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SIDEWISE FORCE EXERTED ON SLOWLY FALLING SPHERES

INSIDE A CIRCULAR CYLINDER

BY

GARY MARK GREENSTEIN

A THESIS

PRESENTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE

OF

MASTER OF SCIENCE IN CHEMICAL ENGINEERING

AT

NEWARK COLLEGE OF ENGINEERING

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Newark, New Jersey

September, 1974

APPROVAL OF THESIS

SIDEWISE FORCE EXERTED ON SLOWLY FALLING SPHERES

INSIDE A CIRCULAR CYLINDER

BY

GARY MARK GREENSTEIN

FOR

DEPARTMENT OF CHEMICAL ENGINEERING NEWARK COLLEGE OF ENGINEERING

BY

FACULTY COMMITTEE

APPROVED:

NEWARK, NEW JERSEY SEPTEMBER, 1974

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ABSTRACT

Numerical values are provided for the forces that must be applied to a reference sphere to prevent its sidewise motion when two equal-sized spheres settle through a viscous liquid bounded by a cylindrical tube. These values are presented for two independent set of circumstances:

(1) - the spheres are translating but not rotating

(2) - the spheres are rotating but not translating Forces have been calculated assuming various distances between the sphere centers and with the line of centers at various angles to the horizontal. The results are discussed for the case where the spheres are both translating and rotating and compared to previous theoretical predictions as well as experimental results. Ī.

а	sphere radius
Ъ	distance from sphere center to cylinder axis
E(7,B)	dimensionless eccentricity function defined by eq. (3.4)
E(4,B)	dimensionless eccentricity function defined by eq. (3.2)
F	frictional force on particle
$\overline{\overline{F}}_{\mathbf{x}}$	component of frictional force on particle in sidewise
• • 1	(or radial) direction
Fz	component of frictional force on particle in z-direction
1, j, k	Cartesian unit vectors
1(7,B)	dimensionless eccentricity function defined in Section 3
1(4,B)	dimensionless eccentricity function defined in Section 3
N(7, B)	dimensionless eccentricity function defined by eq. (4.4)
N(4, B)	dimensionless eccentricity function defined by eq. (4.2)
n(7, B)	dimensionless eccentricity function defined in Section 4
n(4,B)	dimensionless eccentricity function defined in Section
р	dynamic pressure
r	distance between sphere centers when the angle between
	the line joining their centers and the horizontal is
	other than 90 ⁰
Ro	radius of circular cylinder
ບັ,ບ	translational velocity and speed, respectively, of sphere
	center relative to cylinder wall
V	local fluid velocity
x,y,z	rectangular Cartesian coordinates
z1	vertical distance between the sphere centers when the
•	angle between the line joining their centers and the

horizontal is 90⁰

dimensionless eccentricity, b/Ro

angle from the vertical of the path of the spheres dimensionless distance between spheres when the angle between the line joining their centers and the . horizontal is 90° , z_1/R_0

fluid viscosity

β

E

27

Ø

angle between line joining the sphere centers and the horizontal

 $\mathbf{\vec{n}} = \mathbf{in}_1 + \mathbf{jn}_2 + \mathbf{kn}_3$ angular particle velocity

SUPERSCRIPTS

 $11S_1$, 12C, $13S_1$, $23S_1$, $21S_2$, 22C, $22S_1$ definition follows in

definition follows in Section 2; see also figure 3

1. INTRODUCTION

Prediction of the frictional force in a radial direction on one of two identical particles settling at small Reynolds numbers through a bounded, quiescent, viscous fluid finds application in the fields of rheology and biomedical engineering. Early work by Smoluchowski⁽¹⁾ identified the radial migration effect when two spheres fall in a viscous fluid by utilizing the "method of reflections", but assuming an unbounded medium. The "method of reflections" has been described in detail by Happel and Brenner⁽²⁾.

A recent investigation by Greenstein and Happel⁽³⁾ broadened Smoluchowski's work by including wall effects in analyzing the two-sphere problem using the "method of reflections". The latter study, however, was primarily concerned with the frictional forces on the two spheres in their direction of motion. Numerical values for the sidewise forces were not given in their work.

In this work, we have concentrated solely on the frictional forces in the sidewise or radial direction. We have used the "method of reflections" and included wall effects to evaluate numerically the analytical expressions for these sidewise forces.

2. DESCRIPTION OF THE PROBLEM

Let us consider the slow translation or rotation of two equally sized spherical particles in a viscous incompressible fluid confined within an infinitely long circular cylindrical tube. Two independent sets of circumstances were investigated:

(1) the spheres move with arbitrary constant translational velocity ($\vec{U} = -\vec{k}U$) but do not rotate

(2) the spheres rotate with arbitrary constant angular velocity $(\hat{\Omega} = j\Omega_{2})$ but do not translate

Figures 1 and 2 illustrate the geometric configurations studied. In Figure 1, the spheres are situated at an angle $\not\!\!\!/$ between the line of centers and the horizontal. The radius of each sphere is a, the cylinder radius is R_0 , and the centers of Sphere-1 and Sphere-2 are situated at distances b from the cylinder axis. The distance between the centers of Spheres 1 and 2 is r.

In Figure 2, the spheres are located in the same vertical plane and the sphere centers are displaced from the cylindrical axis by a distance b. Again, the radius of each sphere is a and the cylinder radius is R_0 . The spheres are separated by a vertical distance, z_1 .

If we consider only translation, we say that the center of each sphere translates with velocity \overline{U} relative to the cylinder wall in the negative z direction, parallel to the cylinder axis $(\overline{U} = -\overline{k}U)$. If we consider only rotation, we say that each sphere rotates with angular velocity $\overline{\Omega}$ relative to the cylinder wall about axes parallel to the y-coordinate axis $(\overline{\Omega} = j\Omega_2)$. In both cases, at $[z] = \infty$, the fluid is at rest.

The translation and rotation problems can be solved

independently, assuming that the fluid motion is governed by the creeping motion and continuity equations:

 $\mu \nabla^2 \vec{\nabla} = \nabla p \qquad (2.1)$ $\nabla \cdot \vec{\nabla} = 0 \qquad (2.2)$

where \overline{V} is the fluid velocity with respect to a coordinate system that moves with Sphere-1, p is the dynamic pressure, and μ the fluid viscosity. The boundary conditions necessary to solve these problems are:

(1) at fluid -solid interfaces there is no relative motion

(2) at $|z| = \infty$ the fluid is at rest

Both of these boundary value problems can be solved by a technique of successive approximations known as the method of reflections (described by Happel and Brenner ⁽²⁾). Figure 3 outlines a schematic representation of this calculation technique. An initial disturbance is reflected from the boundaries involved and produces smaller and smaller effects with each successive reflection.

The following superscript notation has been adopted to describe the velocity fields

♥ijS_{i and} ♥ijC (3):

i = the sphere at which a disturbance originates

j = the number of times that reflections have occurred to produce this disturbance

S1 = the latest disturbance reflected from Sphere-1

 S_2 = the latest disturbance reflected from Sphere-2

C = the latest disturbance produced by the cylinder wall As shown in Figure 3, Sphere-2 will disturb the motion of Sphere-1 in two ways:

1) by a direct reflection of its own Stokes field, \vec{V}^{21S}_{2}

2) by the reflection of this field from the cylinder wall

and then to Sphere-1, $\sqrt[7]{22C}$

Since the equations of motion and boundary conditions are linear, the frictional force \mathbf{F}_{II} exerted on the reference sphere (Sphere-1) is obtained by adding the frictional forces resulting from each of the individual fields. Hence,

 $\overline{F}_{II} = \overline{F}^{11S_1} + \overline{F}^{13S_1} + \overline{F}^{22S_1} + \overline{F}^{23S_1} + \dots$ where \overline{F}^{ijS_1} is the frictional force associated with the field \overline{V}^{ijS_1} . This work has considered only the component of \overline{F}_{II} in the radial direction perpendicular to the cylindrical axis, i.e., \overline{F}_{X} .

3. RESULTS FOR THE SIDEWISE FORCE EXERTED ON ONE OF

TWO SLOWLY TRANSLATING SPHERES INSIDE A CIRCULAR

CYLINDER

If the spheres are only translating, let us consider the system shown in Figure 1. Two equally sized spheres situated at an angle # between the line of centers and the horizontal settle through a quiescent fluid in a direction parallel to the cylindrical axis. The frictional force in the radial direction \overline{F}_{x}^{23S} 1, exerted by the fluid on the spheres, due to the wall-sphere interaction effect is given by

$$\vec{F}_{x}^{23S} = \vec{1} 6 \pi \mu a UE(\vec{\mu}, \beta) \frac{a}{R_{0}} + 0 \left(\frac{a}{R_{0}}\right)^{3}$$
(3.1)
$$E(\vec{\mu}, \beta) = \frac{3}{2\pi} 1(\vec{\mu}, \beta)$$
(3.2)

where

and ψ is the angle between the line connecting the sphere centers and the horizontal, and β equals b/R_0 .

If we consider the system shown in Figure 2, the two spheres are located in the same vertical plane, separated by a distance z_1 , and displaced from the cylindrical axis by a given distance as they settle through a quiescent fluid in a direction parallel to the cylindrical axis. In this case, the frictional force in the radial direction \overline{F}_x^{23S} due to the wall-sphere interaction effect is given by

$$F_{x}^{23S_{1}} = 16\pi \mu aUE(\eta, \beta) \frac{a}{R_{o}} + 0(\frac{a}{R_{o}})^{3}$$

where

$$E(\eta, \beta) = \frac{3}{2\pi} I(\eta, \beta)$$
 (3.4)

(3.3)

and $\gamma = z_1/R_0$, and $\beta = b/R_0$ Note that the functions $l(\Psi,\beta)$ and $l(\Psi,\beta)$, as well as $E(\Psi,\beta)$ and $E(\Psi,\beta)$, are comparable and differ only in the manner in which the first parameter is expressed. The first parameter is necessary to fully define the location of the spheres. A detailed expression for $l(\Psi, \beta)$ or $l(\Psi, \beta)$ was derived based upon Greenstein's⁽⁴⁾ expression for the ∇^{22C} velocity field. $l(\Psi, \beta)$ or $l(\Psi, \beta)$ was found to be a complicated function of (Ψ, β) or (Ψ, β) involving modified Bessel functions of the first and second kind and requiring numerical evaluation. A high speed computer was employed to accomplish this numerical evaluation and the program is shown in Appendix 1.

Values of $E(\mathcal{V}, \mathcal{B})$ and $1(\mathcal{V}, \mathcal{B})$ vs. \mathcal{B} are tabulated in Table 1 for values of \mathcal{V} equal to 30°, 45°, and 60°. Similarly, values of $E(\mathcal{N}, \mathcal{B})$ and $1(\mathcal{N}, \mathcal{B})$ vs. \mathcal{B} are tabulated in Table 2 for values on \mathcal{N} ranging from 0.00001 to 20.

Based on previous work by Greenstein⁽⁴⁾, it can be shown that, if the spheres are translating parallel to the cylinder axis only:

 $F_x^{11S_1} = F_x^{13S_1} = 0$ (3.5) $F_x^{22S_1}$ is also equal to 0 for the system shown in Figure 2. However, for the system shown in Figure 1, it can be shown that:

 $F_x^{22S_1} = 16\pi\mu a(-\frac{3aU}{8b}\cos^2 \#\sin \#) \qquad (3.6)$ This expression becomes for values of $\# = 30^\circ$, 45° and 60°:

Ý		
300	$F_{x}^{22S_{1}} = 16 \pi \mu a(-0.140625 \frac{aU}{b})$	
45 ⁰	$F_{x}^{22S_{1}} = 16 \pi (-0.132582 \frac{aU}{b})$	(3.7)
60 ⁰	$F_{x}^{22S_{1}} = 16\pi\mu a(-0.0811898\frac{aU}{b})$	

The total sidewise force exerted by the fluid on the reference sphere for the system described by Figure 1 can be computed as the sum of $\overline{F_x}^{22S_1}$ and $\overline{F_x}^{23S_1}$:

$$\overline{F}_{x} = \overline{F}_{x}^{22S_{1}} + \overline{F}_{x}^{23S_{1}}$$

$$= \overline{16\pi\mu a} \left(-\frac{3}{8} \frac{aU}{b} \cos^{2} \# \sin \# + UE(\#\beta) \frac{a}{R_{0}}\right) + O\left(\frac{a}{R_{0}}\right)^{3} \quad (3.8)$$
Values of $E(\#\beta)$ can be obtained from Table 1.

The total sidewise force exerted by the fluid on the reference sphere for the system shown in Figure 2 is $F_x^{23S_1}$ (given by equation (3.3)). Values of $E(\gamma,\beta)$ can be obtained from Table 2.

These sidewise forces cause radial migration when the spheres are freely suspended in the fluid. To prevent the spheres from moving sidewise, a force, $-\overline{F}_{x}$, equal in magnitude but opposite in direction to \overline{F}_{x} would have to be exerted on the spheres. Forces resulting from successive reflections have been neglected, since they are of the same order of magnitude as the error term (on the order of $\left(-\frac{a}{R_{0}}\right)^{3}$).

4. RESULTS FOR THE SIDEWISE FORCE EXERTED ON ONE OF

TWO SLOWLY ROTATING SPHERES INSIDE A CIRCULAR

CYLINDER

Now let us again consider the system in Figure 1, but this time with the spheres only rotating. Two equally sized spheres situated at an angle # between the line of centers and the horizontal rotate but do not translate in a quiescent fluid. The frictional force in the radial direction $F_{x,r}^{23S}$ due to the wall-sphere interaction effect is given by

where $N(\Psi, \beta) = \frac{3}{2\pi} n(\Psi, \beta)$ (4.1) (4.2)

and \checkmark is the angle between the line connecting the sphere centers and the horizontal, and β equals b/R_0 .

If we consider the system shown in Figure 2, the two spheres are located in the same vertical plane, separated by a given distance z_1 and displaced from the cylindrical axis by a given distance as they rotate in the y-direction (but do not translate) in a quiescent fluid. The frictional force in the radial direction $\overline{F}_{x,r}^{23S1}$ due to the wall-sphere interaction effect can be expressed as

$$\overline{F}_{x,r}^{23S_1} = -i8\pi \mu a^2 \Omega_2 (\frac{a}{R_0})^2 N(\eta, \beta) + O(\frac{a}{R_0})^4 \qquad (4.3)$$

where

$$N(\eta, \beta) = \frac{3}{2\pi} n(\eta, \beta)$$
 (4.4)

and

 $\eta = z_1/R_0$, and $\beta = b/R_0$.

The functions $n(\mathcal{V},\mathcal{B})$ and $n(\mathcal{N},\mathcal{B})$, and similarly $N(\mathcal{V},\mathcal{B})$ and $N(\mathcal{N},\mathcal{B})$, are directly comparable and differ only in the form of the first parameter necessary to define the location of the spheres.

A detailed expression for $n(\mathcal{Y},\beta)$ or $n(\mathcal{Y},\beta)$ was developed based upon Greenstein's⁽⁴⁾ expression for the $\overline{\mathbb{V}}^{22\mathbb{C}}$ velocity field. As in the translation problem, $n(\mathcal{V}, \mathcal{B})$ or $n(\mathcal{N}, \mathcal{B})$ was determined to be a complicated function of $(\mathscr{V}, \mathcal{B})$ or $(\mathscr{N}, \mathcal{B})$ involving modified Bessel functions of the first and second kind. This function was evaluated on a high speed computer using numerical techniques and the program used is shown in Appendix 2.

Values of $N(\mathcal{V}, \mathcal{B})$ and $n(\mathcal{V}, \mathcal{B})$ vs. \mathcal{B} are tabulated in Table 3 at values of Ψ equal to 30°, 45° and 60°. Values of N(η , B) and $n(\eta, \beta)$ vs. β are tabulated in Table 4 for values of η ranging from 0.00001 to 20.

