,4,2X,'ZKP1(K)=',E11,4,2X,
     1'ZKP2(K)=',E11.4,2X,'ZKM1(K)=',E11.4,/,2X,'J=',I2,2X,'YJ123(J)=',
     2E11,4,2X,'YJP1(J)=',E11,4,2X,'YJM1(J)=',E11.4,2X,/,2X,'I=',I2,
     32X,'XI123(I)=',E11,4,2X,'XIP1(I)=',E11.4,2X,'XIM1(I)=',E11.4)
   24 FORMAT(1H ,'I=',I2,1X,'J=',I2,1X,'K=',I2,1X,
     1'P(I,J,K)=',E11.4,1X,'P(I-1,J,K)=',E11,4,1X,'P(I+1,J,K)=',E11.4,
     21X,'P(I,J-1,K)=',E11,4,/,7X,'P(I,J+1,K)=',E11,4,1X,'P(I,J,K-1)=',
     3E11,4,1X,'P(I,J,K+1)=',E11,4,1X,'RHS(I,J,K)=',E11.4,1X,'A(I,J,K)='
     4,E11,4,/,7X,'B=',E11,4,2X,'D=',E11,4)
  100 CONTINUE
   40 RETURN
      END
```

```
FORTRAN IV (VER 45 ) SOURCE LISTING: SI      FUNCTION    03/31/77  22:00:10  PAGE 0032

          FUNCTION SI(XTBL,YTBL,X,N)
C         LINEAR INTERPOLATICN OR EXTRAPOLATION OF SINGLE VARIABLE FUNCTION
C         XTBL = INDEPENDENT VARIABLE TABLE
C         YTBL = DEPENDENT VARIABLE TABLE
C         X    =VALUE OF THE INDEPENDENT VARIABLE
C         N     = NO. OF POINTS IN TABLE
C         0=NO EXTRAPOLATION, 1=LOWER EXTRAPOLATION, 2=UPPER EXTRAPOLATION
          DIMENSION XTBL(30),YTBL(30)
C         CHECK TO SEE IF EXTRAPOLATION IS NEEDED
          IF(X-XTBL(1)) 120,130,150
   120    WRITE(6,202)
   202    FORMAT(1H0,5X,' DENSITY TABLE OUT OF RANGE AT THE LOWER END')
    130   II = 2
          GO TO 254
    150   IF(XTBL(N)-X) 160,180,210
   160    WRITE(6,201)
   201    FORMAT(1H0, 5X,' DENSITY TABLE OUT OF RANGE AT THE UPPER END ')
    180   II = N
          GO TO 254
C         FIND X IN TABLE
    210   DO 220 IK=2,N
          II = IK
          IF(XTBL(IK)-X) 220,254,254
    220   CONTINUE
    254   X1 = XTBL(II-1)
          X2 = XTBL(II)
          Y1 = YTBL(II-1)
          Y2 = YTBL(II)
          SI = Y1+(Y2-Y1)*(X-X1)/(X2-X1)
          RETURN
          END
```

```
      BLOCK DATA
      COMMON/TABLE/TTBL,RHOTBL
      REAL * 4 TTBL(26)/70.,72.,74.,76.,78.,80.,82.,84.,86.,88.,90.,92.,
     *94.,
     *96.,98.,100.,102.,104.,106.,108.,110.,112.,114.,116.,118.,120./
      REAL * 4 RHOTBL(26)/62.3029595,62.28977202,62.27425582,62.25487144
     *,62.23937263,62.22000969,62.20065933,62.18132073,62.16199416,
     *62.13881812,62.11565936,62.09251785,62.06939358,62.04628653,
     *62.02319668,61.99628022,61.96938712,61.94251734,61.91567086,
     *61.88884763,61.86204763,61.83144747,61.80469716,61.77415369,
     *61.74364041,61.71315725/
      END
```

ENRUN = 0.2000E 01

TBEG = 0.0000E 00   TEND = 0.1000E 04   ENITER= 0.5000E 04   NPRNT= 0.2500E 02

UI = 0.4000E 00   VI = 0.0000E 00   TI = 0.7500E 02

U0= 0.2000E 01   ANGLE= 0.1571E 01   T0= 0.9000E 02   D0= 0.5000E 02   XNUZ= 0.8250E-05

WINDX = 0.0000E 00   WINDY = 0.0000E 00   RHOA = 0.7630E-01   E = 0.7400E 02   XK = 0.1000E 03

CFX = 0.0000E 00   CFY = 0.0000E 00

DH = 0.1350E 04   DV = 0.4000E 00   XNUH = 0.1350E 04   XNUV = 0.4000E 00   CP= 0.1000E 01

EIMAX = 0.1000E 02   EJMAX = 0.7000E 01   EKMAX = 0.5000E 01

DEBUG1= 0.0000E 00   DEBUG2= 0.0000E 00   DEBUG3= 0.0000E 00   DEBUG4= 0.0000E 00

EIRUN= 0.0000E 00   EIEND= 0.1000E 01

EIM= 0.4000E 01   EML= 0.4000E 01   EMK= 0.5000E 01   SLIP= 0.0000E 00   HYD= 0.0000E 00

HEAT= 0.0000E 00   XIIP= 0.5200E 02

I DELX(I)

1	0.6500E 02	2	0.6000E 02	3	0.5500E 02	4	0.5000E 02
5	0.5000E 02	6	0.5500E 02	7	0.6000E 02	8	0.6500E 02
9	0.7000E 02	10	0.7000E 02				

J DELY(J)

1	0.5000E 02	2	0.5000E 02	3	0.5500E 02	4	0.6000E 02
5	0.6500E 02	6	0.7000E 02	7	0.7000E 02		

K DELZ(K)

1	0.3500E 01	2	0.4000E 01	3	0.5000E 01	4	0.4000E 01
5	0.3500E 01						

TTBL	RHOTBL
0.7000000E 02	0.6230296E 02
0.7200000E 02	0.6228976E 02
0.7400000E 02	0.6227426E 02
0.7600000E 02	0.6225487E 02
0.7800000E 02	0.6223938E 02
0.8000000E 02	0.6222002E 02
0.8200000E 02	0.6220065E 02
0.8400000E 02	0.6218132E 02
0.8600000E 02	0.6216199E 02

0.8800000E 02	0.6213882E 02
0.9000000E 02	0.6211566E 02
0.9200000E 02	0.6209251E 02
0.9400000E 02	0.6206940E 02
0.9600000E 02	0.6204628E 02
0.9800000E 02	0.6202319E 02
0.1000000E 03	0.6199628E 02
0.1020000E 03	0.6196939E 02
0.1040000E 03	0.6194252E 02
0.1060000E 03	0.6191566E 02
0.1080000E 03	0.6188885E 02
0.1100000E 03	0.6186205E 02
0.1120000E 03	0.6183145E 02
0.1140000E 03	0.6180470E 02
0.1160000E 03	0.6177415E 02
0.1180000E 03	0.6174364E 02
0.1200000E 03	0.6171315E 02

IMAX = 10 JMAX = 7 KMAX = 5

I1 = 9 J1 = 6 K1 = 4

I2 = 8 J2 = 5 K2 = 3

I3 = 7 J3 = 4 K3 = 2

RHOZ= 0.6211566E 02

I	J	K	SUGGESTED DT	I	J	K	SUGGESTED DTT
5	2	2	0.4511E 00	5	2	2	0.4511E 00

I	J	KMAX	SUGGESTED DTS	I	J	KMAX	SUGGESTED DTTS
5	2	5	0.4465E 00	5	2	5	0.4465E 00

********************NON-DIMENSIONAL INPUT VALUES ********************

