AN ANALYSIS OF THE GENEVA MECHANISM AS A TIMING DEVICE

BY

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ABSTRACT

A mathematical analysis of the Geneva mechanism was conducted to determine the potential of this device as a timing mechanism, and the feasibility of replacing the widely used pallet and starwheel escapement with a Geneva mechanism.

The primary objective of the study was to determine if the Geneva mechanism will attain a terminal velocity, and if so, how much time is required to reach this velocity.

To accomplish this objective, the differential equation of motion for the system was derived using Lagrangian dynamics. The equation was programmed and solved on a digital computer.

The study indicated that the Geneva mechanism does reach a terminal velocity, and consequently can be used as a timing device. The mechanism, however, requires as much as fifty milliseconds to attain this terminal velocity.

ACKNOWLEDGEMENT

The author wishes to acknowledge the guidance and assistance of Dr. Benjamin Stevenson of the Physics Department at Newark College of Engineering. Also, he would like to thank Dr. Frederick R. Tepper of Picatinny Arsenal, who proposed the problem and provided moral support.

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INTRODUCTION

The runaway escapement of figure 1 is the classic timing device in wide use today. Many watches, clocks, industrial timers, ordnance fuzes, and countless other mechanical timing devices utilize this mechanism. Although this device can be finely tuned to a high degree of accuracy, the system is inherently inaccurate, dictating that an accurate system be an expensive system.

The pallet and starwheel mechanism is inherently inaccurate since its operation depends on impacts of the starwheel with the pallet. Not only are losses involved as with any impact device, but angular accelerations of the members become inefficiently high. With these problems in mind, Dr. Frederick R. Tepper of Picatinny Arsenal proposed the Geneva mechanism of figure 2 as a possible replacement. This is the basic mechanism to which this study is addressed.

The objective of this study is to describe mathematically the Geneva mechanism, and demonstrate its feasibility as a timing mechanism. In order to function as an accurate timing device, the mechanism must attain a terminal, or steady state, velocity. Also, the mechanism must reach this terminal velocity in a time frame consistent with the required accuracy of the timer (this is defined as the rise time). Therefore, the existence of the terminal velocity and the length of the rise time will determine the applicability of the Geneva mechanism as a timing device.