Based on previous work by Greenstein⁽⁴⁾, it can be shown that, if the spheres are rotating only:

 $F_{r}^{11S_1} = F_{r}^{13S_1} = 0$ (4.5)An expression for the values of $\overline{F_{x,r}}^{22S_1}$ can also be derived from Greenstein's work. For the geometric system described in Figure 1, this expression is:

$$\overrightarrow{F_{x,r}}^{22S_1} = \widehat{16} \pi \mu a (\frac{a^3 \Omega_2}{4b^2} \cos^2 \psi \sin \psi) \quad (4.6)$$
which, if evaluated at values of $\psi = 30^\circ$, 45° and 60° , becomes:

$$\frac{\psi}{30^\circ} = \widehat{F_{x,r}}^{22S_1} = \widehat{16} \pi \mu a (0.09375 \frac{a^3 \Omega_2}{b^2})$$

$$45^\circ = \widehat{F_{x,r}}^{22S_1} = \widehat{16} \pi \mu a (0.0883883 \frac{a^3 \Omega_2}{b^2}) \quad (4.7)$$

$$60^\circ = \widehat{F_{x,r}}^{22S_1} = \widehat{16} \pi \mu a (0.0541265 \frac{a^3 \Omega_2}{b^2})$$

A similar expression for the system shown in Figure 2 is:

60⁰

$$\overline{F}_{x,r}^{22S_1} = 16 \pi \mu a \left(\frac{a^3 \Omega_2}{z_1^2}\right)$$
(4.8)

indicating that this component of the sidewise force increases as the spheres are moved closer together.

The total sidewise force on a reference sphere for either of the geometries discussed is the sum of $F_{x,r}^{22S_1}$ and $F_{x,r}^{23S_1}$.

Forces resulting from successive reflections have been neglected, since they are of the same order of magnitude as the error term (on the order of $(\frac{a}{R_0})^4$).

5. QUALITATIVE DISCUSSION OF RESULTS WHEN SPHERES ARE TRANSLATING AND ROTATING

Let us examine in a qualitative manner the total sidewise force on one sphere when the spheres are translating and rotating. By comparing equation (3.1) with equation (4.1) and equation (3.3) with equation (4.3), it is clear that the value of $\overline{F}_{x,r}^{23S}$ 1 due to rotation is much smaller than the value of \overline{F}_x^{23S} 1 due to translation. This is true because the value of a/R_0 is quite small. As a matter of fact, $\overline{F}_{x,r}^{23S}$ 1 due to rotation is of the order of magnitude $(\frac{a}{R_0})^2$, whereas the order of magnitude of \overline{F}_x^{23S} 1 due to translation is $(\frac{a}{R_0})$. Hence, when the spheres are translating and rotating, the value of $\overline{F}_{x,r}^{23S}$ 1 due to rotation can be neglected in computing the total sidewise force.

The $\overline{F}_{x,r}^{22S}$ l component of the sidewise force due to rotation however, cannot be neglected in computing the total sidewise force on the reference sphere when the spheres are translating and rotating. In particular, for the system shown in Figure 2, the value of \overline{F}_{x}^{22S} l due to translation is zero, whereas the value of $\overline{F}_{x,r}^{22S}$ l due to rotation must be computed from equation (4.8). It should be noted, however, that the value of $\overline{F}_{x,r}^{22S}$ l computed from equation (4.8) is of the order of $(\frac{a}{z_1})^2$ which is still an order of magnitude less than the value of \overline{F}_{x}^{23S} l due to translation. For the system described by Figure 1, the value of \overline{F}_{x}^{22S} l due to rotation is of the order of $(\frac{a}{b})$ whereas the value of $\overline{F}_{x,r}^{22S}$ l due to rotation is of the order of $(\frac{a}{b})^2$. Again, the sidewise component due to rotation is of a lower order of magnitude. In summary, it can be shown that when two equally sized spheres situated at an angle # between the line of centers and the horizontal translate and rotate through a quiescent fluid, the sidewise force on the reference sphere is influenced much more strongly by the translation of the spheres than by the rotation of the spheres. This conclusion is based on a qualitative inspection of the orders of magnitude of the sidewise components of the frictional forces. ふつ

6. COMPARISON OF THEORY WITH OTHER THEORETICAL AND EXPERIMENTAL RESULTS

The problem of the motion of two spheres in an infinite medium was first investigated by Smoluchowski⁽¹⁾⁽⁵⁾. In his work, Smoluchowski used the method of reflections but did not consider wall effects. In addition, the spheres were considered to be only translating. Based on expressions derived for the component of force tending to diminish the resistance in the direction of motion along the line connecting the centers of the two spheres, Smoluchowski developed the relationship

 $\sin \epsilon = \frac{3}{4} \frac{a}{r} \left(1 - \frac{3}{2} \frac{a}{r}\right) \cos \psi \sin \psi$ (6.1) for the angle ϵ through which the spheres would be deflected from the vertical towards the line of centers. In other words, the spheres would exhibit motion in a radial direction as well as in a vertical direction.

By not considering wall effects, Smoluchowski only considered those components of the frictional force referred to in this paper as \overline{F}^{11S} l and \overline{F}^{22S} l. As shown in Figure 3, the other components of the frictional force on Sphere-1 result from wall-sphere interactions, which Smoluchowski did not consider. Equation (6.1) can be derived from Greenstein's⁽⁴⁾ equation for the \overline{V}^{21S} 2 velocity field by making a number of approximations. This derivation is shown in Appendix 3.

It should be noted that Smoluchowski would predict radial migration only for the system shown in Figure 1. For the geometry in Figure 2, Smoluchowski's \in would be zero. This is totally consistent with the derivation presented in Section 3 of this paper which states that $\overline{F}_x^{11S}l = \overline{0}$ in both cases and that $\overline{F}_x^{22S}l$ exists for the system shown in Figure 1 but is equal to $\overline{0}$ for the system in Figure 2.

From the results presented in this paper, the total sidewise force (\overline{F}_{X}) on the reference sphere can be calculated. This could be coupled with the appropriate value for the total vertical force (\overline{F}_{Z}) on the sphere based on Greenstein's(4) expressions to obtain an angle of deflection, $\boldsymbol{\varepsilon}$, which would include the effect of wall-sphere interactions. This would therefore extend Smoluchowski's work and result in more theoretically correct values of $\boldsymbol{\varepsilon}$.

Eveson, Hall and Ward⁽⁶⁾ carried out a series of experiments to study the motion of two spheres settling through a quiescent fluid bounded by a circular cylinder. When the spheres were positioned as in Figure 1, it was found that the paths of the spheres were deflected from the vertical by a small angle. The experimental values for these angles were stated to be in agreement with Smoluchowski's relationship (equation (6.1)), although numerical values were not published.

Since the values of **€** tend to be quite small, the difference between equation (6.1) and the method proposed in this paper to obtain **€** may have been beyond the accuracy of Eveşon, Hall and Ward's experiments. This would explain the statement that experimental values are in agreement with Smoluchowski's relationship. Nevertheless, the importance of Eveson, Hall and Ward's experiments was to demonstrate that

radial migration does occur when two spheres settle through a quiescent fluid bounded by a circular cylinder, as discussed in this paper. -----

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displaced from the axis of a circular cylinder by a given distance



fields

TABLE 1

 $1(\Psi, B)$ AND $E(\Psi, B)$ VS. β FOR VARIOUS VALUES OF Ψ

B	$1(\Psi = 30^{\circ}, B)$	E(#=30°, B)	1(4=45°, B)	E(4=450, B)
0.05	·-0.00229853	-0.00109747	-0.00397989	-0.00190026
0.15	-0.0211509	-0.0100938	-0.0363889	-0.0173744
0.20	-0.0381173 -0.0602832	-0.0181997 -0.0287831	-0,0649391 -0,100969	-0.0310061 -0.0482091
0.30	-0.0873666	-0.0417145 -0.0566387	-0.142730	-0.0681484 -0.0895707
0.40	-0.153010	-0.0730568	-0.232452	-0.110988
0.50	-0.189158 -0.224951	-0.107406	-0.309214	-0.147639
0.55 0.60	-0,258674 -0,288304	-0.123508 -0.137655	-0,336020 -0,353282	-0.160438 -0.168680
0.65	-0.312224	-0.149076 -0.157155	-0 .360951	-0.172342
0.75	~0. 338382	-0.161566	-0.352182	-0.168154
0.85	-0.334593	-0,159756	-0.369575	-0.176459

B	$1(\Psi = 60^\circ, B)$	$E(\Psi=60^\circ, B)$
0.05	-0.00688600	-0.00328782
0.10	-0.0275898	-0.0131731
0.20	-0.105708	-0.0504716
0.30	-0.203116	-0. 0969805
0.35	-0.243321	-0.116177
0.45	-0.287007	-0.137035
0,50	-0.290352	-0.138633 -0.135822
0.60	-0.272576	-0.130146
0,65	-0.257639 -0.241873	-0.123013 -0.115486
0.75	-0.226672	-0.108228
0.80	-0.200199	-0.09558 79

TABLE 2

1(n, B)	AND	E(n, B)	vs.	B	FOR	VARIOUS	VALUES	OF	27

B	10 ³ x <u>1(7=. 60 60 1, B)</u>	10 ³ х <u>Е(9=.00001, В)</u>	1(7=0.01,B)	E(7=0.01, B)
0.01 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.00401767 -0.00834350 -0.0133963 -0.0198913 -0.0292634 -0.0449186 -0.0766169 -0.162935 -0.278494	-0.00191830 -0.00398373 -0.00639626 -0.00949738 -0.0139723 -0.0214470 -0.0365819 -0.0777957 -0.132971	-0.000396754 -0.00401491 -0.00833766 -0.0133866 -0.0198761 -0.0292391 -0.0448753 -0.0765204 -0.162591 -0.277579	-0.000189436 -0.00191698 -0.00398094 -0.00639164 -0.00949015 -0.0139607 -0.0214264 -0.0365358 -0.0776315 -0.132534
ß	$1(\eta = 0.05, B)$	E(7=0.05, B)	1(7=0.1,B)	E(9=0.1, B)
0.01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.00197782 -0.0200127 -0.0415487 -0.0666740 -0.0989042 -0.145253 -0.222218 -0.376244 -0.783428 -1.30630	-0.000944339 -0.00955535 -0.0198380 -0.0318345 -0.0472253 -0.0693532 -0.106101 -0.179643 -0.374059 -0.623715	-0.00391872 -0.0396417 -0.0822317 -0.131745 -0.194869 -0.284721 -0.431302 -0.714598 -1.40060 -2.15616	-0.00187105 -0.0189275 -0.0392628 -0.0629034 -0.0930429 -0.135944 -0.205932 -0.341196 -0.668740 -1.02949
B	1(1=0.2, B)	E(7=0.2, B)	1(7=0.4,B)	$E(\eta=0.4,\beta)$
0.01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.00754993 -0.0762977 -0.157749 -0.251124 -0.367302 -0.526337 -0.767524 -1.17378 -1.86931 -2.25217	-0.00360483 -0.0364295 -0.0753195 -0.119903 -0.175374 -0.251307 -0.366466 -0.560441 -0.892533 -1.07533	-0.0130292 -0.131169 -0.267873 -0.416489 -0.584808 -0.781093 -1.00452 -1.20476 -1.14872 -0.876416	-0.00622098 -0.0626287 -0.127900 -0.198859 -0.279225 -0.372944 -0.479625 -0.575233 -0.548474 -0.418458

TABLE 2 (cont.)

B	1(n=0.6, B)	E(7:06, B)	1(7=0.8, B)	E(7=0.8, B)
0.01 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.0153857 -0.154035 -0.308977 -0.464524 -0.616680 -0.751776 -0.832850 -0.780844 -0.502578 -0.297600	-0.00734613 -0.0735461 -0.147526 -0.221794 -0.294443 -0.358946 -0.397657 -0.372826 -0.239963 -0.142093	-0.0148619 -0.147828 -0.290474 -0.420571 -0.525693 -0.584588 -0.565223 -0.436937 -0.219556 -0.114174	-0.00709601 -0.0705827 -0.138691 -0.200808 -0.251000 -0.279120 -0.269874 -0.208622 -0.104831 -0.0545139
B	$1(\eta \in [0, B)$	E(9=1.0, B)	1(9=1.2,B)	E(7=1.2, B)
0.01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.0125061 -0.123574 -0.237817 -0.331828 -0.391901 -0.401882 -0.348185 -0.234392 -0.101027 -0.0488484	-0.00597122 -0.0590022 -0.113549 -0.158436 -0.187119 -0.191884 -0.166246 -0.111914 -0.0482367 -0.0233234	-0.00947619 -0.0930529 -0.175655 -0.237019 -0.266462 -0.255590 -0.203542 -0.124259 -0.0484453 -0.0374640	-0.00452454 -0.0444295 -0.0838689 -0.113168 -0.127226 -0.122035 -0.0971848 -0.0593294 -0.0231309 -0.0178877
B	1(9=1.4,B)	<u>E(9=1.4, B)</u>	<u>1(9=1.6, B)</u>	E(9=1.6, B)
0.01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85	-0.00659991 -0.0644487 -0.119604 -0.156798 -0.169142 -0.153770 -0.114801 -0.0652537 -0.0236983 -0.0216344	-0.00315122 -0.0307720 -0.0571067 -0.0748654 -0.0807594 -0.0734195 -0.0548135 -0.0311563 -0.0113151 -0.0103297	-0.00428087 -0.0416026 -0.0760840 -0.0973271 -0.101455 -0.0883161 -0.0626521 -0.0336713 -0.0115458 -0.00502704	-0.00204396 -0.0198638 -0.0363274 -0.0464702 -0.0484413 -0.0421678 -0.0299142 -0.0160768 -0.00551271 -0.00240023

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TABLE 2 (cont.)

B	$1(\eta=2.0,\beta)$	$E(\eta=2.0,\beta)$	$1(\eta_{=}2.5, B)$	E(7=2.5, B)
0.01	-0.00149058	-0.000711700	-0.000265598	-0.000126814
0.1	-0.0143710	-0.00686162	-0.00253374	-0.00120977
0.2	-0.0256571	-0.0122503	-0.00437855	-0.00209060
0.3	-0.0315330	-0.0150559	-0.00508548	-0.00242814
0.4	-0.0310896	-0.0148442	-0.00460517	-0.00219881
0.5	-0.0252129	-0.0120383	-0.00329796	-0.00157466
0.6	-0.0164206	-0.00784026	-0.00177841	-0.000849129
0.7	-0.00797553	-0.00380804	-0.000620513	-0.000296273
0.8	-0.00241961	-0.00115531	-0.0000800211	-0.0000382073
B	10 ³ x	10 ³ x	10 ³ x	10 ³ x
	1(7=3.0, B)	<u>E(n = 3, 6)</u>	1(7=3.5,B)	E(٦= 3.S, B)
0.01	-0.00769027	-0.00367183	-0.0168686	-0.00805417
0.1	-0.0645746	-0.0308321	-0.164753	-0.0786638
0.2	-0.0631400	-0.0301471	-0.305920	-0.146066
0.3	-0.0295674	-0.0141174	-0.400714	-0.191327
0.4	-0.180887	-0.0863674	-0.431750	-0.206146
0.5	-0.313453	-0.149663	-0.393039	-0.187662
0.6	-0.352302	-0.168212	-0.296252	-0.141450
0.7	-0.273958	-0.130806	-0.173153	-0.0826745
0.8	-0.132197	-0.0631196	-0.0666206	-0.0318090
B	10 ³ x	10 ³ x	1.0 ⁶ x	10 ⁶ x
	1(7:4.0,B)	E(7=4.0, B)	1(7=8.0, <u>&</u>)	E(7=8.0,B)
0.01	-0.00848402	-0.00405082	-0.000486454	-0.000232265
0.1	-0.0820588	-0.0391802	-0.00118708	-0.000566790
0.2	-0.147932	-0.0706325	-0.000945605	-0.000451493
0.3	-0.185286	-0.0884676	-0.00617888	-0.00295020
0.4	-0.188229	-0.0898728	-0.000446258	-0.000213073
0.5	-0.159769	-0.0762842	-0.00162187	-0.000774384
0.6	-0.112096	-0.0535220	-0.00355178	-0.00169585
0.7	-0.0604338	-0.0288550	-0.00434382	-0.00207402
0.8	-0.0216215	-0.0103235	-0.0206333	-0.00985169
B	10 ⁶ x 1(7=20, B)	$E(\gamma=20, B)$	<u>)</u>	
0.01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8	-0.0000990008 -0.000923821 -0.00146969 -0.00362426 -0.00348456 -0.00690809 -0.00641325 -0.0115289 -0.00121437	-0.000047269 -0.000441092 -0.000701723 -0.00173046 -0.00166375 -0.00329837 -0.00306210 -0.00550462 -0.000579818	2	