```
ENRUN = 0.2000E 01

TBEG  = 0.0000E 00     TEND  = 0.4000E 02     NITER=  5000     NPRINT=    25

UI    = 0.2000E 00     VI    = 0.0000E 00     TI    = 0.8333E 00

U0= 0.1000E 01         ANGLE= 0.1571E 01      T0= 0.1000E 01         D0= 0.5000E 02        XNUZ= 0.1000E 01

WINDX = 0.0000E 00     WINDY = 0.0000E 00     RHOA  = 0.1228E-02

DH    = 0.1636E 09     DV    = 0.4848E 05     XNUH  = 0.1636E 09     XNUV  = 0.4848E 05

G     = 0.3220E 02     DT    = 0.8000E-02     E= 0.8222222E 00

I   DELX(I)

 1   0.1300E 01     2   0.1200E 01     3   0.1100E 01     4   0.1000E 01

 5   0.1000E 01     6   0.1100E 01     7   0.1200E 01     8   0.1300E 01

 9   0.1400E 01    10   0.1400E 01

J   DELY(J)

 1   0.1000E 01     2   0.1000E 01     3   0.1100E 01     4   0.1200E 01

 5   0.1300E 01     6   0.1400E 01     7   0.1400E 01

K   DELZ(K)

 1   0.7000E-01     2   0.8000E-01     3   0.1000E 00     4   0.8000E-01

 5   0.7000E-01

RO    = 0.1212E 08     FO    = 0.4984E-01     SO    = 0.9317E-05
```

TTBL	RHOTBL
0.7777777E 00	0.1003015E 01
0.8000000E 00	0.1002803E 01
0.8222222E 00	0.1002553E 01
0.8444444E 00	0.1002240E 01
0.8666666E 00	0.1001991E 01
0.8888888E 00	0.1001679E 01
0.9111111E 00	0.1001368E 01
0.9333333E 00	0.1001057E 01
0.9555555E 00	0.1000746E 01
0.9777777E 00	0.1000373E 01
0.1000000E 01	0.1000000E 01
0.1022222E 01	0.9996273E 00
0.1044444E 01	0.9992552E 00
0.1066667E 01	0.9988830E 00
0.1088886E 01	0.9985113E 00
0.1111111E 01	0.9980780E 00
0.1133333E 01	0.9976451E 00
0.1155555E 01	0.9972126E 00
0.1177777E 01	0.9967802E 00
0.1200000E 01	0.9963486E 00
0.1222221E 01	0.9959170E 00
0.1244444E 01	0.9954244E 00
0.1266666E 01	0.9949939E 00
0.1288888E 01	0.9945021E 00
0.1311110E 01	0.9940107E 00
0.1333333E 01	0.9935200E 00

*******************END OF NON-DIMENSIONAL INPUT VALUES*******************

XNUR = 0.1350E 02 ZNUR = 0.4000E-02 XDHR = 0.1350E 02 XDVR = 0.4000E-02

TIME= 0.0000E 00 DT = 0.8000E-02 ITERATION NUMBER= 0

I	J	K	U	V	W	T	P	H	I	J	K	U	V	W	T	P	H
1	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	4	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
3	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		4	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
5	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
3	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		4	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
5	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		6	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
7	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		8	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
9	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		10	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
1	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
3	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		4	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
5	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		6	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
7	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		8	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
9	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		10	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
1	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
3	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		4	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
5	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		6	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
7	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		8	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
9	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		10	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
1	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
3	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		4	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	
5	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		6	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	

7	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	8	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
9	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	10	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
1	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	2	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
3	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	4	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
5	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	6	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
7	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	8	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
9	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	10	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
1	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	2	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
3	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	4	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
5	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	6	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
7	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	8	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
9	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556	10	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.2556
1	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556	2	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556
3	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556	4	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556
5	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556	6	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556
7	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556	8	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556
9	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556	10	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.2556
1	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	5	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	6	3	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.1654