RUNAWAY ESCAPEMENT

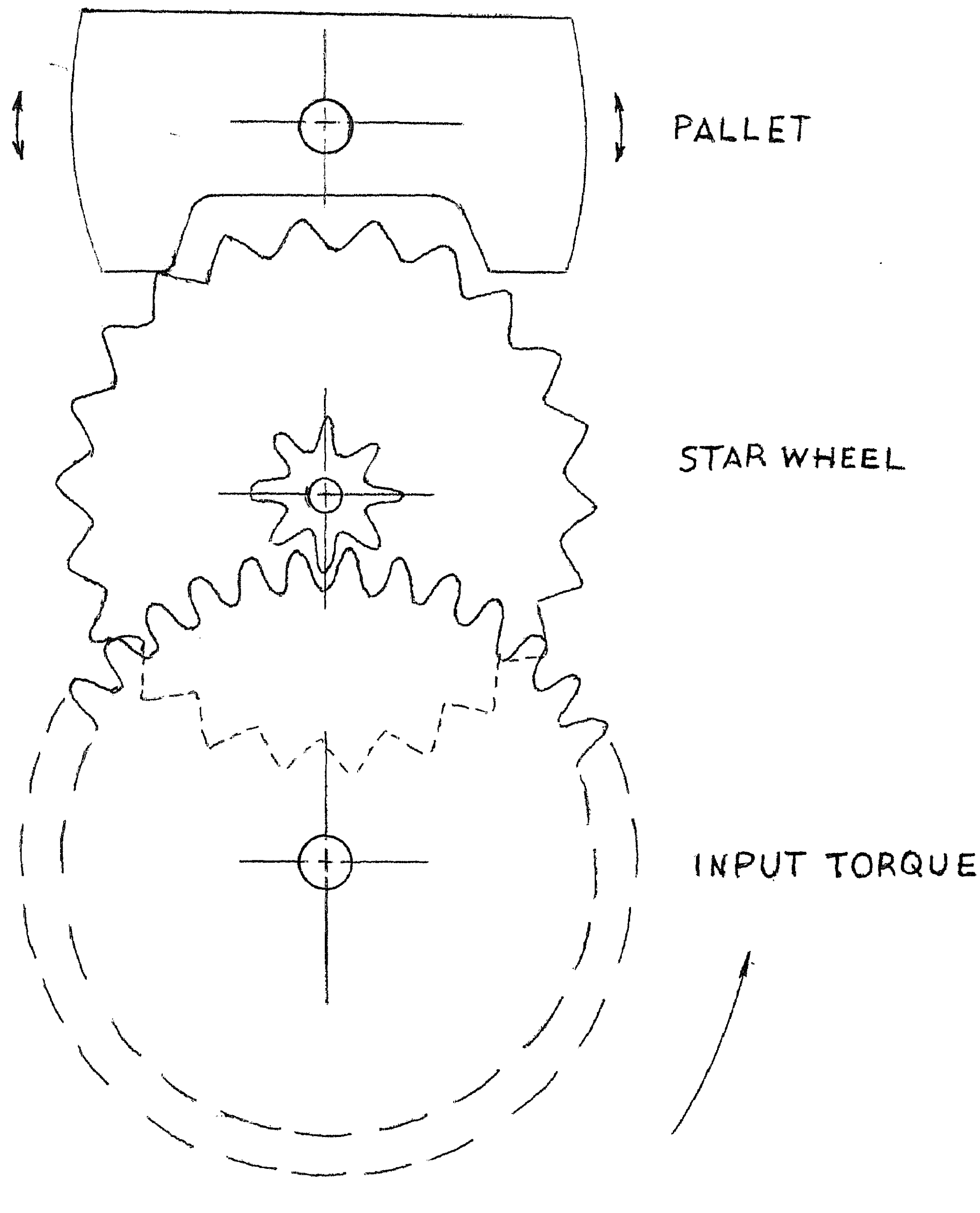
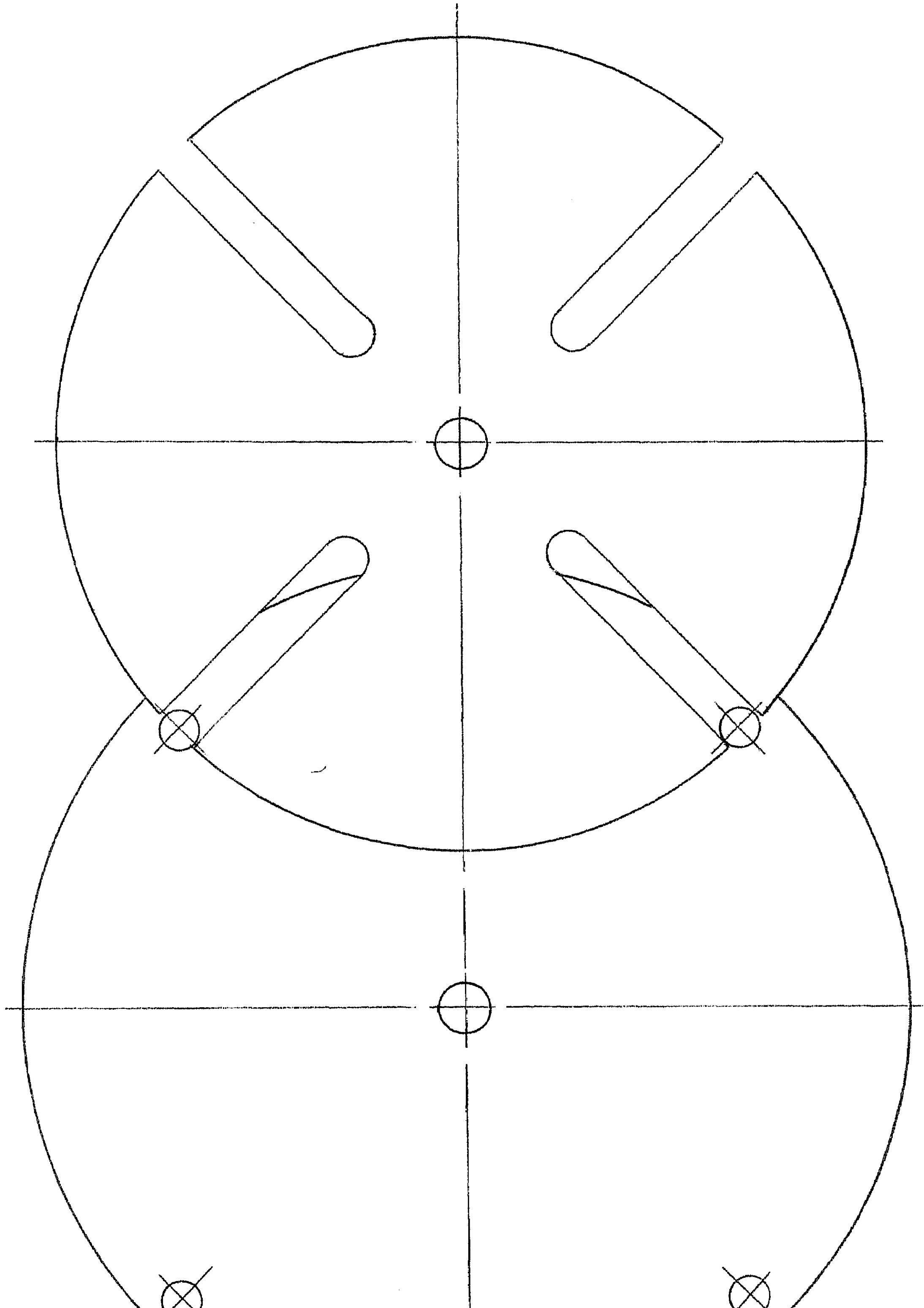