TABLE 3

		1				
ß	<u>n (1=36,8)</u>	N(42309,B)	n (% 45°, B)	N (P= 459, B)	n.(\$460°,5)	N (4=609B)
0.05	0.202471	0.0966727	0.348894	0.166584	0.595091	0.284135
0.10	0.408763	0.195170	0.693459	0.331102	1,12953	0. 539308
0.15	0.622293	0.297123	1.02791	0.490792	1. 55069	0.740399
0.20	0.845613	0.403750	1.34385	0.641641	1.82234	0.870102
0.25	1.08034	0.515825	1.63037	0.778445	1. 93221	0.922561
0.30	1.32699	0.633591	1,87459	0.895049	1.89340	0.904032
0.35	1.58413	0.756365	2.06234	0.984693	1.73975	0.830670
0.40	1.84802	0.882365	2.18116	1.04143	1.51550	0.723601
0.45	2.11408	1.00940	2,22273	1.06127	1.26412	0.6035 7 2
0.50	2.37177	1.11.324	2.18438	1.04297	1. 01997	0.486998
0.55	2,60902	1,24572	2.07126	0.988952	0.804948	0.384334
0.60	2.80744	1.34045	1.89557	0.905066	0.629114	0.300380
0.65	2,94702	1.40710	1.67505	0.799776	0.493532	0.235644
0.70	3,00770	1.43607	1.42992	0.682737	0.393672	0.187965
0.75	2.97364	1.41981	1.17982	0,563323	0.322461	0.153964
0.80	2.83966	1.35584	0.941555	0.449559	0.272430	0.130076
0.85	2,61165	1.24697	0.727704	0.347453	0.236986	0.113153
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n(4, B) AND N(4, B) VS. B FOR VARIOUS VALUES OF 77

TABLE 4

n(7, B) AND N(7, B) VS. B FOR VARIOUS VALUES OF 22

B	10 ³ x n (%.00001, B)	10 ³ x <u>N(7=.00001,B)</u>	n (7=0.01, B)	N(7=0.01, B)
0.1	-0.0380197	-0.0181531	-0.0379953	-0.0181414
0.2	-0.0482421	-0.0230339	-0.0482104	-0.0230188
0.3	-0.0697369	-0.0332969	-0.0696893	-0.0332742
0.4	-0.113109	-0.0540056	-0.113027	-0.0539666
0.5	-0.207279	-0.0989686	-0.207116	-0.0988907
0.6	-0.446028	-0.212963	-0.445624	-0.212699
0.7	-1.23239	-0.588422	-1.23096	-0.587740
0.8	-5.38621	-2.57173	-5.37607	-2.56688
0.85	-15.7216	-7.50649	-15.6763	-7.48486
B	n (7:005,6)	$N(\eta_z 0.05_AB)$	n(n=0.1,B)	<u>N(9=0.1, B)</u>
0.1	-0.189603	-0.0905286	-0.376882	-0.179948
0.2	-0.240489	-0.114825	-0.477484	-0.227982
0.3	-0.347420	-0.165881	-0.688482	-0.328726
0.4	-0.562942	-0.268785	-1.11232	-0.531093
0.5	-1.02997	-0.491775	-2.02544	-0.967077
0.6	-2.20982	-1.05511	-4.30795	-2.05689
0.7	-6.06717	-2.89686	-11.6098	-5.54323
0.8	-26.0465	-12.4363	-47.4312	-22.6467
0.85	-74.1781	-35.4174	-126.007	-60.1640
B	n(7=0.2, B)	<u>N(9=0.2,B)</u>	n(7=64,B)	N(7=0.4,B)
0.1	-0.735556	-0.351202	-1.33664	-0.638200
0.2	-0.927698	-0.442943	-1.65732	-0.791313
0.3	-1.32768	-0.633921	-2.30645	-1.10125
0.4	-2.12057	-1.01250	-3.53213	-1.68647
0.5	-3.79065	-1.80990	-5.91249	-2.82300
0.6	-7.79962	-3.72404	-10.8854	-5.19737
0.7	-19.6231	-9.36934	-22.2343	-10.6161
0.8	-67.1287	-32.0516	-49.3058	-23.5418
0.85	-145.065	-69.2633	-72.5444	-34.6374

TABLE 4 (cont.)

B	n(N=0.6, B)	N(7=0.6, B)	n(7=0.8, B)	N(7=0.8,B)	n(7=1.0,B)	N (9=1.0,B)
0.1	- 1.72009	-0.821282	- 1.87375	-0.894649	-1.84040	-0.878724
0.2	- 2.07976	-0.993013	- 2.19939	-1.05013	-2.09488	-1.00023
0.3	- 2.77897	-1.32686	- 2.80357	-1.33860	-2.54516	-1.21522
0.4	- 4.01145	-1.91532	- 3.78998	-1.80958	-3.22609	-1.54034
0.5	- 6.15745	-2.93996	- 5.31759	-2.53896	-4.16849	-1.99031
0.6	- 9.92209	-4.73745	- 7.57105	-3.61491	-5.35452	-2.55660
0.7	-16.4018	-7.83127	-10.5831	-5.05305	-6.61903	-3.16035
0.8	-26.1424	-12.4821	-13.6925	-6.53769	-7.51153	-3.58649
0.85	-31.1948	-14.8944	-14.6842	-7.01120	-7.59212	-3.62497
B	n (n=1.2, B)	<u>N(7:1.2, B)</u>	n(n=1.4, B)	N(91.4,8)	n(721.6, B)	<u>N(7:1.6,8)</u>
0.1	-1.68612	-0.805064	-1.47360	-0.703590	-1.24898	-0.596346
0.2	-1.86398	-0.889985	-1.58706	-0.757767	-1.31568	-0.628190
0.3	-2.16468	-1.03356	-1.77082	-0.845506	-1.41926	-0.677645
0.4	-2.58768	-1.23552	-2.01222	-0.960766	-1.54631	-0.738306
0.5	-3.11486	-1.48724	-2.28406	-1.09056	-1.67440	-0.799467
0.6	-3.68642	-1.76014	-2.53589	-1.21080	-1.77065	-0.845424
0.7	-4.16918	-1.99064	-2.68917	-1.28398	-1.79377	-0.856460
0.8	-4.34316	-2.07370	-2.64677	-1.26374	-1.70306	-0.813153
0.85	-4.23562	-2.02236	-2.52540	-1.20579	-1.60689	-0.767233

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β	n(9=2.0, B)	<u>N(7=2.0, B)</u>
0.1	-0.859034	-0.410158
0.2	-0.876634	-0.418562
0.3	-0.901297	-0.430338
0.4	-0,926101	-0.442181
0.5	-0.941909	-0.449728
0.6	-0.938415	-0.448060
0.7	-0,906276	-0.432715
0.8	-0.840154	-0.401144
0.85	-0.794591	-0.379389

TABLE 4 (cont.)

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B	n(n=2.5,B)	N(9=2.5,B)	n(7=3.0,B)	N(7=3.0,B)
0.1	-0.539317	-0,257505	-0,360533	-0.172142
0.2	' -0。 539883	-0.257775	-0.359113	-0.171464
0.3	-0,539550	-0.257616	-0.356560	-0.170245
0.4	-0. 536645	-0.256229	-0,352671	-0.168388
0.5	-0.529326	-0.252734	-0.347304	-0.165825
0.0	-0.516047	-0.246394	-0.340477	-0.162500
	-0.496125	-0.230002	-0.552475	-0.158745
	-0.470220 0.455699	=0.224515	-0.310655	-0.152624
0.02	-0.499000	••U•21/5/5	-0.019000	-0.1J2024
R		27 (m 0 0 0)		
<u> </u>	n(7=3.5,6)	N(N=3,5,B)	n(n=4.0,B)	N(1/2 4.0, B)
0.1	-0.259014	-0.123670	-0.196586	-0.0939108
0.2	-0.258116	-0.123241	-0,196255	-0,0937047
0.3	-0.256634	-0.122534	-0,195558	-0.0933720
0.4	-0,254609	-0.121567	-0.194633	-0 .0929305
0.5	-0.252117	-0.120377	-0.193530	-0. 0924035
0.6	-0.249285	-0.119025	-0.192309	-0.0918208
0.7	-0.246296	-0.117598	-0.191041	-0.0912154
8.0	-0.243380	-0.116206	-0.189795	-0.0906206
0.85	-0,242030	-0.115561	-0.189200	-0,0903361
B	n(7=8.0,B)	N(7=8.0,B)	$n(7=20, \beta)$	N(7=20, B)
0.1	-0.0491615	-0.0234729	-0.00785602	-0.00375097
0.2	-0.0491445	-0.0234647	-0.00785557	-0.00375076
0	-0.0491197	-0.0234310	-0.00785485	-0.00375041
0 5	-0.0490750 -0.0400251	-0.0234320 -0.0234077		-U.UU574992
0.6	-0.0490201	-0 0233790	-0.00705020	-0.00574929
0.7	-0.0488008	-0.0233/02	0 00702007	-U.UU3/4852
0.8	-0.0488083	-0.0233042	-0 00704097	-U.UUJ/4/01
0.85	-0.0487623	-0.0232823		-U,UU3/4055
	0.000020			=0.003/4002

C ·	MAIN PRUGRAM	TG 0002
	TAPLICIT SEAL*B(A-H,O-Z)	OP DP
	READ (5,1111) MAXJ, MAXN, JUNCT, PMAX, DELPO, TESTI,	TG 0003
	1 TESTS	TG 0004
1111	FORMATUI2, 3X,12, 2X, 11, 9X, 4F10.8)	TG 0005
	READ (5,33) NN	TG 0006 -
- 33	F0PM4T(15)	TG 0007
	00 2222 J=1,NN	TG 0008
	READ (5,200) XSUBI, YSUBI, ZSUBI, XSUBJ, YSUBJ, ZSUBJ	TG 0009
200	E09467(2F10.9)	TG 0010
2222	V=CVEL(XSUBI, YSUBI, ZSUBI, XSUBJ, YSUBJ, ZSUBJ, MAXJ, PMAX,	TG 0011
	EDELPO; MAXN, TESTI, TESTS, JUNCT)	TG 0012
1029	9072	TG 0013
•	F (1)	TG 0014
C .	PROGRAM COMPUTES EBESI	TG 0015
	FUNCTION FRESI(N,X)	TG 0016
	IMPLICIT REALABLA-H,O-Z)	DP DP
	PSU ¹ = 1.0	TG 0017
	FEMP1 = 1.0	TG 0018
	TEMP2 = 1.0	TG 0019
	$T_{2}(AP3) = 3 \cdot 0$	TG 0020
	К = Э	TG 0021
1	X=X+1	TG 0022
	TEMP1 = TEMP1 本((0.5* X)**2)	TG 0023
	РК=К	TG 0024
	TEMP2 = TEMP2 * PK	TG 0025
	$\Delta K = N + K$	TG 0026
	FEMP3 = TEMP3 * AK	TG 0027
	TFRN = TEMP1 / (TEMP2 * TEMP3)	TG 0028
	RATIO = TERMZPSUM	TG 0029
	$DIF = RATIO - \bullet 10-09$	DP 0030
	PSUM = PSUM + TERN	TG 0031
	TF (01F) 2+2+1	TG 0032
2	FBESI = PSUM	TG 0033
	RETURN	TG 0034
	END	TG 0035
C	PROGRAM COMPUTES FBESK	TG 0036
	FUNCTION FRESK(N,X) -	TG 0037
	IAPLICIT REAL*8(A-H, O-Z)	DP DP
•	P S11M=1.0	TG 0038
	TEMF1=1.0	TG 0039
	TEMP2 = 1.0	TG 0040
	TEMPB = 1.0	TG 0041
	TEMP4=1.0	TG 0042
	r', = ()	TG 0043
1	K= K + 1	TG 0044
	TEMP1 = TEMP1+((0.5+X)++2)	TG 0045
	PK=K	TG 0046
	TEMP2 = TEMP2 * PK	TG 0047
	3 K = N - K	TG 0048
	$T \in MP3 = TEMP3 \Rightarrow BK$	TG 0.049
	754P4 = -TEMP4	TG 0050
	TERM = (TEMP4 * TEMP1)/(TEMP2*TEMP3)	TG 0051
	PATIO = DABS (TERM)/PSUM	DP 0052
	DIE = RATIO10-09	DP 0053
	PSUM=PSUM++ TERM	TG 0054
	IF (K-N+1) 2.3.3	TG 0055
. 2	IF (01F) 3.3.1	TG 0056
2	ENERGE 0.5 * PSUM	TG 0057
		· · · ·

COMPUTER PROGRAM USED TO EVALUATE $1(\psi, \beta)$ AND $E(\psi, \beta)$ 0R $1(\eta, \beta)$ AND $E(\eta, \beta)$

		• TG1 0058
	¢ FNO	TG* 0059
	C CVEL FUNCTION SUBPROGRAM	TG 0060
	FUNCTION CVEL (XSUBI, YSUBI, ZSUBI, XSUBJ, YSUBJ, ZSUBJ, MAXJ)	TG 0062
	1PMAX,DELPO,MAXN, TESTI, TESTS, JUNCT)	TG 0063
	IMPLICIT REAL*8(A-H,O-Z)	DP DP
	C NON-EXECUTABLE STATEMENTS	TG 0064
	DIMENSION CINT(200), ACINT(200)	TG 0065
	II FORMAT (48H1 SPHERE GENERATING CYLINDER VELOCITY FIELD/	TG 0066
	156HO XSUBI YSUBI ZS	TG 0067
	2031/10 ,13X, E15.8, 4X, E15.8, 4X, E15.8// 46HO POSITION AT WHICH	TG 0068
	3FIELD STRENGTH IS MEASURED / 56HO XSUBJ YSU	TG 0069
	48J ZSUBJ/1H + 13X, E15.8, 4X, E15.8, 4X, E15.8/ 59H0	TS 0070
	5 SPHERE AND FIELD POSITIONS IN CYLINDRICAL COORDINATES/ 55HO	TG 0071
	6 P SUBI RSUBJ PHI/1H , 13X, E15.	TG 0072
	78, 4X, E15.8, 4X, E15.8//35HO COEFFICIENTS OF FILON INTEGRATION/55	TG 0073
	BHO ALPHA BETA GAMMA/1H,13X,	TG 0074
	$9E15*R_14X_1$ E15*R_1 4X_1 E15*R]	16 0075
	12 FURMATCHOUZZHU BESSEL FUNCTIONS FUR N=127 4HOQI=EI5.8, 2X	16 0076
	1 = 8H = IN(UI) = C10.8 + 2X + 4H = INI(UI) = E10.8 + 2X + 8H = KN(UI) = E10.87	
	2 = 135 0.4 (1002-515 0.28) 0.01 0.025 0.0000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.0000	16 0078
	$2 = 12 \cdot 07 + 470 \cdot 03 = e12 \cdot 87 + 28 + 101 \cdot (03) = e12 \cdot 87 + 101 \cdot (03) = e12 \cdot 87 + 12 \cdot 101 \cdot $	16 0074
	15 FURMATE 4500 NUMERICAL EVALUATION OF INTEGRAL FUR N= 12_{1}	TG 0080
		TC 0081
	$10 \text{CONFAULT 4500} 10 \text{CONFOUT-CUTTURE} \qquad P = A \left(\text{DEP} \left(\text{D} = \text{E13.64} \right) \right)$	TC 0002
	$\frac{1100}{100} + \frac{1100}{100} + 10$	TG 0005
	18H TINTC= $515,82,12$ HO TODINT= $515,8,7$ H BETA= $515,8,7$ H	TG 0085
		TG 0086
	17 EDRMAT(140/22H0 TOTA) INTEGRAL (N=12, 2H)=E15,8, 19H SUM DE INT	TG 0087
	1EGRAL SEE15.8. 9H RATIOSEE15.8)	TG 0088
	18 FORMAT(47HO INTEGRAND=HSUBN(P)*COSF(P*Z) P=A(DELP)B=E15.8.	TG 0089
	11H(,E15,8,1H),E15,8//(8H 7E16,8))	TG 0090
	19 FORMAT(1H0/10HC CESUM=E15.8, 8H COSUM=E15.8, 9H CINTG=E15.8,	TG 0091
	1 9H TINTG=E15.8/ 12HO TOPINT=E15.8, 8H RECTA=E15.8,	TG 0092
	2 8H RATIOI=E15.8)	TG 0093
	101 FORMAT(30HOCYLINDER VELOCITY FIELD AFTER 13, 14H INTEGRATIONS=	TG 0094
	1E15.8)	TG 0095
	102 FORMAT(1H0/28H0 FRESSEL FUNCTIONS FOR N=12/6H0 Q1=E15.8,2X,	TG 0096
	1 9H FIN(Q1)=E15.8,11H FIN1(Q1)=E15.8, 9H FKN(Q1)=E15.8/	TG 0097
	2 5H0 Q2=E15.8, 9H FIN(Q2)=E15.8, 1H FIN(Q2)=	TG 0098
	3 E15.8 /4HCO3=E15.8, 9H FIN(03)=E15.8,10H FIN1(03)=E15.8)	TG 0099
C	, THIS SECTION COMPUTES FILCN CUEFFICIENTS, ALPHA, BETA, GAMMA	.16 0100
	$R_{SOBI} = DSOR (XSOBI * XSOBI + YSOBI * YSOBI)$	DP 0101
	K 2047 = 02041 (X2087 ★ X2087 + A2081 ★ A2081)	
	2 IF(TSUBIE 19)30 E DUTE - A	TC 0104
		TG 0105
	$6.7 + 10^{-1} = 3 + 1415926572$	TC 0107
		TG 0108
	4 PHIL =- 3-14159265/2	TG 0109
	GO TO 7	TG 0110
	1 PHIL = DATAN (YSUBI/XSUBI) + 3.14159265	DP 0111
	60 10 7	TG 0112
	3 PHII = DATAN (YSUBI/XSUBI)	DP 0113
	7 IF(XSURJ) 8,9,10	TG 0114
	9 IF(YSUKJ) 20, 21, 22	TG 0115
	21 PHIJ= 0.	TG 0116
	SM TO 23	TG 0117
	22 - 111 Jan - 1415 1202 120 - 1	TG 0118