1	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	2	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
3	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	4	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
5	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	6	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
7	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	8	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654
9	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654	10	7	3	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.1654

1	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	1	4	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0752	
5	1	4	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0752		6	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	2	4	0.2000E 00	0.0000E 00	0.4610E-01	0.8333	0.0752	
5	2	4	0.2000E 00	0.0000E 00	0.4610E-01	0.8333	0.0752		6	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	2	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
5	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	3	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
5	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	4	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
5	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	5	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
5	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	6	4	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
5	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	2	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
3	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	4	1	5	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0175	0.3500E-01
5	1	5	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0175	0.3500E-01	6	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
7	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	8	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
9	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	10	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
1	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	2	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
3	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	4	2	5	0.2000E 00	0.0000E 00	0.8110E-01	0.8333	0.0175	0.3500E-01
5	2	5	0.2000E 00	0.0000E 00	0.8110E-01	0.8333	0.0175	0.3500E-01	6	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
7	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	8	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
9	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	10	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01

I	J	K	U	V	W	T P	H	I	J	K	U	V	W	T P	H
1	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	2	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
3	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	4	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
5	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	6	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
7	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	8	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
9	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	10	3	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
1	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	2	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
3	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	4	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
5	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	6	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
7	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	8	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
9	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	10	4	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
1	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	2	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
3	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	4	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
5	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	6	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
7	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	8	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
9	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	10	5	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
1	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	2	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
3	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	4	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
5	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	6	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
7	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	8	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
9	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	10	6	5	0.2000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
1	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	2	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
3	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	4	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
5	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	6	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
7	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	8	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01
9	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01	10	7	5	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.0175	0.3500E-01

TIME= 0.8000E-02 DT = 0.8000E-02 ITERATION NUMBER= 1

I	J	K	U	V	W	T P	H	I	J	K	U	V	W	T P	H
1	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		2	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
3	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		4	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
5	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		6	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
7	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		8	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