FIGURE 1

FOUR PIN MECHANISM



SCALE: 2½ to 1

FIGURE 2

CHAPTER I

DERIVATION OF THE EQUATION OF MOTION

The Geneva mechanism, in its usual application, consists of a driver with a single pin, and a follower with from three to eighteen slots. The most common application of this type of mechanism is as an indexing device, where one revolution of the driver will index the follower 1/n revolutions, where n is the number of slots in the follower. The driver is usually assumed to rotate at a uniform angular velocity.

The single pin mechanism is relatively simple to analyze. However, the intermittent motion of this device is undesirable for a timing mechanism. One requirement for the Geneva escapement now becomes evident: the follower must engage at least one pin at all times. In order to meet this condition, certain geometrical constraints are necessary.

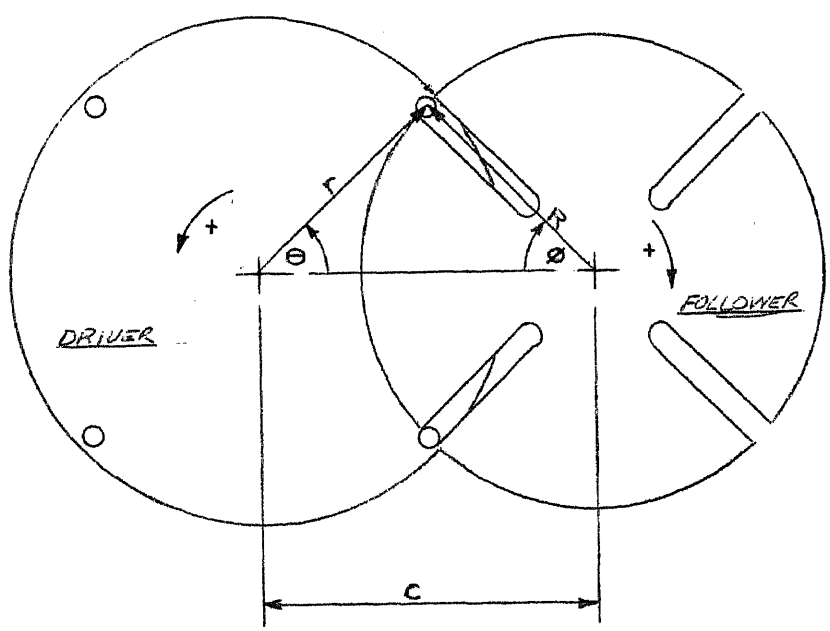


Figure 3. Geometry of the Geneva escapement (four pin, four slot configuration shown).

In the Geneva escapement of figure 3, at the instant the first pin begins to leave the slot, the next pin must be entering the next slot. In order to assure continuous motion, the velocity of the pin at the point of entry and exit must be along the centerline of the slot. Figure 4 represents the mechanism at the instant of exit of the lead pin and entry of the next pin.

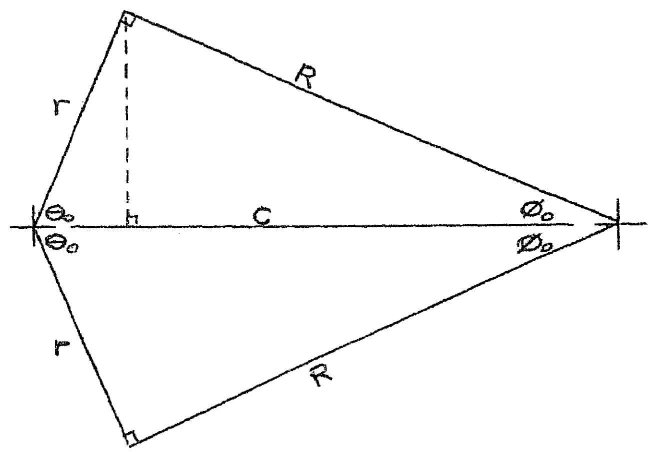


Figure 4. Geometry at pin entry and exit.