Ca 16 73		* @	กมีกก	Ŷ
		10	0114	
$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} + \frac{1}{2} \right) \left(\frac{1}{2}$		70	0120	
		13	0121	
8 PHID = DATAM (APDRDAX20RD1 + 3-14124595		00	0122	
GD 10 23 1 1		TG	0123	
IO PHIJ = DATAN (YSUBJ/XSUBJ)		DP	0124	
23 PHI = PHII - PHIJ		TG	0125	
Z = Z SURI + Z SURJ		TG	0126	
AZ = OABS (Z)		() P	0127	
DELP = DELPO/(1.0 + DSIN(AZ * DELPO))		DP	.0128	
THETA = 7 * DELP		TG	0129	
LETTHETAL 24-25-24		TG	0130	
		T0	0131	
$\mathcal{L}_{\mathcal{L}} = P(\mathcal{L}) P(\mathcal$		TC	0133	
		10	0122	
		16	0133	
6.0 10 28		16	0134	
24 IH-TA2 = THETA + THETA		16	0135	
THETA3 = THETA' * THETA2		TG	0136	
STHETA = DSIN(THETA)		DP	0137	
CITHEITA = DCOS(THEITA)		ÐP	0138	
SINCOS = STHETA * CTHETA		TG	0139	
ALPHA =(THETA2 + THETA*SINCOS-2.*STHETA*STHETA)/ THETA3		TG	0140	
BETA =2.**(THETA*(1.+ CTHETA*CTHETA)-2.*SINCOS)/THETA3		TG	0141	
GAMMA = 4.5(STHETA-THETA*)(THETA)/THETA3		TG	0142	
26 CO TO (27.28.27), HART		TC	0143	
	7 010 1	10	0143	
27 WRITE (0,11) ASUBI, TSUBI, ZSUBI, ASUBJ, TSUBJ,	22001	10	0144	
IR SUBL, RSUBJ, PHI, ALPHA, BETA, GAMMA		- 16	0145	
C THIS SECTION COMPUTES THE INTEGRALS FOR N=O (1) MAXN		TG	0146	
28 MAXNP= MAXN +1		TG	0147	
1 + LXAM = PLXAM		TG	0148	
MAXJP1 = MAXJP-1		TG	0149	
TNNTG = 0.		TG	0150	
10031 NP=1. MAXNP		TG	0151	
N = ND - 1		TC	0152	
		to	0152	
		TC	01 13	
		10	0104	
11416 = 0		16	0155	
IF(N) 29, 29, 30		16	0156	
C INTEGRAL FOR N=O		16	0157	
29 DO 32 JP=1, MAXJP		TG	0158	
J = J P - I		TG	0159	
D = J		TG	0160	
P=A + DJ*DELP		TG	0161	
IF(P) 40, 41, 40		TG	0162	
41 CINT(JP) = 0.0		GG ·	1	
FGOEP=0.0		GG	1 Å	
60 10 32		TG	0164	
		TC	0165	
		10	0165	
		10	0100	
		16	0157	
02 = P # R SUB 1		TG	0168	
03=P &R SU3 J		TG	0169	
CALL BESI ('QI, NPLUSI, BIN, IER)		TS	0170	
MPLUS1 = NPLUS1 + 1		ŤG	0171	
CALL BESI (Q1, MPLUSI', BINI, 1ER)		ŦG	0172	
CALL BESK (Q1. NPLUS1. BKN. IER)		TG	0173	
CALL BESK' (QL. MPLUSI . BKNI. TER)		TG	0174	
IE(02) 43. 44. 43		ŤG	0175	
		TC	0174	
j – marty i krystik, mi k.a. La distributik sa		10	0110	
M 1 3 1 0 1 - 70 •		10	0177	
GC T() 45		T G	U178	

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43 CALL HEST (+424 NPLUSI, BINRI, TER)	TG 01)79
CALL BESI (Q2, MPLUSI, BINIRI, IER)	TG OP80
45 [F(03] 46, 47, 46	TG 0181
47 BINPJ = 1.	TG 0182
BINIRJ = 0.	TG 0183
	16 0184
46 CALL BESI (Q3, NPLUSI, BINRJ, IER)	TG 0185
CALL BEST (Q3, MPLUS1, BINIRJ, IER)	TG 0186
48 GO TO (49, 49, 50) ,JUNCT	16 0187 ·
50 WRITE (6,12) N, Q1, BIN, BINI, BKN, Q2, BINRI,	16 0188
18 INIRI, Q3, BINRJ, BINIRJ	16 0189 TC 0189
C COMPOSES INTEGRAND CINT(J)	16 0190
49 A1 = 81 Ne (751 N)	16 0191 TC 0103
A3 = BINIRI/BINI	16 0192 TC 0102
A4 = BINIRJ/BINI	16 0195 TC 0104
Ab = A 3 + A 4	16 0194 TC 0105
A T = A I + A C	16 0195
A9 = BIN/BIN(16 0196
A10 = A9*A9	16 0197
A11 = A10 - 2 * A9/P - 1	10 0148
A12 = BINI73IN	
A13 = BKN * BINI	
A18 = 410RJ/BIN	
$A_2 \mathcal{E} = -A_1 \mathcal{E} A_1 \mathcal{E}$	
$A \ge 7 = R \le 0.31 * A 6 * A 1 2$	
A28 = RS0HJ * A18 * A1 * A9	
$A \geq 9 \Rightarrow -R \leq 0 \leq J \times R \leq S \geq 1 \Rightarrow A \Rightarrow A \Rightarrow A \Rightarrow A = A \geq 0$	<u> </u>
A 30= (A26+A27+A28+A29)/A11	
$A_{31} = 1^{2} \times 1^{3} \times 1^{4}$	
	GG 4
	00 5
$A_{34} = A_{31} * (A_{32} + A_{33})$	
	66 00
A 3 / ** D X R L * D L * L * A * A * A * A * A * A * A * A *	66 10
	66 11 66 11
	66 12
	GG 13
$A \uparrow I \rightarrow I \downarrow I \downarrow I \downarrow I \downarrow I$	CG 14
N72-N77N71 N/3-N30\$AA3	GG 15
	GG 16
	GG 17
	66 18
$1054 - CINT(IR) = COEPARSIN(P \times 7)$	66 19
	TG 0201
	TG 0202
	TG 0203
	TG 0204
33 CESH4 = CESH4 + CINT (.1P)	TG 0205
CESIM = CESIM - (CINT(1) + CINT(MAX)P)/2	TG 0206
0.14 ± 10^{-2} MAX 191.2	TG 0207
34 COSUM = COSUM + CINT(JP)	TG 0208
TE (THEIA) 54. 55. 54	TG 0209
55 CINTS = $DE = D + (B + TA * C + S) + CAMMA*COS(M)$	TG 0210
	TG 0211
$\theta = x + \theta d + \theta E t P$	TG 0212
	TG 0212
	TG 0214
A = A = A = A + A + A + A + A + A + A +	TG 0215
5 - A T 07 - 176 ET STD7A = 155 N.C.AAN	DP 0216
a filita iza ili uzi dati za za ili uzi te	

STNZB (=DSÎN(Z*B)	JP 0214
COSZA = OCOS(Z * A)	DP 0218
COS7B = DCOS(Z*B)	DP 0219
CINTG = DELP*(-ALPHA*(CINT(MAXJP)*COSZB/SINZ)	B - FGOFP*COSZA) GG 20
1 + BETA*CESUM + GAMMA*COSUM)	GG 21
56 TINTG = TINTG + CINTG	TG 0222
INNIG = INNIG + CINIG	TG 0223
50.35 $10=1.4$ MAX 1P	TG 0224
35 ACINT(IP) = DARS(CINT(IP))	DP 0225
$\frac{1}{100}$	TC 0226
	10 0220 10 0227
1F (10F1N1 - ACIN(3F)) = 60, 37, 37	TG 0228
60 Inpin = ACIN(3P)	16 0229
37 CONTINUE	16 0230
LXAMELR	TG 0231
RECTA = DELP * BJ*TOPINT*2.	TG 0232
RATIOI = RECTA /DABS(TNNTG)	DP 0233
GO TO (61, 62, 61), JUNCT	TG 0234
61 WRITE (6,13)N	TG 0235
WRITE (6,15) A, DELP, B, (CINT(J)	P), JP=1, MAXJP) TG 0236
WPITE (6.16) CESUM. COSUM. CINTG.	TINTG. TOPINT. RECTA. TG 0237
1347101	TG 0238
62.3 = 9	TG 0239
	TG 6240
	TC 0241
	TC 0242
1F (3- PMAX) 53, 53, 54	10 0242
63 TH (TESTI - RATION 29, 29, 64	16 0243
64 CYL = TINTG	16 0244
CYLPHI = TINTG	TG 0245
IF(RSUBI * RSUBJ) 31, 65, 31	TG 0246
C INTEGRAL FOP M=1,2,3,	TG 0247
30 DP 38 JP=1, MAXJP	TG 0248
1 - 1 = 1	TG 0249
L=LA	TG 0250
$P = A + 0.1 \pm 0.$	TG 0251
1E(P) 66. 67. 66	TG 0252
67 (1NT(JP) = 0.0	GG 22
EHUEPEO.O	GG 22A
	TC 0254
	TC 0254
66 IF (N-10) 68, 69, 69	TC 0255
84 IF (P/FN = 0.2) 70, 70, 68	
C CALL BESSEL FUNCTIONS	16 0257
68 NPLUS1 = N	TG 0258
Q1 = P	TG 0259
$Q_2 = P \times R S UB I$	TG 0260
$0.3 = P \neq P SUBJ$	TG 0261
CALL BEST (Q1; NPLUS1, BIN, IER)	TG 0262
MPLUS1 = NPLUS1 + 1	TG 0263
CALL BEST (Q1. MPLUSI. BINI. IER)	TG 0254
CALL RESK [ALL MULLISI _ RKN, TERN	TG 0265
CALL DESK & GLY NEUSLY DEBY LENT CALL RECT / A2. NOTHER, ATMDT, TEDT	TC 0266
CALL DEST & WEY MELOSLY DIMPLY LEAK	TC 0267
CALL DECT (AC NOLDEL DINIKI) ICKI	TC 0240
CALL MEST I WAY NYLUSI & DINKJY IEKJ	
CALL HEST (03, MPLUSI, BINIRJ, IER)	16 0209
CALL BESK(01, MPLUS1, BKN1, IER)	
GO TO (71, 71, 72), JUNCT	TG 0270
72 WPITE (6,12) N. Q1, BIN, BIN1, BK	KN, Q2, BINRI, TG 0271
13INIRI, Q3, BINRJ, BINIRJ	TG 0272
C COMPUTES INTEGRAND CINT(J)	TG 0273
	TG 0274
t = t + h + h	
$\frac{11}{12} = \frac{1}{12} + \frac{1}{12} \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{12} + \frac{1}{12}$	TG 0275

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A 3 1 144 A 3 1	T. 0. 276 *
	TC 0377
$A4 = \{P(N N) \neq P(N)$	TO 0277
A5 = A47P	16 0278
$\Lambda 6 = \Lambda 4 + \Lambda 4$	16 0279
A7 = BINRI / BIN	TG 0280
AB = BINRJ / BIN	TG 0281
A12 = BIN1RI/BIN	TG 0282
A g = 412 * R S H B T	TG 0283
	TC 0284
	TC 0205
A = A + A + A + A + A + A + A + A + A +	
A14 = A10 * A7	16 0286
A15 = A12 * AB	TG 0287
$A16 = A4 \star A14$	TG 0288
$\Delta 17 = A9 * 410$	TG 0289
A18 = -A3 + A4 + A6	TG 0290
$A_{10} = A_{5} + A_{5}$	TG 0291
	TC 0202
$A_{20} = A_{2} + (A_{3} - A_{1}) + (A_{2} + A_{3} + (A_{3} - A_{1}))$	10 0292
$A22 = A1 + \{HN - 2 \cdot HN + HN + A5 - 4 \cdot HN + A5\}$	16 0293
A23 = A18 + A20	TG 0294
$\Lambda 24 = \Lambda 23 + \Lambda 22$	TG 0295
$\Delta 27 = 8KN \times BIN$	TG 0298
A = B = B SHB + *A + 1	
	cc
V90=5 *V23	66 23
A = A + A + A + I	, ·
A 62=A 15#R SUBI/RSUBJ	
A 63=F14☆(A6]+A14+A62) · · · · · · · · · · · · · · · · · · ·	GG 24
$\Delta 64=2.0 \times 100 \times 1000 \times 100 $	GG 25
A (5 = A 27 * (A (0 + A (3 + A (4))))	66 26
	66 27
	CC 20
A 67=+ N#BKN1+KI N#ALI7KSUBJ	66 20
A68=A66+A67	66 29
\$69=-PSUNI *RSUBJ*A15	GG 30
A 70=A 58*A4	GG 31
$\Lambda 71 = \Delta 17 * \Delta 4$	GG 32
\wedge 72 = - \wedge 1.6 * \wedge 4	66 33
A 1	CC 34
	CC 35
A 74=31R731N1	00 99
A 75=(-2.0*416+A61*A4+A62*A4+2.0*A1/+A58+A69*A/4)	66 36
A 76=FN*A1*A4*A75/424	-GG 37
A 77=(-A14+A61+A62+A17*A74)	GG 38
$A 78 = F N \times F N + P \times A 4 + A 77 / A 24$	GG 39
A 79 = A 17 - A 4 * A 14	GG 40
	66 41
	CC 42
HUF P=A65+A68+A73+A76+A78+A80+A81	66 43
IF(JP-1) 1055,1055,1056	66 44
1055 FHOFP = HOFP	GG 45
1056 CINT(JP) = HOFP*DSIN(P*Z)	GG 46
C(1) T(1) 3.8	TG 0306
	TG 0307
	TC 0309
70.01 = P	10 0308
$02 = P \star RSOBI$	16 0309
U3=P*RSUBJ	TG 0310
FIN = FBESI(N, 01)	TG 0311
FINRI = FBFSI(N, Q2)	TG 0312
16(02-03) 73, 74, 73	TG 0313
	TG 0314
(1+1) + (1+1	10 UJLT 10 0JLE
73 FINEJ = FBESI(N, 03)	16 0316
75 FINI = FBESI (N+1,Q1)	TG 0317
	• •