9	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		10	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
1	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		2	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3309	
3	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		4	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3309	
5	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3309		6	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
7	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		8	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
9	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		10	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
1	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		2	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
3	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		4	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
5	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		6	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
7	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		8	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	
9	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308		10	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.83330.3308	

1	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	2	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
3	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	4	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
5	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	6	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
7	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	8	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
9	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	10	4	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
1	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	2	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3305
3	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	4	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
5	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	6	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
7	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	8	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
9	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	10	5	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
1	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	2	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
3	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	4	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
5	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	6	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
7	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	8	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
9	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	10	6	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
1	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	2	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
3	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	4	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
5	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3306	6	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
7	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	8	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
9	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308	10	7	1	0.0000E 00 0.0000E 00 0.0000E 000.83330.3308
1	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556	2	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2557
3	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556	4	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2557
5	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2557	6	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556
7	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556	8	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556
9	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556	10	1	2	0.0000E 00 0.0000E 00 0.0000E 000.83330.2556
1	2	2	0.2000E 00 0.0000E 00 0.0000E 000.83330.2556	2	2	2	0.1764E 00 0.1169E-03 0.6792E-030.83330.2557
3	2	2	0.1763E 00-0.5988E-04 0.1119E-040.83330.2556	4	2	2	0.1761E 00 0.1910E-03-0.1785E-040.83330.2557
5	2	2	0.1765E 00 0.1906E-03-0.2106E-040.83330.2557	6	2	2	0.1765E 00-0.6188E-04 0.1025E-040.83330.2556
7	2	2	0.1763E 00 0.1966E-06 0.6213E-050.83330.2556	8	2	2	0.1763E 00-0.5441E-06-0.1428E-050.83330.2556
9	2	2	0.1763E 00-0.1731E-05 0.7238E-070.83330.2556	10	2	2	0.1763E 00-0.1731E-05 0.7238E-070.83330.2556
1	3	2	0.2000E 00 0.0000E 00 0.0000E 000.83330.2556	2	3	2	0.1974E 00 0.1038E-03 0.7779E-040.83330.2556
3	3	2	0.1974E 00-0.5242E-04-0.1770E-050.83330.2556	4	3	2	0.1974E 00 0.1653E-03 0.6504E-050.83330.2556
5	3	2	0.1974E 00 0.1651E-03 0.6236E-050.83330.2556	6	3	2	0.1974E 00-0.5454E-04-0.1947E-050.83330.2556
7	3	2	0.1974E 00-0.7965E-06 0.6880E-070.83330.2556	8	3	2	0.1974E 00-0.2788E-05 0.8248E-070.83330.2556
9	3	2	0.1974E 00-0.1513E-05-0.3083E-070.83330.2556	10	3	2	0.1974E 00-0.1513E-05-0.3083E-070.83330.2556
1	4	2	0.2000E 00 0.0000E 00 0.0000E 000.83330.2556	2	4	2	0.1974E 00-0.1057E-05 0.7685E-040.83330.2556
3	4	2	0.1974E 00-0.2474E-06-0.1546E-050.83330.2556	4	4	2	0.1974E 00-0.1103E-05 0.5816E-050.83330.2556
5	4	2	0.1974E 00-0.1277E-05 0.5578E-050.83330.2556	6	4	2	0.1974E 00-0.6820E-06-0.1720E-050.83330.2556
7	4	2	0.1974E 00-0.2341E-05 0.3409E-070.83330.2556	8	4	2	0.1974E 00-0.2226E-05-0.1873E-060.83330.2556
9	4	2	0.1974E 00-0.6755E-06-0.8487E-070.83330.2556	10	4	2	0.1974E 00-0.6755E-06-0.8487E-070.83330.2556
1	5	2	0.2000E 00 0.0000E 00 0.0000E 000.83330.2556	2	5	2	0.1974E 00-0.4116E-04 0.7551E-040.83330.2556
3	5	2	0.1974E 00-0.1857E-05 0.2295E-060.83330.2556	4	5	2	0.1974E 00 0.5352E-06 0.3336E-060.83330.2556
5	5	2	0.1974E 00 0.5352E-06 0.8313E-070.83330.2556	6	5	2	0.1974E 00-0.1105E-05 0.8852E-070.83330.2556
7	5	2	0.1974E 00-0.7059E-06-0.2049E-060.83330.2556	8	5	2	0.1974E 00 0.3872E-06-0.1524E-060.83330.2556
9	5	2	0.1974E 00-0.2960E-06-0.2647E-070.83330.2556	10	5	2	0.1974E 00-0.2960E-06-0.2647E-070.83330.2556