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	\tilde{T} IN1RI = \tilde{F} HE \tilde{S} I (N+1), Q2)	ŶG	0318	
	IF(02 - Q3) 76;77;76	TG	0319	
77	FIN1RJ = FIN1RI	TG	0320	
_	GO TO 78	- 1G	0321	
76	FINIRJ = FBESI (N+1, Q3)	16	0322	
78	FKN = FBESK(N, QI)	10	0323	
	FKNI = FNESK(N+1,QI)	TG	0324	
8.0	$\frac{1}{10} \frac{1}{10} \frac$	TC	0325	
00	UNITE (0)102) IN NY NIY FINIY FINIY KANA KANA Y	TG	0326	
· · · · ·	CONDITES INTEGAND CINT(1)	TG	0327	
79		TG	0328	
		ŤĞ	0329	
	A3 = 8 \$1181 **N	TG	0330	
	A4 = K SUB J **N	ŤG	0331	
	A5 = FINRI /FIN	TG	0332	
	A6 = FINRJ / FIN	TG	0333	
	A7 = FIN1RI/FIN	TG	0334	
	AB = FINIRJ / FIN	TG	0335	
	A9 = FIN1 / FIN	TG	0336	
	$\Lambda 10 = \Lambda 9 * \Lambda 9$	TG	0337	
	A11 = A5 * A6	TG	0338	
	A12 = A7 * A3	ΤG	0339	
	A13 = A7 * A6	τs	0340	
	$A14 = A8 \times A5$	TG	0341	
	A15 = FKN *FIN	TG	0342	
	$A1 \circ = P * P$	TG	0346	
	$\Lambda 20 = \Lambda 19 + \Lambda 19$	TG	0347	•
	$\Lambda 21 = \Lambda 19 * \Lambda 20$	TG	0348	
	A22 = N+1	TG	0349	
	A23 = A22 * A22	TG	0350	
	A24 = A22 * A23	16	0351	
	$A_{25} = -A_{21} *A_{9} * A_{10} (8 * A_{24})$	16	0352	
	$A_{26} = A_{20*}(0.5*A_{3}/A_{22} - 0.5*A_{10}/A_{23} - 0.75*FN*A_{10}/A_{23})$	16	0353	
	A27 = A19*(-2.*FN*A9/A22 + (FN*FN*(FIN - FINI) + FN* FIN)/(A22*FIN))	1 16	0374	
	AA27 = A25 + A26	10	0372	
	$A \ge B = AA \ge I + A \ge I$	16	0350	
	$A29 = A3 \times A4$	15	1000	
				•
		66	4.8	
	п Эл-г тніл тн э///ч» тнсэ/ Л 20-Л 39жЛЗК	UU	-70	
	A 4 U = A 2 + A 2 7 人 4 1 = E N A 2 + A 人 A 1 = E N A A A 1 = E N A A A 1 = E N A A A 1 = E N A A A A A A A A A A A A A A A A A A	GG	49	
	A + 1 = A + A + A + A + A + A + A + A + A + A			
	A + 2 = A + A + 1 + 3			
	$A \neq \Delta = R \leq 118 \pm 28 \leq 118 \pm 24 \leq 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < 148 < $			
	A + A + A + A + A + A + A + A + A + A +			
	A + A = D + A + 3 T	GG	50	
	A 4 7 = A 4 6 * A 1 1	GG	51	
	A 48=2.*FN*FN*A40*A11/P	GG	52	
	449=A15/FN			
· •	A 50=A 49# (A 39+ A 45+ A 47+ A 48)			
	A51=-2.*FKN1*FIN*(A46*A14/(2.*A22)+FN*A40*A11/P)	GG	53	
	A52=A21*A46*A10*A36/(16.*A24*A22*A28)			
	A53=1./A9·			
	A 54=FN*P+A20*A40*A10/(8.*A24*A28)	GG 👘	54	
. •	A 55=A 43+A 42			
	4 56 =R SUB J *R SUB J * 453 * A3 5			

	ヘッド=ヘッチャップ ▲ 5 3 = 0 ± 4 3 7 ± 4 3 7 ± 4 3 7 ± 4 3 3 ± 4 3 8 ± 4 3 8 ± 4 3 2 ± 4 3 8 ± 4 3 2 ± 4 3 4 3 2 ±	66 55
	A マラード でみどいかれつし かねづし 1 かねどうか Aとつか Aとつき A との + A マラ / A マウ	GG 56
	A 0 U = A 3 7 / A 2 2	66 57
	401=0.34/N22 A 42=4 50 × 1 A 44 A 41 = 455 1	66 58
		66 59
	4 83-FN4FN9F741744947734494773445742374287	66 60
		GG 61
	A 0 J - A 0 J - A 0 J - A 0 J A 0 A 0 A 0 A 0 A 0 J () . # A 2 2 # A 2 # A A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 2 # A 4 A 2 # A A 2 # A A 2 # A A 2 # A A A 2 # A A 2 # A A A A	66 62
		GG 63
	A 6 3 = A 55 / A 2 2	GG 64
		GG 65
•	A 7 D = R SUB J + R SUB J + A 11	GG 66°
	$A71 = 41 \neq RSUBJ \neq RSUBJ$	GG 6 7
1	A 72=A 71 *A1 3*A53	GG 68
	A73=A66*(A67+A68-A69+A70-A72)	GG 69
	HOFP=A50+A51+A52+A58+A62+A65+A73	GG 71A
	IF(JP-1) 1057,1057,1058	GG 72
1057	FH0FP =H0FP	6G 73
1058	$CINT(JP) = HOFP \neq DSIN(P \neq Z)$	GG 74
38	CONTINUE	TG 0364
	CESUM =0.	TG 0365
	COSUM =0.	TG 0366
	DD 39 JP=1, MAXJP,2	TG 0367
39	CESUM = CESUM + CINT(JP)	TG 0368
	CESUM = CESUM -(CINT(1) + CINT(MAXJP))/2.	TG 0369
	DO R1 JP = 2, MAXJP1, 2	TG 0370
81	COSUM = COSUM + CINT(JP)	TG 0371
	1F (THETA) 82, 83, 82	TG 0372
83	CINTG = DELP *(BETA*CESUM + GAMMA * COSUM)	TG 0373
	B J=MA J	TG 0374
	H= A + BJ *DELP	TG 0375
	SU TO 34	TG 0376
82	LX AM=L F	16 0377
	B = A + BJ * DELP	16 0378
	SINZA = DSIN(Z*A)	DP 0379
	$SIN_2R = DSIN(Z*R)$	08 0380
	LOSZA = DCOS(Z * A)	DP 0301
	CINTG = DELP*(-ALPHA*(CINT(MAXJP)*CUSZB/SINZB - FHUFP*CUSZA)	· 66 75
1	+ HETA*CESUM + GAMMA*CUSUM]	
84	11N16 = 11N16 + CINIG	
	$\frac{1}{10} = 10016 + 01016$	10 0000 TC 0207
0.5	JU = 0 J = T + MAAJE	10 0301
85	AUININIT - DADSICININITI TODINT - D	TC 0300
	10/21/1 = 0	TG 0307
1	JU 00 JF-1, MAKJF 16/10/NF-ACTNF/10/187, 04 04	TC 0301
07	$\frac{1}{10} \frac{1}{10} \frac$	TG 0391
87	IUPINI = ALINI IJP)	TC 0393
50	5 JIN 1 1 NOC D 1+ NA Y 1	TG 0395
	DJ=H4AJ DCCTA→DCID#DI DCCTA→DCID#DI	TG 0394
1	NELIA-DELYTOJ TIUNINITIIOTZO TINDITADUDJI DATIOI - DECTA (DARCITANICA	A020 01
	CO TO IQU. 9 DECLA / DADATININIO/	TG 0390
0.0	30 11 10010710011 JUNUT JOTTE 12 121 N	- 16 0398
កដ	MALIE (0)107 N 	10 0390 16 0390
	AND THE TOTAL THE COULD DE COUNTAINE DE LE TANGE TOTALE TOTALE TOTALE TOTALE TOTALE TOTALE.	TG 0400
•	MILIC COLLAT CERTAIN COUNT CTURES LINEST COLLARS	TG 0400
1	ካርርር የዓም ኮቶተቱ የተቆረጉ እ. ለ ሐርን	TG 0401
0 4 1	4;) 2 1ΜΛΥΙ	TC 0403
1		10 0400

*, A		ំឯ។ 🖁	0404
	TE (8-PMAX) 90.90.91	ŤĞ	0405
90	F(T) = ST =	TG	0406
01		TG	0407
91		TC	0408
	TL = UTL TZ + TING	- 10	0400
	$YLPHI = CYLPHI + 2.0 \times DCUS(FN \times PHI) \times INTG$		0404
	ATIOS = 2.0*(1.0+4.0*RSUBI*RSUBJ)*DABS(1INTG/CYLL)	UP	0410
	30 TO, (92,93, 92), JUNCT	1 G	0411
92	APITE (6,17) N. TINTG, CYL, RATIOS	TG	0412
93	LF (TE ST S-RATIOS) 31,65,65	TG	0413
31 (CONTINUE	TG	0414
C (COMPUTES OVEL	TG	0415
- 65 (VFI = 3.7(2.*3.14)592651* CYIPHI		
	PROD = (1 - RSUBI) * CVEI	GG	77
		22	78
	THE ALL ASUBLIACTERIAL VEHAL VEHAL VEHAL VEHAL VEHAL 70181	TC	0418
	ACTE TOTIL ATTACANT ACTION ACT	TC	0410
17	(SOEL, KSOBJ, PHI, ALPHA, BELA, GAMMA	66	70
	$\frac{1}{10} \frac{1}{10} \frac$	00	17
1234 6	DF MAT (6H0PROD=E15.8,3X,7H PRODU=E15.8)	66	80
1	VRITE (6,555) TNNTG,TINTG,CYLPHI,CYLL,CYL,N,	TG	0422
15	RATIOS, R, RATIOI, CVEL, PROD, RSUBI	GG	81
555 /	ORMAT(1H0/7HOTNNTG=E15.8,3X, 7H TINTG=E15.8,3X, 8H CYLPHI=E15.8	TG TG	0424
1	6HOCYLL=E15.8,3X, 5H CYL=E15.8,3X, 3H N=I2/ 8HORATIDS=E15.8,	TG	0425
2	3X. 3H 8=115.8. 3X. 8H PATIOL=F15.8/ 6HOCVEL=F15.8.5X.	TG	0426
2,	5H PRAD = 515, 9, 3X, 7H RSUB(= 515, 8)	GG	82
	ADTE (6.685) MAY I.MAYN, HINCT, DELP, TESTS, PMAY	TG	0428
105 1	m_{1} (c)	TC	0420
000 1	$\frac{1}{10} = \frac{1}{10} $		0420
15	1,3X, /H (FSI)=E13.8,3X, /H (ESIS=E13.0,3X, ON PMAA=E13.0)	10	0430
	10 10 (94,95, 94), JUNCI	10	0431
94 \	IRITE (6,101) N, CVEL	16	0432
95 H	(ETURN)	16	0433
C • 1	THIS PROGRAM COMPUTES EDEBETA		•
. F	ND	TG	0434
С		ΤG	0435
C ,		TG	0436
Ċ		TG	0437
C	SUBROUTINE BESK	TG	0438
č		TG	0439
Č ·	COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	TG	0440
c c	SUBJECT THE R DEGLE FOR THE RANGE FOR A STEEL AROUTERT AND UNDER	τG	0441
0° C		TC	0442
	USAVE CALL DEEVLY N. DV. TEDA	TC	0442
L A	CALL DESK(X+N+DK+IEK)	10	0449
C .		10	0444
C	DESCRIPTION OF PARAMETERS	16	0445
C	X - THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED	TS	0446
С	N -THE ORDER OF THE K BESSEL FUNCTION DESIRED	TG	0447
C	BK -THE RESULTANT K BESSEL FUNCTION	TG	0448
C	IER-RESULTANT ERROR CODE WHERE	TG	0449
-	IFR=0 N1 FRROR	TG	0450
C.		TG	0451
C C	EN REAL AND AN AND AND AND AND AND AND AND AND	TC	0452
C C C	I = D = 2 Y IS ZERD OF NECATIVE		•/ : ./ 5
C C C	IER=2 X IS ZERD OR NEGATIVE	TC	0453
C C C	IER=2 X IS ZERD DR NEGATIVE	TG	0453
с с с с	IER=2 X IS ZERD OR NEGATIVE REMARKS	TG TG	0453 0454
с с с с с с с	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO	TG TG TG	0453 0454 0455
с с с с с с с с с с с с с	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE OF	TG TG TG TG	0453 0454 0455 0456
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE DF THE MACHINE	TG TG TG TG TG	0453 0454 0455 0456 0457
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE OF THE MACHINE	TG TG TG TG TG TG	0453 0454 0455 0456 0457 0458
	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE DF THE MACHINE SUBROUTINES AND EUNCTION SUBPROGRAMS REQUIRED	TG TG TG TG TG TG	0453 0454 0455 0456 0457 0458 0459
	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE DF THE MACHINE SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	TG TG TG TG TG TG	0453 0454 0455 0456 0457 0458 0459 0460
	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE DF THE MACHINE SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED NONE	TG TG TG TG TG TG TG TG	0453 0454 0455 0456 0457 0458 0459 0460 0461
	IER=2 X IS ZERD OR NEGATIVE REMARKS N MUST BE GREATER THAN OR EQUAL TO ZERO X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE DF THE MACHINE SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED NONE	TG TG TG TG TG TG TG TG TG	0453 0454 0455 0456 0457 0458 0459 0460 0461 0462

.

	1 = • 57721566 + 101.0G (B)	DP 0523
		TG 0524
r	1F(N-1)3/,43,3/	16 UD2D TC 0526
ř	COMPUTE KO USTNG SEDTES EXPANSION	TG 0527
č		TG 0528
Ŭ	37 60=-4	TG 0529
	X2J=1.	TG 0530
	FACT=1	TG 0531
	H.J=_0	TG 0532
	00 40 J=1,6	TG 0533
	RJ=1./FL()AT(J)	TG 0534
	$X \ge J = X \ge J * C$	15 0535
	FALI=FALI#KJ#KJ U 1-U 1-0 1	10 0030 TC 0537
		TG 0578
		TG 0539
	42 BK=G0	TG 0540
	RETURN	TG 0541
С		TG 0542
С	COMPUTE K1 USING SERIES EXPANSION	TG 0543
С		TG 0544
	43 X2J=B	TG 0545
		[G UD46 ΤC 0647
		TG 0547
	D(1 = 50, 1 = 2 + 8)	TG 0549
	X2J=X2J*C	TG 0550
	PJ=1./FLUAT(J)	TG 0551
	FACT=FACT*RJ*RJ	TG 0552
	HJ=HJ+RJ	TG 0553
	50 G1=G1+X2J*FACT*(.5+(A-HJ)*FLOAT(J))	TG 0554
	1 + (N - 1) + (1 + 52) + 31	
	FND	TG 0558
С		TG 0559
č		TG 0560
С		TG 0561
C :	SUBROUTINE BESI	TG 0562
C		TG 0563
C	PURPOSE	TG 0564
с c	COMPUTE THE I BESSEL FUNCTION FOR A GIVEN ARGUMENT AND URDER	TC 0544
r		TG 0567
č	CALL REST(X-N-RT-TER)	TG 0568
č		TG 0569
Ĉ	DESCRIPTION OF PARAMETERS	TG 0570
Ċ	X - THE ARGUMENT OF THE I BESSEL FUNCTION DESIRED	TG 0571
С	N - THE ORDER OF THE I BESSEL FUNCTION DESIRED	TG 0572
C	BI -THE RESULTANT I BESSEL FUNCTION	TG 0573
C	IER-RESULTANT ERROR CODE WHERE	TG 0574
C	IFR=0 ND ERROR	16 0575 TC 0577
C C	IERTIN IS NEGATIVE	10 0070 TC 0577
č	ICK-Z X IS NEGALIVE	TG 0578
č	REMARKS	TG 0579
С	N MUST BE GREATER THAN OR EQUAL TO ZERO	T3 0580
C	X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE OF	TG 0581
С	THE MACHINE	TG 0582
C		TG 0583

```
SUBROUTINE'S AND FUNCTION SUBRROGRAMS REQUIRED
5-C
                                                                 JG 0584
        NONE
С
                                                                TG 0585
С
                                                             TG 0586.
С
        ME THOD
                                                                TG 0587
        COMPUTES ZERO ORDER AND FIRST ORDER RESSEL FUNCTIONS USING TO 0588
С
С
           SERIES APPROXIMATIONS AND THEN COMPUTES N-TH ORDER FUNCTION TG 0589
      USING RECURRENCE RELATION.
С
                                                               TG 0590
С
                                               TG 0591
С
     •••••• TG 0592
С
                                                                TG 0593
     SUBROUTINE BESI(X,N,BI,IER)
                                                                TG 0594
     IMPLICIT REAL*8(A-H,O-Z)
                                                               DP DP
С
                                                               TG 0595
С
     CHECK FOR ERRORS IN N AND X AND EXIT IF ANY ARE PRESENT
                                                             TG 0596
С
                                                                TG 0597
     IFR=0
                                                                TG 0598
     BI=1.0
                                                                TG: 0599
     IF(N)150,15,10
                                                                TG 0600
   10 TF(X)160,20,20
                                                                TG 0601
   15 IF(X)160,17,20
                                                                TG 0602
  17 RETURN
                                                                TG 0603
С
                                                                TG 0604
С
     DEFINE TOLERANCE
                                                                TG 0605
С
                                                                TG 0606
  20 TOL=1.0-6
                                                                CP 0607
                                                           TG 0608
TG 0609
TG 0610
С
     IF ARGUMENT GT 12 AND GT N, USE ASYMPTOTIC FORM
С
  IF(X-12.)40,40,30
30 IF(X-FLDAT(N))40,40,110
С
                                                               TG 0611
                                                                TG 0612
  TG 0613
COMPUTE FIRST TERM OF SERIES AND SET INITIAL VALUE OF THE SUM
40 XX=X/2.
C
С
С
     FACTN=1.
                                                                TG 0617
     IF(N-1)70,70,50
                                                                IG 0618
   50 DO 60 1=2,N
                                                                TG 0619
     FI=I
                                                                TG 0620
  60 FACTN=FACTN*FI
                                                                TG 0621
  70 TERM=(XX**N)/FACTN
                                                                TG 0622
   • BI=TERM
                                                                TG 0623
     XX = XX * XX
                                                                TG 0624
                   С
                                                                TG 0625
С
     COMPUTE UP TO 30 TERMS, STOPPING WHEN ABS(TERM) LE ABS(SUM OF TERM TG 0626

      TIMES TOLERANCE
      TG 0627

      DO 90 K=1,30
      TG 0628

      IF(DABS(TERM)-DABS(BI*TOL))100,100,80
      DP 0630

С
С
  IF(DABS(TERM)-DABS(BI*TOL))100,100,80
80 FK=K*(N+K)
TERM=TFRM*(XX/FK)
                                                                DP 0630
                                                                TG 0631
     TERM=TERM*(XX/FK)
                                                                TG 0632
 3
  90 BI=BI+TERM
                                                                TG 0633
С
                                                                TG 0634
С
     RETURN BI AS ANSWER
                                                                TG 0635
                                                          TG 0636
TG 0637
С
    RETURN
X GT 12 AND X GT N, SO USE ASYMPTOTIC APPROXIMATION
100 RETURN
                                                            TG 0638
TG 0639
TC 0
C
С
                 С
                                                                TG 0640
 110 FN=4*N*N
                                                                TG 0641
     XX=1./(8.*X)
                                                                TG 0642
     TERM=1.
                                                                TG 0643
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Service and Carlos and Carlo	T5 8544
D() 130 K=1,30	TG 0649
IF(DABS(TERM)-DABS(TOL*BI))140,140,120	DP 0646
120 FK=(2*K-1)**2	TG 0647
TERM=TERM*XX* {FK-FN}/FLOAT(K)	TG 0648
130 BI=BI+TERM	TG 0649
140 PI=3.141592653	TG 0650
BI=BI*DEXP(X)/DSQRT(2.0*PI*X)	DP 0651
GU TO 100	TG 0652
150 IER=1	TG 0653
GU TO 100 ·	TG 0654
160 IER=2	TG 0655
GO TO 100	TG 0656
END · ·	TG 0657 *
	i ź

С	MAIN PROGRAM	TG	0002
	READ (5.1111) MAXI, MAXN, MAXN, DEPO, TESTI,	UP TG	0003
	ITESTS	TG	0004
1111	FORMAT(12, 3X,12, 2X, 11, 9X, 4F10.8)	TG	0005
	READ (5,33) NN	TG	0006
33	FORMAT(15)	TG	0007
	DD 2222 141,NN		8000
200	READ 1945061 X20014 X20014 X20014 X20014 X20014 X20014 X2001	- 10 TC	0009
2222	V=CVEL(XSUB1, YSUB1, ZSUB1, XSUB1, YSUB1, ZSUB1, MAX1, PMAX.	rG	0010
	DELPD, MAXN, TESTI, TESTS, JUNCT)	TG	0012
1028	STUP	TG	0013
	EN()	TG	0014
С	PROGRAM COMPUTES FRESI	TG	0015
	FUNCTION FRESI(N,X)	TG	0016
	1 MPLILIF REAL+8(A-H, U-Z)	UP TC	0017
	PS00 = 1.0	TC	0.01.9
	TEMP2 = 1.0	TG	0010
	TEMP3 = 1.0	ŤĞ	0020
	$\mathbf{K} = 0$	TG	0021
1	K = K + 1	ŤG	0022
	TEMP1 = TEMP1 * ((0.5* X)**2)	TG	0023
	PK=K	TG	0024
	TEMP2 = TEMP2 * PK	TG	0025
	AK = N + K TEND2 - TEND2 + AK	16	0026
	TEMPS → TEMPS → AN TEMPS → TEMPS / (TEMPS)	10	0027
	PATIO = TERM/PSUM	TG	0029
•	DIF = RATIO1D-09	DP	0030
	PSUM = PSUM + TERM	TG	0031
	IF (01F) 2:2.1	TG	0032
2	FBESI = PSUM	TG	0033
	RETURN	TG	0034
~ ·	ENI) DROCRAM COMONTEE ERECK	16 TC	0035
ι.	FRUGRAM COMPUTED FORSE FUNCTION FRESKINLYN	TG	0030
	THE TOTT REAL *8(A-H.O-7)	DP	0007
	PSUM=1.0	TG	0038
	TEMP1=1.0	TG	0039
	TEMP2 = 1.0	TG	0040
	$TEMP3 = 1 \cdot O$	TG	0041
	TEMP4=1.0	TG	0042
		TG	0043
1	N= N + 1 TCH01 - TCH01+//A 5+V1++21	16	0044
	DK#K	TC	0045
	TEMP2 = TEMP2 * PK	TG	0047
	BK ≠N−K	TG	0048
	TEMP3 = TEMP3 * BK	TG	0049
	TFMP4 = -TEMP4	TG	0050
	TERM = (TEMP4 * TEMP1)/(TEMP2*TEMP3)	TG	0051
	RATIO = DAUS (TERM)/PSUM.	DP	0052
	DIF = KATLU - •10-09 DSTN=DSTN + TEDM	UP TO	0053
	NAUNG-NAUNART TERMI TR TRANSF 2.3.3	16 TC	0054
2	11 1N= 4517 69797 TF {N3F} 3.3.1	10	0055
4. S	$FR_{\rm E}SK=0.5$ \Rightarrow $PSUM$	TG	0057

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COMPUTER PROGRAM USED TO EVALUATE $m(\psi, \beta)$ AND $M(\psi, \beta)$ $m(\eta, \beta)$ AND $M(\eta, \beta)$

•

- *	The Provide RETURN To a construction of the second s	TG	0\$58
	END I FOR THE REPORT OF A CONTRACT	TG	0059
C	CV-L FUNCTION SUBPROGRAM	TG	0060
-	ENMETTION OVEL LYSURT, YSURT, YSURT, YSURT, YSURL, YSURL, TSURL, MAYL,	TG	0062
	10MAY DELDON MAYNE TECTLE TECTLE HINGT FOR A SUBJY LOODLY LOODLY TO BE	TC	0063
	THE ADDRESS OF A BALLARY TESTSY JUNCTIVE AS A STATE AS A STATE AND A ST	00	0003
	137211111 REAL*A(A-H,U-Z)	08	
C	NON-CXECUTABLE STATEMENTS	16	0064
	DIMLASION CINT(200), ACINT(200)	TG	0065
	11 FORMAT (4PH1 _ SPHERE GENERATING CYLINDER VELOCITY FIELD/	ŤG	0066
	166HO XSUBI YSUBI ZS	G TG	006 7
	2URI/1H +137, E15.8, 4X, E15.8, 4X, E15.8// 46H0 POSITION AT WHICH	I TG	006 8
	BETCLO STRENGTH IS MEASURED/ 56HO XSUBJ YSU	I TG	0069
	481 7 SUBJ/1H - 13X - F15-8 - 4X - F15-8 - 4X - F15-8// 59H0	TG	0070
٠	5 SPHERE AND FIELD POSITIONS IN CVI INDRICAL COOPDINATES / 55HO	TG	0071
		τ-	0072
	σ = $r_{2}\sigma_{1}$ = $r_{2}\sigma_{1}$ = $r_{2}r_{1}r_{1}r_{1}r_{2}r_{2}r_{3}r_{1}r_{1}r_{2}r_{3}r_{3}r_{3}r_{3}r_{3}r_{3}r_{3}r_{3$		0072
	/0, 4X, ED.0, 4X, ED.0// SHU CUEFICIENTS UP FILDN INTEGRATION/S	10	0075
	HO ALPHA BEIA GAMMA/IH , 13X,	16	0074
	9F15.8,4X, E15.8, 4X, E15.8)	TG	0075
	12 FORMATCIHO/27HO BESSEL FUNCTIONS FOR N=I2/ 4H0Ql=E15.8, 2X,	TG	0076
	1 8H IN(Q1)=E15.8, 2X, 9H IN1(Q1)=E15.8,2X, 8H KN(Q1)=E15.8/	TG	007 7
	2 4H002=E15.8,2X, 8H IN(02)=E15.8,2X,9H IN1(02)=	TG	0078
	3 515.87 4H003=515.8. 2X. HH IN(03)=515.8. 9H IN1(03)=515.8)	TG	0079
	13 FORMATE 4540 MUMERICAL EVALUATION OF INTEGRAL FOR N= 12.	Ta	0080
	1 SHOW AT THE AN ACTION ACTION OF THE OWNER THE ATTENT AT THE AND A THE ATTENT AT THE	тс	0000
		10	0001
	$15 FURMAT(431'O INTEGRAND=G(P)*COSF(P*Z) P=A(UELP)B=E15.8_{+}$	16	0082
	11H(,E15,R,1H),E15,877(8H) 7E16,8)	TG	0083
	16 FORMAT(1HO/7HOCESUM=E15.8,8H COSUM=E15.8, 7H CINTG=E15.8,	TG	0084
	18H TINTG=015.8/ 12H0 TOPINT=015.8, 7H RECTA=015.8, 10H RATIO	TG	0085
	21=F15.8)	TG	0086
	17 FORMAT(1H0/22H0 TOTAL INTEGRAL (N=12, 2H)=E15.8, 19H SUM OF INT	ŤĠ	0087
	1EGRAL S=E15.8. 9H RATIOS=E15.8)	TG	0088