5	6	2	0,5283E-01	0.2369E-01	0,1508E-020,83330,2573	6	6	2	[illegible]	[illegible]	[illegible]
7	6	2	0,4627E-01	0.1169E-01	0,8584E-030,83330,2563	8	6	2	0.4119E-01	0,6343E-02	0.5409E-030.83330.2560
9	6	2	0,3697E-01	0.3273E-02	0,2498E-030,83330,2558	10	6	2	0.3697E-01	0,3273E-02	0.2498E-030.83330.2558
1	7	2	0,0000E 00	0.0000E 00	0,0000E 000,83330,2556	2	7	2	0.0000E 00	0,0000E 00	0.0000E 000.83330.2605
3	7	2	0,0000E 00	0.0000E 00	0,0000E 000,83330,2577	4	7	2	0,0000E 00	0,0000E 00	0,0000E 000.83330.2575
5	7	2	0.0000E 00	0.0000E 00	0,0000E 000,83330,2571	6	7	2	0,0000E 00	0,0000E 00	0.0000E 000.83330.2565
7	7	2	0,0000E 00	0,0000E 00	0,0000E 000,83330,2561	8	7	2	0,0000E 00	0,0000E 00	0,0000E 000.83330.2559
9	7	2	0,0000E 00	0.0000E 00	0,0000E 000.83330,2557	10	7	2	0,0000E 00	0,0000E 00	0,0000E 000.83330,2556
1	1	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1654	2	1	3	0.0000E 00	0,0000E 00	0.0000E 000.83330.1720
3	1	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1644	4	1	3	0,0000E 00	0,0000E 00	0.0000E 000.83330,1863
5	1	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1835	6	1	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1629
7	1	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1668	8	1	3	0,0000E 00	0,0000E 00	0.0000E 000.83330,1653
9	1	3	0,0000E 00	0.0000E 00	0,0000E 000.83330,1654	10	1	3	0.0000E 00	0,0000E 00	0,0000E 000.83330,1654
1	2	3	0.2000E 00	0.0000E 00	0,0000E 000,83330,1654	2	2	3	0,7214E-01	0,3074E-01	0,1179E-010.83330.1711
3	2	3	-0,1406E-01	0.4601E-01	0,7703E-030,83330,1648	4	2	3	-0,3996E-02	0,1474E 00	-0,2324E-010.83330.1805
5	2	3	0,1249E 00	0.1398E 00	-0.2234E-010,83330,1783	6	2	3	0,1315E 00	0,3911E-01	0.5003E-030.83330,1634
7	2	3	0,6143E-01	0.1772E-01	0,3719E-020,83330,1667	8	2	3	0.5345E-01	0,4376E-02	0,5023E-030.83330.1655
9	2	3	0,4335E-01	0.1654E-02	0,3579E-030,83330,1655	10	2	3	0,4335E-01	0,1654E-02	0.3579E-030.83330.1655
1	3	3	0.2000E 00	0.0000E 00	0,0000E 000,83330.1654	2	3	3	0.9961E-01	0,4083E-01	0.1012E-010.83330.1699
3	3	3	0,5126E-01	0.6376E-01	0.1209E-020,83330,1682	4	3	3	0.6225E-01	0,1608E 00	-0,4140E-020.83330.1714
5	3	3	0,1323E 00	0.1527E 00	-0,4567E-020,83330,1704	6	3	3	0.1444E 00	0,5620E-01	-0.1836E-030.83330,1668
7	3	3	0,1081E 00	0.2944E-01	0,2302E-020,83330,1669	8	3	3	0.9259E-01	0,1042E-01	0.9472E-030.83330.1660
9	3	3	0,7921E-01	0.4641E-02	0,3848E-030,83330,1656	10	3	3	0.7921E-01	0,4641E-02	0.3848E-030.83330.1656
1	4	3	0.2000E 00	0.0000E 00	0,0000E 000,83330,1654	2	4	3	0.1189E 00	0,3434E-01	0.9354E-020.83330.1693
3	4	3	0,9065E-01	0.6362E-01	0,2776E-020,83330,1683	4	4	3	0.9593E-01	0,1077E 00	0.2509E-020.83330.1701
5	4	3	0,1338E 00	0.1044E 00	0,1573E-020,83330,1694	6	4	3	0.1432E 00	0.6081E-01	0.8266E-030.83330.1671
7	4	3	0,1234E 00	0,3519E-01	0,2080E-020,83330.1667	8	4	3	0.1091E 00	0,1603E-01	0.1157E-020.83330.1660
9	4	3	0.9705E-01	0.7677E-02	0,4613E-030,83330,1656	10	4	3	0.9705E-01	0,7677E-02	0.4613E-030.83330.1656
1	5	3	0,2000E 00	0.0000E 00	0,0000E 000.83330,1654	2	5	3	0,1248E 00	0,1697E-01	0,8705E-020.83330.1689
3	5	3	0,1067E 00	0.4344E-01	0,4353E-020,83330,1679	4	5	3	0,1022E 00	0,6853E-01	0.3885E-020.83330.1683
5	5	3	0.1144E 00	0,6846E-01	0,2936E-020,83330,1679	6	5	3	0.1175E 00	0,4686E-01	0.2300E-020.83330.1669
7	5	3	0,1080E 00	0.2932E-01	0,2225E-020,83330,1664	8	5	3	0.9795E-01	0.1504E-01	0.1436E-020.83330.1659
9	5	3	0,8937E-01	0.7659E-02	0,6154E-030,83330,1656	10	5	3	0.8937E-01	0,7659E-02	0.6154E-030.83330.1656
1	6	3	0,2000E 00	0.0000E 00	0,0000E 000,83330,1654	2	6	3	0.1009E 00	0,1630E-02	0.8830E-020.83330.1698
3	6	3	0,8498E-01	0.1827E-01	0,4619E-020,83330,1675	4	6	3	0,7177E-01	0,2967E-01	0.4783E-020.83330.1675
5	6	3	0,7142E-01	0,3101E-01	0,3835E-020,83330,1671	6	6	3	0.6997E-01	0,2384E-01	0,2910E-020.83330.1665
7	6	3	0,6478E-01	0.1592E-01	0.2225E-020,83330,1661	8	6	3	0.5906E-01	0,8799E-02	0.1398E-020.83330.1658
9	6	3	0,5428E-01	0.4690E-02	0,6608E-030,83330,1656	10	6	3	0.5428E-01	0,4690E-02	0.6608E-030.83330.1655
1	7	3	0.0000E 00	0.0000E 00	0,0000E 000,83330,1654	2	7	3	0.0000E 00	0,0000E 00	0.0000E 000.83330.1704
3	7	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1675	4	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1674
5	7	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1669	6	7	3	0.0000E 00	0,0000E 00	0.0000E 000.83330.1664
7	7	3	0.0000E 00	0.0000E 00	0,0000E 000,83330,1659	8	7	3	0.0000E 00	0,0000E 00	0.0000E 000.83330.1657
9	7	3	0.0000E 00	0.0000E 00	0,0000E 000,83330,1655	10	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1654

1	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
3	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		4	1	4	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0752	
5	1	4	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0752		6	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	1	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
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7	5	4	0.2000E 00	-0.1138E-06	-0.4365E-06	0.8333	0.0752		8	5	4	0.2000E 00	0.0000E 00	-0.3189E-06	0.8333	0.0752	
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3	6	4	0.1888E 00	-0.8533E-06	0.1226E-05	0.8333	0.0752		4	6	4	0.1888E 00	-0.1448E-06	0.4911E-05	0.8333	0.0752	
5	6	4	0.1888E 00	-0.1446E-06	0.3656E-06	0.8333	0.0752		6	6	4	0.1888E 00	-0.4241E-06	-0.1136E-06	0.8333	0.0752	
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9	6	4	0.1888E 00	0.2121E-06	-0.6036E-07	0.8333	0.0752		10	6	4	0.1888E 00	0.2121E-06	-0.6036E-07	0.8333	0.0752	
1	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		2	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
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5	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		6	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
7	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		8	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
9	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752		10	7	4	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0752	
1	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	2	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
3	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	4	1	5	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0175	0.3500E-01