	18 EORMAT(47H) INTEGRAND=HSUBN(P)*COSE(P*7) $P=\Delta(DE(P)B=E15, 8, \cdots)$	TG	0.089
	111(, E15, B. 14), E15, B// (BH) 7616, B/)	TG	0000
	10 - 10 - 10 - 10 - 10 - 10 - 10 - 10 -	TC	0001
	17 (1864) (107) (100) (200)	10	0001
	1 90 1100 ELD.87 1280 TUPINIELD.8, 88 RECIAHELD.8,	10	0092
	2 3H KA)101 = E10.8)	16	0093
	TOT FORMATCOOPCIVETNDER VELOCITY FIELD AFTER 13, 14H INTEGRATIONS=	16	0094
	1615,8)	TG	0095
	102 FORMAT(1H0/28H0 FBESSEL FUNCTIONS FOR N=12/6H0, Q1=E15.8,2X,	TG	0096 .
	1 9H FIN(Q1)=E15.8,11H FIN1(Q1)=E15.8, 9H FKN(Q1)=E15.8/	TG	0097
	2 5H0 Q2=E15.8, 9H FIN(Q2)=E15.8, 11H FIN1(Q2)=	TG	0098
	3 E15.8 /4H0Q3=E15.8, 9H FIN(Q3)=E15.8,10H FIN1(Q3)=F15.8)	TG	0099
C	THIS SECTION COMPUTES FILON COEFFICIENTS, ALPHA, BETA, GAMMA	ΤG	0100
	BSUBT = DSORT (XSUBT * XSUBT + YSUBT * YSUBT)	DP	0101
	RSUBJ = DSORT (XSUBJ + XSUBJ + YSUBJ + YSUBJ)	nP	0102
		TG	0103
		TC	0104
		70	0104 .
	5 PH11 = 0.	16	0105
	60 10 7	16	0106
	6 PHII = 3.14159265/2.	TG	0107
	GO TO 7	TG	C108
	4 PHII =-3.14159265/2.	TG	0109
	GU TU 7	TG	0110
	1 PHII = DATAN (YSUBI/XSUBI) + 3.14159265	DP	0111
	GU TU 7	TG	0112
	3 PHIL = DATAN (YSUBI/XSUBI)	nP	0113
	7 TELXSUB (1, 8, 9, 10)	TC	0114
		10	0115
		10	0119
·		10	0117
		10	0110
	22 MHJF 3•1415725572•	16	0119

art a star						
	\$	<u>e</u>	≪ # €€ OT€ D.5		ŤG 0119	
		20	PHI = -3.14199265/2.		TG 0120	
		20			TG 0121	
		2	он на 23 рит – Патал (усла //усла // 415 4 3. 14159265		DP 0122	
		0			TG 0123	
		10			DP 0124	
		20	PHIG = DATFN T SOUGTASOUT		TG 0125	
		23	$P(\mathbf{n}) \neq P(\mathbf{n}) = P(\mathbf{n})$		TG 0126	
			z = 25001 - 25003		DP 0127	
			AL = DADS (L)		DP 0128	
			HELP = UPEPUVII.0 + USINIAZ +DELPUVI		TC 0129	
			$141 = 2 \neq 0 = 0$		TC 0130	
			LF(1HE1A) 24, 25, 24		TC 0121	
		25	$\mathbf{A} [\mathbf{P} + \mathbf{A}] = \mathbf{O}_{\bullet}$		TC 0133	
			$BETA = 2 \cdot / 2 \cdot$		TC 0132	
			$GAMMA = 4 \cdot 1 \cdot 1$		TC 0135	
		÷ .	GU 10 26		10 0125	
		24	THE TAZ = THE TA * THE TA		10 0195 TC 0196	
			$TH_1TA3 = THETA + THETA2$			
			STHETA = DSIN(THETA)		DP 0137	
			C THE TA = DCOS(THE TA)		UP 0138	
			SINCOS = STHETA * CTHETA		IG 0139	
			ALPHA = (THETA2 + THETA*SINCOS-2.*STHETA*STHETA]/ THETA3		16 0140	
			BETA =2.*(THETA*(1.+ CTHETA*CTHETA)-2.*SINCOS)/THETA3		16 0141	
			GAMMA = 4.*(STHETA-THETA* CTHETA)/THETA3		TG 0142	
		26	GA TO (27+28+27)+JUNCT		TG 0143	
		27	WRITE (6,11) XSUBI, YSUBI, ZSUBI, XSUBJ, YSUBJ,	Z SUBJ,	TG 0144	
			IRSUBI, RSUPJ, PHI, ALPHA, BETA, GAMMA		TG 0145	
	С		THIS SECTION COMPUTES THE INTEGRALS FOR N=O (1) MAXN		TG 0146	
		28	MAXNP = MAXN + 1		TG 0147	
			MAXJP = MAXJ +1		TG 0148	
			MAXJP1 = MAXJP-1		TG 0149	
		•	TNHTG = 0.		TG 0150	
			DO 31 NP=1, 44XNP		TG 0151	
			N = NP-1		TG 0152	
		•	FM =N		TG 0153	
			Λ=0.		TG 0154	
			TINTG = 0		ŤG 0155	
			TE(N) 29. 29. 30.		TG 0156	
	c		INTEGRAL FUR N=0		TG 0157	
	v	20	$D(1, 32, 1) = 1$, $MAX_{1}P$		TG 0158	
	•	<u>.</u>			TG 0159	
					TG 0160	
					TG 0161	
			PRA + DJADELP		TG 0162	
					CC 19	
		41			GG 1A	
					TC 0164	
	_				TC 0165	
	C		CALL RESSEL FUNCTIONS,		TC 0165	
		40	NPLUSI = N			
			Q1 = P		16 0167	
			Q2 =P*RSUBI		16 0168	
			Q3=P*RSUBJ		16 0169	
			CALL BESI (Q1, NPLUS1, BIN, IER)		16 0170	
			MPLUS1 = NPLUS1 + 1		15 0171	
			CALL BESI (Q1, MPLUS1, BIN1, IER)		TG 0172	
			CALL BESK (Q1, NPLUSI, BKN, IER)		IG 0173	
			CALL BESK / Q1, MPLUS1, BKN1; IER)		TG 0174	
			IF(Q2) 43, 44, 43		TG 0175	
		44	BIMBI = 1.		TG 0176	
			3IN1RI =0.		TG 0177	
			GO TO 45		TG 0178	

\$3 CĂLLEBPSI & Q2°, NPLUSI, BINRI, IER)	TS 01791
CALL BESI (Q2, MPLUSI, BINIRI, IER)	TG OEBO
45 [F(03) 46, 47, 46	TG 0181
47 BINRJ = 1.	TG 0182
BIN1RJ = 0.	TG 0183
GU TO 48	TG 0184
46 CALL BEST (03. NPLUST. BINR. TER)	TG 0185
CALL BEST (D3. MPLUST. BINDEL TER)	TG 0186 ·
$\begin{array}{c} \text{Green} \\ Gr$	TG 0197
50 WR ITE (6.12) N. 01, RIN, RINI, RKN, 02, RINPL, 1	TG 0188
IRTIGATION AND L. RANDAL RANDAL BINA DIALY DRAY WAY DIALY	
C COMPACT AND CONTRACT	
$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{1}{$	00 ZR
	66 3K
	66 4K
A4=BINIKJ/EINI	66 5K
A D=6 374 4	GG 6R
A H = A 2 * A 3	GG 7R
A9 = i I N / B I N I	GG 8R
A10=A9#A9	GG 9R
A11=A10-2.0*A9/P-1.0	GG 10R
A12=BIN1/BIN	GG 11R
A 1 3 = B KN * B 1 N 1	GG 12R
A26 = -A6 * A12	GG 13R
A 2 7=R SUB J×A 8★A9	GG 14R
A = (A = (A = A = A = A = A = A = A = A	GG 15R
$\Lambda 2 2 = -P \times \Lambda 1 2 \times \Lambda 1 2 \times \Lambda 6$	GG 16R
$A_{30} = -2_{0} O((P + A))$	GG 178
A = 30 + 46	GG 18R
53 COE D= A 2 9+ A 2 9+ A 3 1	CC 10P
I = (10-1) + 1053 + 1054	66 17
$\frac{10}{2} = \frac{10}{10} + \frac{10}{10} = \frac{10}{10} = \frac{10}{10} + \frac{10}{10} = \frac{10}{10} = \frac{10}{10} + \frac{10}{10} = \frac{10}$	
	16 0202
$C_{11}^{(1)}SUM = 0.$	16 0203
DC_{33} JP=1, MAXJP,2	16 0204
33 CESUM = CESUM + CINT (JP)	TG 0205
CESUM = CESUM - (CINT(1)+ CINT(MAXJP)//20	TG 0208
D() 34 JP=2, MAXJP1,2.	TG 0207
34 COSUM = COSUM + CINT(JP)	TG 0208
1F (THETA) 54, 55, 54	TG 0209
55 CINTG =DELP*(BETA*CESUM + GAMMA*COSUM)	TG 0210
B J = MA X J	TG 0211
$\theta = A + BJ * \Theta ELP$	TG 0212
GU TU 56	TG 0213
54 B J=MA X J	TG 0214
$B = A + BJ \neq DELP$	TG 0215
SINZA = 0 SIN(Z * A)	DP 0216
$SINZR = DSIV(Z \neq R)$	DP 0217
COSZA = DCOS(Z * A)	DP 0218
C(S, Z) = C(S, Z, Z, Z)	00 0210
1 CIPERS - DULT A CONTACTIVE AND A CONTACT STORE STATES - FOULT FOUSTARY -	
I TOLIATELSOTI GAMMATEUSUMI	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	15 UZZZ TO 0222
$\frac{1}{10} \frac{1}{10} \frac$	16 0223
DU 35 JP=1, MAXJP	15 0224
35 AUINI(JP) = DABS(CINT(JP))	UP 0225
10PINT = 0	rg 0226
90 37 JP = 1, MAXJP	TG 0227
IF (TOPINT -ACINT(JP)) 60, 37, 37	TG 0228

					1	1
		60	TOTOTOS ACTINTISIPI	ੇ ਹਰ	0229	ŝ
		27		TC	0230	
		21		70	0230	
			R J = MA X J	16	0251	
			RECTA= DELP * BJ*TOPINT*2.	TG	0232	
			RATIOI = RECTA /DABS(INNTG)	DP	0233	
			60 TO (61, 62, 61), HINCT	ΤG	0234	
		4.1		TC	0225	
		01	WK112 (0,12)N	10	0233	
			WRITE $(6, 15)$ A, DELP, B, (CINT(JP), JP=1, MAXJP)	16	0236	
			WPITE (6,16) CESUM, COSUM, CINTG, TINTG, TOPINT, RECTA,	TG	0237	
			IRATIOI	TG	0238	
		62	A = B	ΤG	0239	
		07.		TC	0240	
			B J=MA XJ	10	0240	
			B = A + BJ * DELP	16	0241	
			IF (B- PMAY) 63, 63, 64	TG	0242	
		63	IF(TESTI - RATIOI) 29. 29. 64	ΤG	0243	
		61.	CYL - TINTC	TG	0244	
		04		70	02.11	
	·		CYLPHI = IINIG	10	0240	
			IF(PSUBI * RSUBJ) 31, 65, 31	16	0246	
С			INTEGRAL FOR N=1,2,3,	TG	0247	
		30	DO 38 JP=1 MAXJP	TG	0248	
				TC	0249	
				70	0277	
		•	D = 1	16	0250	
			P= A+DJ*DELP	TG	0251	
			IF(P) 06, 67, 66	TG	0252	
		67	(1) (1)	66	208	
		0.		<u> </u>	224	
				00	224	
			SU TO 38	16	0254	
		66	IF (N-10) 68, 69, 69	TG	0255	
		69	IE(P/EN=0.2) 68.68.68	GG	93R	
r		•	CALL RESSEL FUNCTIONS	TG	0257	
C			CALL DESSEL FUNCTIONS	10	0221	
		08	MPLUSI = N	10	0228	
	•	•	Q1 = P	TG	0259	
			$O_2 = P * RSUPI$	TG	0260	
			$O_{i} = P \times S S (IB_{i})$	TG	0261	
				TO	0262	
			CALL DESL & WIP NPLUSIP DINF ICKI	1.0	02.02	
	·		MPLUSI = NPLUSI + 1	16	0263	
			CALL BEST (Q1, MPLUS1, BIN1, IER)	TG	0264	
			CALL BESK (Q1 . NPLUSI . BKN . IER)	TG	0265	
			CALL REST (1)2. NOTHST, BINRT, TERM	TG	0266	
			CALL BEET F AZY NECULY DINNY TENY	TC	0200	
			CALL BEST (W2, MPLUST, MINIKI, TER)	10	0201	
			CALL BESI (Q3, NPLUSI, BINRJ, IER)	16	0268	
			CALL BESI (03, MPLUSI, BINIRJ, IER)	TG	0269	
			CALL BESK(01.MPLUS1.BKN1.TEB)			
				TC	0270	
		-		TC	0210	
		12	WRITE (6,12) N, OI, BIN, BINL, BKN, Q2, BINKI,	16	0271	
		1	HINIRI, Q3, HINRJ, HINIRJ	TG	0272	
С			COMPUTES INTEGRAND CINT(J)	TG	0273	
-		71	A1=P*P	22	21R	
		12		00 c c	220	
				00	228	
			A 3=P*A1	GG	23R	
			A4=BIN1/BIN	6 6	24R	
			Δ 5=Δ 4 / P	GG	25R	
			A 6 A 4 A 4	2.2	268	
				čč	270	
				00	218	
			A 8=BINR J/BIN	6G	28R	
			Α °= ΝΙΝΙΚΙ/ΡΙΝ	GG	29R	
			∧10=A9*R511P1	66	308	
				66	210	
				00	714	
			$A_1 \angle = A / A_1$	66	32R	
			A13=A11*A7	GG	33R	
			A14=A9*A8	GG	34R	···
					•	-

* A 15=PR+A ##FN *	66 × 398 -
ΛI 6=A4*ΛI 3	GG 368
A 17=A 10≠A 11	GG 37R
A18=-A3*A4*A6	GG 38R
A19=A5*A5	GG 39R
A 20=A 2*(A5-A19*(2.+3.*FN))	GG 40R
A 2 2 = A 1*(FN-2.*FN*FN*A5-4.*FN*A5)	GG 41R
A 2 3 = 1 19 + A 2 Q	GG 42R
A 24=A 23+A 22	GG 43R
A 2 7=13 K N × H I N	GG 44R
A 3 D = A 1 4 / A 1 5	GG 45R
A 31 = -f N * F H * A 30 / R SUB J	66 46R
A32 = A12/A15	GG 47R
A 3 3=-2. C#F N#F N#F N/(P*R SUB 1 #RSUB 1)	GG 48R
A 34=1 32*A 33	GG 498
A 35=A 9+A11	GG 50R
$A_{36} = -P \neq A_{35}$	66 518
$A 37 = -F N \pm A1 + /R SUB.$	66 52B
A 3 3 - 7, 0×F N+A 1 3 / R SUBI	66 538
A 39=A 33/FN	66 548
A 4 0 = A 3 9 * A 1 2	66 55B
A 41 = A 27* (A 31+ A 3 4+ A 3 6+ A 37+ A 38+ A 4 0)	66 56R
	CC 57P
A 4 3 = EN★P≭A 30/8 SUB.1	60 57K
$A 44=2$, $0 \times (N \times N \times A 32)$ (RSUB) * RSUB 1)	66 59P
$\Delta \Delta 5 = \Delta \delta 2 \times (\Delta \delta 3 + \Delta \delta 4)$	904 93
A 46 = A 3 + A 4 / A 2 4	CC 610
A 4 7 = - A 4 X A 3 5	CC 629
	CC 630
A 49= A 66% (A 47+ A 48)	ACO 00
	CC 459
(5) = (4 + 1) 4	CC 669
	00 00R
A 53 = - 2 . 0 A 3 A / R SHR T	00 07K CC 400
A = 5 - 2 = 0 A $O = 0$ A $O = 0$ A $O = 0$	CC . 699
A 55-2 CON (A) CONTACT (SOUT	CC 709
	66 718
$\Delta \leq 7 = -2 + 0 \times 1 \times 4 \times 1 \times 1$	CC 729
	00 728
	CC 749
	00 14K
	CC 740
$A \cap J = A \cap $	
A = A = A = A = A = A = A = A = A = A =	00 77R
	66 78K
A 65+A 63±1 A 64 ± A 60 ± 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	00 198
4 01742 • 0777 747 F 147 F 147 A 24 A 67 - A 66 + 1 - A 66 + A 69 A	
	GG 82R
	GG 83R
$A = \frac{1}{2} A = $	GG 84R
$A = U = Z + 0^{-1} A = Z + 0^{-1} $	66 85R
$A / I = A / O / (R > 0 B I \neq R > 0 B J)$	GG 86R
$A / Z = A S H \pi (A S H A / I)$	GG 87R
A (J=Z+07+N*FN*FN*FN/AZ4	GG 88R
	GG 89R
A /5=A /4 /(RSUB1 #RSUBJ)	GG 90R
A 76 = A 73 * A 75	GG 91R
HDFP=A41+A-5+A49+A55+A60+A62+A65+A67+A72+A76	GG 92R
IF(JP-1) 1055,1055,1056	GG 44
1055 FRIFP = HIFP	6G 45
1056 CINICUP = HOFP*0SIN(P*Z)	66 ,46 . · ·
•	



	·	e g . 4
87 กายโมโก =∜ ง(โงกั∿เเตี)	้อา	0392 1
86 CONTINUE	ŤĠ	0393
B. J.= MAX.I	TG	0394
	TG	0395
DATION - DECTA /AASCITANTAA	00	0396
RATIO = RECTA / DASTINUTS	UF TC	0307
	10	0200
	10	0398
$WRITE((6, 18)) \qquad A \in DELP(\mathbf{B} \in CINTTJP)(JP=I,MAXJP)$	10	0399
WRITE (8,19) CESUM, CUSUM, CINIG, TUPINI,	16	0400
IRECTA, RATIOI	IG	0401
89 A = A	TG	0402
$\mathbf{L} \mathbf{X} \mathbf{A}^{\mathbf{P}} = \mathbf{L} \mathbf{B}$	TG	0403
B = A + BJ *DELP	TG	0404
IF(R-PMAX)>0,90,91	TG	0405
90 IF(TESTI-RATIOI)30,30,91	TG	0406
91 CYLL = CYL	TG	0407
$CYL = CYL + 2 \cdot * TINTG$	TG	0408
CYLPHI = CYLPHI + 2.0*DCOS(FN*PHI)*TINTG	DP	0409
$BATIOS = 2 \cdot 0 \times (1 \cdot 0 + 4 \cdot 0 \times B SUBI \times RSUBJ) \times DABS (TINTG/CYLL)$	DP	0410
GD TD (92.93, 92), HINCT	TG	0411