5	1	5	0.6036E-05	0.1000E 01	0.0000E 00	0.8333	0.0175	0.3500E-01	6	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
7	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	8	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
9	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	10	1	5	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01
1	2	5	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01	2	2	5	0.1789E 00	0.0000E 00	0.2721E-02	0.8333	0.0175	0.3500E-01
3	2	5	0.1789E 00	0.0000E 00	0.2668E-04	0.8333	0.0175	0.3500E-01	4	2	5	0.1789E 00	0.1054E 00	0.7732E-01	0.8333	0.0175	0.3565E-01
5	2	5	0.1789E 00	0.1054E 00	0.7742E-01	0.8333	0.0175	0.3565E-01	6	2	5	0.1789E 00	0.0000E 00	0.1350E-03	0.8333	0.0175	0.3500E-01
7	2	5	0.1789E 00	0.0000E 00	0.1487E-04	0.8333	0.0175	0.3500E-01	8	2	5	0.1789E 00	0.0000E 00	-0.3423E-05	0.8333	0.0175	0.3500E 01
9	2	5	0.1789E 00	0.0000E 00	0.1426E-06	0.8333	0.0175	0.3500E-01	10	2	5	0.1789E 00	0.0000E 00	0.0000E 00	0.8333	0.0175	0.3500E-01

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7	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	1	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
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3	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3304		4	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3455	
5	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3434		6	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3290	
7	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3320		8	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3309	
9	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3309		10	2	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	3	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3352	
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5	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3332		6	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3323	
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9	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3310		10	5	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3352	
3	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3329		4	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3329	
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9	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3309		10	6	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		2	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
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5	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		6	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
7	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		8	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
9	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308		10	7	1	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.3308	
1	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2620	
3	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2545		4	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2753	
5	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2726		6	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2529	
7	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2568		8	1	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2555	
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1	2	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556		2	2	2	0.6389E-01	0.2101E-01	0.5704E-02	0.8333	0.2613	
3	2	2	-0.2045E-01	0.2558E-01	0.8642E-03	0.8333	0.2552		4	2	2	-0.1300E-01	0.1095E 00	-0.9524E-02	0.8333	0.2704	
5	2	2	0.1069E 00	0.1020E 00	-0.9140E-02	0.8333	0.2683		6	2	2	0.1121E 00	0.1818E-01	0.6971E-03	0.8333	0.2538	
7	2	2	0.4603E-01	0.6755E-02	0.1893E-02	0.8333	0.2569		8	2	2	0.3902E-01	-0.4089E-03	0.3348E-03	0.8333	0.2557	
9	2	2	0.3006E-01	-0.6560E-03	0.2161E-03	0.8333	0.2557		10	2	2	0.3006E-01	-0.6560E-03	0.2161E-03	0.8333	0.2557	

1	3	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	2	3	2	0.8566E-01	0.2867E-01	0.4890E-02	0.8333	0.2601
3	3	2	0.3433E-01	0.3993E-01	0.6851E-03	0.8333	0.2584	4	3	2	0.4129E-01	0.1224E 00	-0.1838E-02	0.8333	0.2616
5	3	2	0.1048E 00	0.1145E 00	-0.2058E-02	0.8333	0.2606	6	3	2	0.1145E 00	0.3161E-01	-0.5649E-04	0.8333	0.2570
7	3	2	0.8030E-01	0.1513E-01	0.1027E-02	0.8333	0.2571	8	3	2	0.6664E-01	0.3573E-02	0.4178E-03	0.8333	0.2562
9	3	2	0.5484E-01	0.1144E-02	0.1777E-03	0.8333	0.2558	10	3	2	0.5484E-01	0.1144E-02	0.1777E-03	0.8333	0.2558
1	4	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	2	4	2	0.1010E 00	0.2400E-01	0.4541E-02	0.8333	0.2595
3	4	2	0.6709E-01	0.4368E-01	0.1255E-02	0.8333	0.2585	4	4	2	0.6820E-01	0.7987E-01	0.9583E-03	0.8333	0.2603
5	4	2	0.1011E 00	0.7639E-01	0.5013E-03	0.8333	0.2596	6	4	2	0.1084E 00	0.3984E-01	0.2170E-03	0.8333	0.2573
7	4	2	0.3984E-01	0.2205E-01	0.8108E-03	0.8333	0.2569	8	4	2	0.7724E-01	0.9168E-02	0.4604E-03	0.8333	0.2562
9	4	2	0.6671E-01	0.4001E-02	0.1768E-03	0.8333	0.2558	10	4	2	0.6671E-01	0.4001E-02	0.1768E-03	0.8333	0.2558
1	5	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	2	5	2	0.1066E 00	0.1065E-01	0.4230E-02	0.8333	0.2591
3	5	2	0.8230E-01	0.3091E-01	0.1913E-02	0.8333	0.2562	4	5	2	0.7480E-01	0.5154E-01	0.1550E-02	0.8333	0.2585
5	5	2	0.8435E-01	0.5118E-01	0.1065E-02	0.8333	0.2581	6	5	2	0.8629E-01	0.3315E-01	0.8251E-03	0.8333	0.2571
7	5	2	0.7738E-01	0.2022E-01	0.8418E-03	0.8333	0.2566	8	5	2	0.6854E-01	0.9991E-02	0.5483E-03	0.8333	0.2561
9	5	2	0.6102E-01	0.4836E-02	0.2284E-03	0.8333	0.2558	10	5	2	0.6102E-01	0.4836E-02	0.2284E-03	0.8333	0.2558
1	6	2	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	2	6	2	0.8820E-01	-0.7134E-03	0.4189E-02	0.8333	0.2600
3	6	2	0.6840E-01	0.1305E-01	0.2000E-02	0.8333	0.2578	4	6	2	0.5406E-01	0.2260E-01	0.1960E-02	0.8333	0.2577
5	6	2	0.5283E-01	0.2369E-01	0.1508E-02	0.8333	0.2573	6	6	2	0.5109E-01	0.1773E-01	0.1117E-02	0.8333	0.2567
7	6	2	0.4627E-01	0.1169E-01	0.8584E-03	0.8333	0.2563	8	6	2	0.4119E-01	0.6343E-02	0.5409E-03	0.8333	0.2560
9	6	2	0.3697E-01	0.3273E-02	0.2498E-03	0.8333	0.2558	10	6	2	0.3697E-01	0.3273E-02	0.2498E-03	0.8333	0.2558