92 (RETE (6.17) N. TINTG, CYL, RATIOS	TG	0412
a tertestepatiosial 65.65	тс.	0413
	10	4440
	10	0415
	1 (1	0413
65 CVEL = 3.7(2.*3.14159265)* CYLPHI		
PKOD = (1 - RSUBI) + CVEL	GG	11
PRODD = (1RSUBI)*CYLPHI	GG	78
WRITE (6,11) XSUBI, YSUBI, ZSUBI, XSUBJ, YSUBJ, ZSU	IBJ, TG	0418
18 SUBI, RSUBJ, PHI, ALPHA, BETA, GAMMA	TG	0419
WRITE (6,1234) PROD, PROD	GG	79
1234 FURMAT (6H0PROD=E15.8,3X.7H PRODU=E15.8)	GG	80
JR (TE (6.555) TNNTG.TINTG.CYLPHI.CYLL.CYL.N.	- TG	0422
18ATIOS-8-RATIOL-CVFL-PRCD-RSUBL	GG	81
555 EORMAT(1H0/7H0TNNTG=E15.8.3X. 7H TINTG=E15.8.3X. 8H CYLPHI=	F15.8 TG	0424
1/ AHOCYLLEETS R.3Y. SH CYLETS R.3Y. AH N=12/ AHOKATIOS=EI	5.8. TG	0425
1 = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0	TG	0426
234, 30 8-115,07, 34, 30 64101-113,07 0000711-113,07	0.0	82
DOT PRODELING JAN IN ROUTELING / DELO TECTI TECTO DIAN	·	0429
	-015 TO	0420
685 FURNARI 115/0HUMAAJEIZ, 37, 6H MAXNEIZ, 37, 7H JUNU-EII/ 6HOUELP	-ET54 10	0727
18,3X, /H 15511=EL5.8,3X, /H 1ESIS=EL5.8,3X, 6H PMAX=EL5.8]	16	0430
GO TO (94,95, 94), JUNCT	TG	0431
94 WRITE (6,101) N, CVEL	TG	0432
95 RETURN	TG	0433
C THIS PROGRAM COMPUTES EDFBETA		
5 N'D	TG	0434
c	TG	0435
Сс	•••• TG	0436
C	TG	0437
	TG	0438
	. TC	0439
CONDITE THE V BECCHI EINCTION EDG & CIVEN APPRIMENT AND	NO NED TO	0440
C CUMPUTE THE N DESSEL FUNCTION FOR A GIVEN AROMENT AND	TC	0441
	10	0443
	16	0442
C CALL BESK(X,N,BK,TER)	• IG	0445
C	TG	0444
C DESCRIPTION OF PARAMETERS	TG	0445
C X - THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED	TG	0446
	TG	0447
C N - THE ORDER OF THE K BESSEL FUNCTION DESIRED		0448
C N - THE ORDER OF THE K BESSEL FUNCTION DESIRED C BK - THE RESULTANT K BESSEL FUNCTION	TG	
C N - THE ORDER OF THE K BESSEL FUNCTION DESIRED C BK - THE RESULTANT K BESSEL FUNCTION C LER-RESULTANT ERPOR CODE WHERE	TG TG	0449
C N - THE ORDER OF THE K BESSEL FUNCTION DESIRED C BK - THE RESULTANT K BESSEL FUNCTION C IER-PESULTANT ERPOR CODE WHERE C IER-D ND FORDS	TG TG TC	0449 0450

S C	TEREL IN 35 DEGATIVE	16	045 045
r c	TEN-C A IS LEND ON NEORITYE	TG	045
ř	REMARKS	TG	045
· C	N MUST BE GREATER THAN OR FOULAL TO ZERO	TG	045
· ~	Y SHOWD BE LESS THAN 60 TO AVOID EXCEEDING THE BANGE DE	TG	045
r r	THE MACHINE	TC	045
č	ing FAGUERE	ŤC	045
č	SUPPORTATION SUPPORTATION SUPPORTATION SUPPORTATION SUPPORTATION	TG	045
Ċ	NONE	TC	045
· · ·		TC	040
· · ·	MC THOD	- TO	040
C C	METHON		040
с с	COMPUTES ZERU URDER AND FIRST URDER BESSEL FUNCTIONS USING	TC	040
C C	SENTES APPROATMATIONS AND THEN COMPDIES N TH ORDER FUNCTION	- TC	040
	USING RECORDENCE RELATIONS DECEMPTICE DELATIONS DOLVMONTAL ADDROVIMATION TECHNIQUE	TC	040
	RECORRENCE RELATION AND FOLLNOWTAL AFFROATMATION FECHNIQUE	T C	040
C C	AS DESCRIPTED BE ASSOMENTICHTCHCHCHCHTNDMEAL APPRUATMATIUNS TO RECELEUNCTIONS OF ODDEA JEDO AND ONE AND TO BELATED	- 10 TC	040
L C	, TO BESSEL FUNCTIONS OF UNDER ZERF AND UNE AND TO RELATED	10 TC	040
	$FUNCTIONS = M_{0} + A_{0}C_{0} + V_{0} + 1 + 1927 + PF + 80 - 50 + AND + 50 N_{0} + WA + 50 N_{0}$	10	040
- L	WA TREATISE ON THE THEORY OF BESSEL FUNCTIONSS, CAMBRIDGE	10	047
L L	UNIVERSITY PRESS + 1958 + P+ 62	10	041
1,		16	047
C		16	047
. C		1.5	047
	SUMROUTINE BESKIX, N, BK, TER)	10	041
	IMPEICII REAL #8(A-H,U-Z)	UP	0
	DIMENSION T(12)	-16	047
	B K=• 0	16	047
	IF(N)10,11,11	TG	047
	10 IER=1	TG	047
	RETURN	TG	048
	11 IF(X)12,12,20	TG	048
	12 IER=2	TG	048
	RETURN	TG	048
	20 IER=0	TG	048
	IF(X-1.)36,36,25	TG	048
	25 A=DEXP(-X)	DP	048
	$B = 1 \cdot X$	TG	048
	C = 0 SQRT(B)	· DP	048
	T(1)=8	TG	04.8
	00.26 t = 2,12	TG	049
	26 T(L)=T(L-1)*B	TG	049
	IF(N-1)27,29,27	TG	049
С		TG	049
C	COMPUTE KO USING POLYNOMIAL APPROXIMATION	T3	049
С		TG	049
	27 G0=A*(1.2533141415656418*T(1)+.098111278*T(2)091390954*T(3)	TG	049
	2+.13445962*T(4)22998503*T(5)+.37924097*T(6)52472773*T(7)	TG	049
	3+.55753684*T(8)42626329*T(9)+.21845181*T(10)066809767*T(11)	TG	049
	4+.009189383*T(12))*C	TG	049
	IF(N)20.28.29	TG	050
		TG	050
	RETURN	TG	050
r		TG	050
с г		TC	050
L r	LESS OT AL DEING FUCHUMIAL AFFRUATION		050
L	20 (1-4+11 25221411 46000270+7111- 14605020+71211 12064346+7121	TC	020
	C3 01-A*11423231441440337710*1117*14003030*1121*112004200*1131	. TO	050
	$z^{-} \circ 1 : 1 : 0 : 4 : 1 : 5^{+} : 1 : 4 : 1 : 0 : 2 : 0 : 1 : 0 : 1 : 0 : 1 : 0 : 0 : 0 : 0$	10	050
	366322954*[(8]+.50502386*[[9]25813038*[(10]+.078800012*[[11]	15	050
	A→★UIU826177#1(J2)1#C 1014 1100 10 20	16	050
	121111111111111111111111111111111111111	15	051

· · ·¥* · · ·	30 9k=d1	TG 0511
	RETURN	TG 0512
С		TG 0513
Ċ	FROM KO-KI COMPUTE KN USING RECURRENCE RELATION	TG 0514
č		TG 0515
Ũ	31 D() 35 1=2-N	TG 0516
	$G = 2 + x (F = 0 \times T (1) - 1 - 1 \times G (1 \times F G))$	TG 0517
	60=61	TG 0518
· .		TG 0519
		10 0520
		TG 0521
		TC 0522
		DD 0523
•		
	1+(N+1)3/,43,3/	
C		16 0526
C	COMPUTE KO USING SERIES EXPANSION	TG 0527
C		TG 0528
	37 GO=-A	TG 0529
	X2J=1.	TG 0530
	FACT=1.	TG 0531
	0.=LH	TG 0532
	00 40 J = 1.6	TG 0533
	$B_{J}=1 \cdot (E_{J}) \Delta T(J)$	TG 0534
	x 2 1= x 2 1 ± C	TG 0535
		TG 0536
		TC 0527
•		
	40 GC=G0+X2JAFACTA(HJ-A)	TG 0538
	IF(N)43,42,43	16 0339
	42 BK=G0	16 0540
· · · ·	RETURN	TG 0541
C C	•	TG 0542
C	COMPUTE KI USING SERIES EXPANSION	TG 0543
. C		TG 0544
	43 X2J=3	TG 0545
	FACT=1.	TG 0546
		TG 0547
· .	$G_{1} = 1 \cdot / X + X - 1 * (\cdot S + \Delta - H_{1})$	TG 0548
		TG 0549
		TC 0550
	FALT=FALT#FJ*RJ	
		TG 0553
	50 S1=G1+X2J*FACT*(•5+(A-HJ)*FLOAT(J))	TG 0554
	1F(N-1)31,52,31	TG 0555
	52 BK=G1	ŤG 0556
	RETURN	TG 0557
	END	TG 0558
С		TG 0559
č		TG 0560
č		TG :0561
		TC 0562
; 0	Subkutthe hest	TC 0562
Ĺ	010 00 00	
Ĺ	PURPUSE - PURPUS	TO 0845
Ċ	CUMPULE THE I BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	
C C		16 0566
C	USAGE	TG 0567
C	CALL BESI(X,N,BI,IER)	TG 0569
С		TG 0569
C	DESCRIPTION OF PARAMETERS	TG 0570
C.	X - THE ARGUMENT OF THE I BESSEL FUNCTION DESIRED	TG 0571
· · ·		

6		A THE DUNCE WE THE I BECOEL FUNCTION DESIDED	TV:	05777
ř		RI - THE VESH TANT I RESELENTION	TC	0573
č			TG	0574
ř		$\frac{1}{2} = \frac{1}{2} $	TC	0576
с. С		f(x) = 0 ind f(x) = 0	10	0574
č		TENTIN IS NEGATIVE	10	0578
С С		IER=2 X IS NEGALIVE	15	0577
C .			16	0578
C		REMARKS	TG	0579
С		N MUST BE GREATER THAN OR EQUAL TO ZERO	TG	0580
С		X SHOULD BE LESS THAN 60 TO AVOID EXCEEDING THE RANGE OF	TG	0581
С		THE MACHINE	TG	0582
С		•	TG	0583
С		SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	TG	0584
C		NONE	TG	0585
С			TG	0586
C		METHOD	TG	0587
č		COMPLITES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING	TS	0588
č		SERIES APPROXIMATIONS AND THEN COMPUTES NOTH ORDER FUNCTION	TG	0589
č		HEAT OF RECEIPTION AND THE CONTREST THE DREET FOR THE	TG	0590
ř			10	0501
č			10	0597
C C			10 TO	0592
L			16	0593
		SUBRUUTINE RESIGNER FIER	16	0594
		IMPLICIT REAL*8(A-H,U-Z)	DP	DP
С			TG	0595
Ç		CHECK FOR ERRORS IN N AND X AND EXIT IF ANY ARE PRESENT	TG	0596
С			TG	0597
		I ER = O	t TG	0598
		R I = 1 • 0	TG	0599
		IE(N)150+15+10	TG	0600
	10	1F(X)160.20.20	TG	0601
	15	1F(X)160-17-20	TG	0602
	17	AFTINN	TG	0603
. r	÷ '		TC	0604
č		DEE INC. TOLADANCE	TC	0605
č		Define Tulekande	10	0005
C	• •		00	0000
•	20	11][=[+D=6	UP	0607
Ľ,			16	0608
C		IF ARGUMEN! GI 12 AND GI N, USE ASYMPTUTIC FORM	15	0609
С			TG	0610
•		1F(X-12.)40,40,30	TG	0611
	30	IF(X-FLUAT(N))40,40,110	TG	0612
С			TG	0613
С		COMPUTE FIPST TERM OF SERIES AND SET INITIAL VALUE OF THE SUM	TG	0614
С			TG	0615
	40	XX=X/2.	TG	0616
		FACTN=1.	TG	0617
		1E(N-1)70.70.50	TG	0618
	50	$\frac{1}{2} = \frac{1}{2} = \frac{1}$	TG	0619
			TC	0620
				0620
	60		10	0621
	70		10	0622
		B1=TERM	16	0623
_		X X = X X * X X	TG	0624
С			TG	0625
C		COMPUTE UP TO 30 TERMS, STOPPING WHEN ABS(TERM) LE ABS(SUM OF TERM	I TG	0626
С		TIMES TOLEPANCE	TG	0627
С			TG	0628
		00 90 K=1,30	TG	0629
		IF(DABS(TEPM)-DABS(BI*TOL))100,100,80	DP	0630
	80	FK=K*(N+K)	TG	0631

è.

90	TERM=TERM*(XX/FK) BI=BI+TERM	TG	0632
-		. TG	0634
	RETURN BL AS ANSWER	TG	0635
•		TG	0636
100	RETURN	TG	0637
		TG	0638
	X GT 12 AND X GT N. SO USE ASYMPTOTIC APPROXIMATION	TG	0639
		TS	0640
110	FN=4*N×N	TG	0641
	XX = 1 / (8 + X)	TG	0642
	TFRM=1	TG	0643
-	BI=1.	TG	0644
	UO 130 K=1.30	TG	0645
	TE(DABS(TERN)-DABS(T() *BL))140-140-120	10	0646
120	FK=(2*K-1)**2	TC	0647
• • •	TERM=TERM=YYX/EK-EN//ELOAT(K)	70	0649
130	NIENT+TERM	TC	0640
140	PT=3,141592653	TO TO	0650
1 /0	R1=81 x12 x2	10	0651
			0652
150	160 - 10 - 100	10 TC	0052
10		10	0655
160	160 10 100 160-0	10	0455
100		10	
		10	0000
		16	1000

APPENDIX 3

DERIVATION OF EQUATION (6.1) FROM GREENSTEIN'S EXPRESSION FOR $\sqrt{21S_2}$ VELOCITY FIELD

Greenstein⁽⁴⁾ provides the following expression for the v^{21S}_{2} velocity field if only translation is considered: $\overline{v}^{21S_2} = \overline{i}(\frac{1}{2} \operatorname{Ar}^{-1} \sin \psi \cos \psi) + \overline{k}(\operatorname{Ar}^{-1} \sin^2 \psi + \frac{A}{2} \operatorname{r}^{-1} \cos^2 \psi)$ (A.3.1) where $A = \frac{3}{2} aU$ (A.3.2) Therefore: $\vec{F}_{x}^{22S_{1}} = 6\pi\mu a(\vec{V}_{x}^{21S_{2}}) = 6\pi\mu a(\frac{3U}{4} = \sin\psi \cos\psi)$ (A.3.3) and $\vec{F}_{n}^{22S_{1}} = 6\pi\mu a(\vec{v}_{n}^{21S_{2}}) = 6\pi\mu a(\frac{3U}{2} = \sin^{2}\psi + \frac{3U}{4} = \cos^{2}\psi)$ (A.3.4) Also, from Stokes law: $F_{\mu}^{11S_1} = 6 \pi_{\mu} a U$ (A.3.5) Computing E, the angle of deflection: $\tan \mathbf{e} = \frac{\mathbf{F}_{\mathbf{x}}^{\mathrm{II}}}{\mathbf{F}_{\mathbf{z}}^{\mathrm{II}}} = \frac{6\pi\mu aU(\frac{3}{4}\frac{a}{r}\sin\psi\cos\psi)}{6\pi\mu aU(1+\frac{3}{2}\frac{a}{r}\sin^{2}\psi+\frac{3}{4}\frac{a}{r}\cos^{2}\psi)}$ (A. 3, 6) The denominator of equation (A.3.6) can be simplified to: $1 + \frac{3}{2} = \frac{a}{r} (\sin^2 \mu + \cos^2 \mu) - \frac{3}{4} = \cos^2 \mu = 1 + \frac{3}{2} = -\frac{3}{4} = \cos^2 \mu$ (A.3.7) Hence, $\tan \epsilon = \frac{\frac{3}{4} \frac{a}{r} \sin \psi \cos \psi}{1 + \frac{3}{4} \frac{a}{r} - \frac{3}{7} \frac{a}{r} \cos^2 \psi}$ (A.3.8) Dividing the denominator into the numerator, we obtain: $\tan \mathbf{e} = \frac{3}{4} \frac{a}{r} \sin \psi \cos \psi - \frac{9}{8} \frac{a^2}{r^2} \sin \psi \cos \psi + \frac{9}{16} \frac{a^2}{r^2} \sin \psi \cos^3 \psi + \dots$ (A.3.9) Neglecting the third term on the right hand side of equation (A.3.9) and subsequent higher order terms, we obtain: $\tan \mathbf{e} \approx \frac{3}{4} \frac{a}{r} \left(1 - \frac{3}{2} \frac{a}{r}\right) \sin \mathbf{\psi} \cos \mathbf{\psi}$ (A.3.10) Furthermore, since E is small, tan E is approximately equal to sin £ . Therefore,

 $\sin \epsilon \approx \frac{3}{4} \frac{a}{r} (1 - \frac{3}{2} \frac{a}{r}) \sin \frac{\mu}{r} \cos \frac{\mu}{r}$ (A.3.11) which is the equation (6.1), given by Smoluchowski.