1	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556	2	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2605
3	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2577	4	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2575
5	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2571	6	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2565
7	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2561	8	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2559
9	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2557	10	7	2	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.2556
1	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654	2	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1720
3	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1644	4	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1863
5	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1835	6	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1629
7	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1668	8	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1653
9	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654	10	1	3	0.0000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654
1	2	3	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654	2	2	3	0.7214E-01	0.3074E-01	0.1179E-01	0.8333	0.1711
3	2	3	-0.1406E-01	0.4601E-01	0.7703E-03	0.8333	0.1648	4	2	3	-0.3996E-02	0.1474E 00	-0.2324E-01	0.8333	0.1805
5	2	3	0.1249E 00	0.1398E 00	-0.2234E-01	0.8333	0.1783	6	2	3	0.1315E 00	0.3911E-01	0.5003E-03	0.8333	0.1634
7	2	3	0.6143E-01	0.1772E-01	0.3719E-02	0.8333	0.1667	8	2	3	0.5345E-01	0.4376E-02	0.5023E-03	0.8333	0.1655
9	2	3	0.4335E-01	0.1654E-02	0.3579E-03	0.8333	0.1655	10	2	3	0.4335E-01	0.1654E-02	0.3579E-03	0.8333	0.1655
1	3	3	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654	2	3	3	0.9961E-01	0.4083E-01	0.1012E-01	0.8333	0.1699
3	3	3	0.5126E-01	0.6376E-01	0.1209E-02	0.8333	0.1682	4	3	3	0.6225E-01	0.1608E 00	-0.4140E-02	0.8333	0.1714
5	3	3	0.1323E 00	0.1527E 00	-0.4567E-02	0.8333	0.1704	6	3	3	0.1444E 00	0.5620E-01	-0.1836E-03	0.8333	0.1668
7	3	3	0.1081E 00	0.2944E-01	0.2302E-02	0.8333	0.1669	8	3	3	0.9259E-01	0.1042E-01	0.9472E-03	0.8333	0.1660
9	3	3	0.7921E-01	0.4641E-02	0.3848E-03	0.8333	0.1656	10	3	3	0.7921E-01	0.4641E-02	0.3848E-03	0.8333	0.1656
1	4	3	0.2000E 00	0.0000E 00	0.0000E 00	0.8333	0.1654	2	4	3	0.1189E 00	0.3434E-01	0.9354E-02	0.8333	0.1693
3	4	3	0.9065E-01	0.6362E-01	0.2776E-02	0.8333	0.1683	4	4	3	0.9593E-01	0.1077E 00	0.2509E-02	0.8333	0.1701
5	4	3	0.1338E 00	0.1044E 00	0.1573E-02	0.8333	0.1694	6	4	3	0.1432E 00	0.6081E-01	0.8266E-03	0.8333	0.1671
7	4	3	0.1234E 00	0.3519E-01	0.2080E-02	0.8333	0.1667	8	4	3	0.1091E 00	0.1603E-01	0.1197E-02	0.8333	0.1660
9	4	3	0.9705E-01	0.7677E-02	0.4613E-03	0.8333	0.1656	10	4	3	0.9705E-01	0.7677E-02	0.4613E-03	0.8333	0.1656

5	3	0,2000E 00	0.0000E 00	0,0000E 000.83330,1654	2	5	3	0.1248E 00	0,1697E-01	0.8705E-020.83330.1689
5	3	0.1067E 00	0.4344E-01	0,4353E-020.83330,1679	4	5	3	0.1022E 00	0,6853E-01	0.3885E-020.83330.1683
5	3	0.1144E 00	0.6846E-01	0,2936E-020,83330.1679	6	5	3	0.1175E 00	0,4686E-01	0.2300E-020.83330.1669
5	3	0.1080E 00	0.2932E-01	0,2225E-020.83330,1664	8	5	3	0,9795E-01	0.1504E-01	0.1436E-020.83330.1659
5	3	0.8937E-01	0.7659E-02	0,6154E-030.83330,1656	10	5	3	0.8937E-01	0,7659E-02	0.6154E-030.83330.1656
6	3	0.2000E 00	0.0000E 00	0,0000E 000,83330,1654	2	6	3	0.1009E 00	0,1630E-02	0.8830E-020.83330.1698
6	3	0.8496E-01	0.1827E-01	0,4619E-020.83330,1675	4	6	3	0,7177E-01	0,2967E-01	0.4783E-020.83330.1675
6	3	0,7142E-01	0.3101E-01	0,3835E-020.83330,1671	6	6	3	0.6997E-01	0,2384E-01	0.2910E-020.83330.1665
6	3	0,6478E-01	0.1592E-01	0,2225E-020.83330,1661	8	6	3	0.5906E-01	0,8799E-02	0.1398E-020.83330.1658
6	3	0,5428E-01	0.4690E-02	0,6608E-030.83330,1656	10	6	3	0.5428E-01	0,4690E-02	0.6608E-030.83330.1655
7	3	0.0000E 00	0.0000E 00	0,0000E 000,83330,1654	2	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1704
7	3	0,0000E 00	0.0000E 00	0,0000E 000,83330.1675	4	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1674
7	3	0,0000E 00	0.0000E 00	0,0000E 000,83330,1669	6	7	3	0.0000E 00	0,0000E 00	0.0000E 000.83330.1664
7	3	0.0000E 00	0.0000E 00	0,0000E 000.83330,1659	8	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1657
7	3	0.0000E 00	0.0000E 00	0,0000E 000,83330.1655	10	7	3	0,0000E 00	0,0000E 00	0.0000E 000.83330.1654
1	4	0.0000E 00	0.0000E 00	0.0000E 000,83330,0752	2	1	4	0.0000E 00	0,0000E 00	0.0000E 000.83330.0827
1	4	0.0000E 00	0.0000E 00	0,0000E 000,83330.0768	4	1	4	0.6036E-05	0,1000E 01	0,0000E 000.83330.0752
1	4	0,6036E-05	0.1000E 01	0,0000E 000,83330.0752	6	1	4	0.0000E 00	0,0000E 00	0.0000E 000.83330.0755
1	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0776	8	1	4	0.0000E 00	0,0000E 00	0.0000E 000.83330.0754
1	4	0.0000E 00	0.0000E 00	0,0000E 000,83330,0753	10	1	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0752
2	4	0.2000E 00	0.0000E 00	0,0000E 000,83330.0752	2	2	4	0.7200E-01	0,6538E-01	0.1643E-010.83330.0810
2	4	-0.1883E-01	0.1574E 00	-0,3379E-020.83330,0743	4	2	4	-0,5890E-02	0,4563E 00	0.6305E-030.83330.0907
2	4	0.1336E 00	0.4591E 00	0,1910E-020,83330,0883	6	2	4	0,1424E 00	0,1578E 00	-0.3965E-020.83330.0730
2	4	0.6719E-01	0.6109E-01	0,3720E-020,83330,0766	8	2	4	0,5872E-01	0,1970E-01	-0.1217E-030.83330.0753
2	4	0.4737E-01	0.7989E-02	0,1526E-030,83330.0752	10	2	4	0.4737E-01	0,7989E-02	0.1526E-030.83330.0752
3	4	0.2000E 00	0.0000E 00	0,0000E 000,83330,0752	2	3	4	0,9885E-01	0,7647E-01	0.1572E-010.83330.0797
3	4	0.4985E-01	0,1504E 00	0.3918E-020,83330.0779	4	3	4	0.6453E-01	0,3182E 00	0.2750E-020.83330.0811
3	4	0.1421E 00	0.3128E 00	0.2552E-020.83330.0801	6	3	4	0,1572E 00	0,1483E 00	0.2134E-020.83330.0766
3	4	0.1185E 00	0.7400E-01	0.4102E-020.83330,0767	8	3	4	0.1015E 00	0,2918E-01	0.1532E-020.83330.0757
3	4	0.6648E-01	0.1331E-01	0.5601E-030.83330.0754	10	3	4	0,8648E-01	0,1331E-01	0.5601E-030.83330.0754
4	4	0.2000E 00	0.0000E 00	0,0000E 000,83330.0752	2	4	4	0.1191E 00	0,6012E-01	0.1489E-010.83330.0791
4	4	0,9194E-01	0.1176E 00	0,6286E-020.83330.0780	4	4	4	0,1007E 00	0,1872E 00	0.8216E-020.83330.0799
4	4	0,1443E 00	0.1849E 00	0,6934E-020,83330,0793	6	4	4	0,1562E 00	0,1183E 00	0.3711E-020.83330.0769
4	4	0.1351E 00	0.6826E-01	0,4389E-020,83330.0765	8	4	4	0.1192E 00	0,3210E-01	0.2318E-020.83330.0758
4	4	0,1058E 00	0.1584E-01	0,8991E-030.83330,0754	10	4	4	0,1058E 00	0.1584E-01	0.8991E-030.83330.0754
5	4	0.2000E 00	0.0000E 00	0,0000E 000.83330,0752	2	5	4	0.1261E 00	0,3213E-01	0.1366E-010.83330.0768
5	4	0.1102E 00	0.7256E-01	0,8034E-020.83330.0777	4	5	4	0,1082E 00	0,1076E 00	0.8225E-020.83330.0781
5	4	0,1234E 00	0.1079E 00	0,6881E-020.83330,0777	6	5	4	0.1280E 00	0,7760E-01	0.5304E-020.83330.0767
5	4	0.1180E 00	0.4894E-01	0,4524E-020.83330,0762	8	5	4	0.1068E 00	0,2554E-01	0.2787E-020.83330.0757
5	4	0.9722E-01	0.1338E-01	0.1223E-020,83330,0754	10	5	4	0.9722E-01	0.1338E-01	0.1223E-020.83330.0754
6	4	0,2000E 00	0.0000E 00	0,0000E 000.83330,0752	2	6	4	0.1025E 00	0,9843E-02	0.1383E-010.83330.0796
6	4	0.8850E-01	0.3163E-01	0,8082E-020.83330.0773	4	6	4	0,7619E-01	0,4574E-01	0.8804E-020.83330.0773

5	6	4	0.7702E-01	0.4679E-01	0.7389E-020.83330.0768		6	6	4	0.7606E-01	0.3658E-01	0.5704E-020.83330.0763	
7	6	4	0.7058E-01	0.2450E-01	0.4269E-020.83330.0759		8	6	4	0.6425E-01	0.1366E-01	0.2638E-020.83330.0755	
9	6	4	0.5895E-01	0.7449E-02	0.1277E-020.83330.0753		10	6	4	0.5895E-01	0.7449E-02	0.1277E-020.83330.0753	
1	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0752		2	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0802	
3	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0773		4	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0773	
5	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0768		6	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0762	
7	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0758		8	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0755	
9	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0753		10	7	4	0.0000E 00	0.0000E 00	0.0000E 000.83330.0752	
1	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
3	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	4	1	5	0.6036E-05	0.1000E 01	0.0000E 000.83330.0175	0.3500E-01
5	1	5	0.6036E-05	0.1000E 01	0.0000E 000.83330.0175	0.3500E-01	6	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
7	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	8	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
9	1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-0110		1	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
1	2	5	0.2000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	2	5	0.6532E-01	0.1831E-01	0.1968E-010.83330.0206	0.4119E-01
3	2	5	-0.7848E-01	0.1496E 00	-0.4531E-020.83330.0169	0.3378E-01	4	2	5	-0.5003E-01	0.4579E 00	0.2038E-010.83330.0255	0.5097E-01
5	2	5	0.2067E 00	0.4685E 00	0.9457E-020.83330.0242	0.4841E-01	6	2	5	0.2190E 00	0.1688E 00	-0.6100E-020.83330.0163	0.3252E-01
7	2	5	0.7888E-01	0.4923E-01	0.6023E-020.83330.0183	0.3651E-01	8	2	5	0.7044E-01	0.1519E-01	-0.3372E-030.83330.0176	0.3507E-01
9	2	5	0.5176E-01	0.5997E-02	0.4268E-030.83330.0176	0.3508E-0110		2	5	0.5176E-01	0.5997E-02	0.4089E-030.83330.0176	0.3507E-01
1	3	5	0.2000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	3	5	0.7844E-01	0.8176E-01	0.1699E-010.83330.0199	0.3975E-01
3	3	5	0.1950E-01	0.1619E 00	0.4832E-020.83330.0189	0.3776E-01	4	3	5	0.4814E-01	0.4007E 00	0.7541E-020.83330.0205	0.4095E-01
5	3	5	0.1853E 00	0.3922E 00	0.3364E-020.83330.0200	0.3989E-01	6	3	5	0.2086E 00	0.1638E 00	0.1581E-020.83330.0182	0.3641E-01
7	3	5	0.1422E 00	0.8004E-01	0.5111E-020.83330.0183	0.3661E-01	8	3	5	0.1181E 00	0.2901E-01	0.1832E-020.83330.0178	0.3558E-01
9	3	5	0.9467E-01	0.1301E-01	0.9612E-030.83330.0176	0.3521E-0110		3	5	0.9467E-01	0.1301E-01	0.9610E-030.83330.0176	0.3521E-01
1	4	5	0.2000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	4	5	0.9954E-01	0.7573E-01	0.1795E-010.83330.0195	0.3912E-01
3	4	5	0.7530E-01	0.1454E 00	0.7700E-020.83330.0189	0.3785E-01	4	4	5	0.9644E-01	0.2444E 00	0.1287E-010.83330.0200	0.3991E-01
5	4	5	0.1746E 00	0.2399E 00	0.9331E-020.83330.0196	0.3920E-01	6	4	5	0.1939E 00	0.1450E 00	0.4067E-020.83330.0184	0.3668E-01
7	4	5	0.1581E 00	0.8296E-01	0.5491E-020.83330.0182	0.3636E-01	8	4	5	0.1350E 00	0.3771E-01	0.2785E-020.83330.0178	0.3562E-01
9	4	5	0.1145E 00	0.1815E-01	0.1311E-020.83330.0177	0.3524E-0110		4	5	0.1145E 00	0.1815E-01	0.1303E-020.83330.0177	0.3525E-01
1	5	5	0.2000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	5	5	0.1130E 00	0.4718E-01	0.1692E-010.83330.0194	0.3875E-01
3	5	5	0.1091E 00	0.1011E 00	0.9627E-020.83330.0188	0.3758E-01	4	5	5	0.1134E 00	0.1521E 00	0.1106E-010.83330.0190	0.3804E-01
5	5	5	0.1429E 00	0.1504E 00	0.8731E-020.83330.0188	0.3754E-01	6	5	5	0.1511E 00	0.1033E 00	0.6391E-020.83330.0183	0.3654E-01
7	5	5	0.1352E 00	0.6451E-01	0.5628E-020.83330.0180	0.3605E-01	8	5	5	0.1188E 00	0.3297E-01	0.3441E-020.83330.0178	0.3554E-01
9	5	5	0.1042E 00	0.1684E-01	0.1665E-020.83330.0176	0.3522E-0110		5	5	0.1042E 00	0.1684E-01	0.1627E-020.83330.0176	0.3521E-01
1	6	5	0.2000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	6	5	0.9742E-01	0.5013E-01	0.1660E-010.83330.0198	0.3966E-01
3	6	5	0.9892E-01	0.6984E-01	0.9132E-020.83330.0186	0.3714E-01	4	6	5	0.8495E-01	0.8645E-01	0.1116E-010.83330.0186	0.3721E-01
5	6	5	0.3959E-01	0.8234E-01	0.9152E-020.83330.0184	0.3673E-01	6	6	5	0.8927E-01	0.6094E-01	0.6928E-020.83330.0181	0.3614E-01
7	6	5	0.8075E-01	0.3948E-01	0.5235E-020.83330.0179	0.3573E-01	8	6	5	0.7116E-01	0.2136E-01	0.3236E-020.83330.0177	0.3538E-01
9.	6	5	0.6299E-01	0.1115E-01	0.1649E-020.83330.0176	0.3517E-0110		6	5	0.6299E-01	0.1115E-01	0.1580E-020.83330.0176	0.3515E-01
1	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	2	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
3	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	4	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
5	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	6	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
7	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01	8	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01
9	7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-0110		7	5	0.0000E 00	0.0000E 00	0.0000E 000.83330.0175	0.3500E-01

VITA

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