At this point in time,

$$\phi = \phi_o = \tfrac{1}{2}\left(\frac{2\pi}{n}\right) = \frac{\pi}{n}$$

and

$$\theta = \theta_o = \tfrac{1}{2}\left(\frac{2\pi}{m}\right) = \frac{\pi}{m}$$

In the large right triangle,

$$r = c \sin \phi_o = c \sin\left(\frac{\pi}{n}\right)$$

or

$$c = a\, r \qquad (1\text{-}1)$$

where

$$a = \frac{1}{\sin(\pi/n)} \qquad (1\text{-}2)$$

Again in the large right triangle of figure 4,

$$\theta_o + \phi_o + \frac{\pi}{2} = \pi$$

$$\theta_o + \phi_o = \frac{\pi}{2}$$

$$\frac{\pi}{m} + \frac{\pi}{n} = \frac{\pi}{2}$$

$$m = \frac{2n}{n-2} \qquad (1\text{-}3)$$

Equations 1-1 and 1-3 are the fundamental geometrical constraints of the Geneva escapement. They assure continuous motion of the driver and follower.

Only certain pin and slot combinations are permitted by equation 1-3, since only integral numbers of pins and slots are possible. The only three possible combinations are:

SLOTS (n)	3	4	6
PINS (m)	6	4	3

Note that as n increases in equation 1-3, m decreases. The next integral number of pins is 2, which will not occur until n reaches infinity. Scale drawings of the three possible configurations are given in Appendix A. Examination of the drawings will show that these three mechanisms meet the requirement that the trajectory of the pin as it enters and exits the slot is coincident with the centerline of the slot.

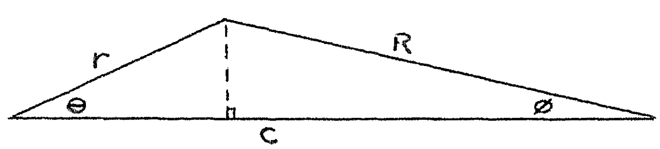


Figure 5. General case for angular relationships.

From the geometry of figure 5,

$$r \cos \theta + R \cos \phi = c$$

$$\cos \phi = \frac{1}{R} (c - r \cos \theta)$$

and substituting equation 1-1,

$$\cos \phi = \frac{1}{R} (ar - r \cos \theta) \qquad (1\text{-}4)$$

In figure 5, by the Law of Sines,

$$\frac{R}{\sin \theta} = \frac{r}{\sin \phi}$$

$$R = \frac{r \sin \theta}{\sin \phi} \tag{1-5}$$

and substituting this relationship into equation 1-4,

$$\cos \phi = \frac{r \sin \phi}{r \sin \theta} (a - \cos \theta)$$

$$\frac{\cos \phi}{\sin \phi} = \frac{a - \cos \theta}{\sin \theta} = \frac{1}{\tan \phi}$$

$$\tan \phi = \frac{\sin \theta}{a - \cos \theta} \tag{1-6}$$

Equation 1-6 is the fundamental angular relationship between the driver and the follower.

To find the relationship between the angular velocities of the two members, differentiate equation 1-6 with respect to time.

$$\frac{d}{dt}(\tan \phi) = \frac{(a-\cos \theta)\frac{d}{dt}(\sin \theta) - \sin \theta \frac{d}{dt}(a - \cos \theta)}{(a - \cos \theta)^2}$$

$$(\sec^2 \phi)\, \dot{\phi} = \frac{(a-\cos \theta)(\cos \theta)\dot{\theta} - \sin \theta\, (\sin \theta)\dot{\theta}}{(a-\cos \theta)^2}$$

$$(1 + \tan^2 \phi)\dot{\phi} = \frac{a \cos \theta - 1}{(a - \cos \theta)^2}\, \dot{\theta}$$

$$\left(1 + \frac{\sin^2 \theta}{(a - \cos \theta)^2}\right)\dot{\phi} = \frac{a \cos \theta - 1}{(a - \cos \theta)^2}\, \dot{\theta}$$

$$\dot{\phi} = \left(\frac{a \cos \theta - 1}{1 + a^2 - 2a\cos \theta}\right) \dot{\theta} \tag{1-7}$$

To find the relationship between the angular accelerations of the two members, differentiate equation 1-7 with respect to time.

$$\ddot{\phi} = \left[\frac{(1+a^2-2a\cos \theta)\frac{d}{dt}(a\cos\theta - 1) - (a\cos \theta - 1)\frac{d}{dt}(1 + a^2 - 2a\cos \theta)}{(1 + a^2 - 2a\cos \theta)^2}\right]\dot{\theta}$$

$$+ \left(\frac{a \cos \theta - 1}{1 + a^2 - 2a\cos \theta}\right)\ddot{\theta}$$

$$\ddot{\phi} = \left[\frac{(1+a^2-2a\cos\theta)(-a\sin\theta)\dot{\theta} - (a\cos\theta - 1)(2a\sin\theta)\dot{\theta}}{(1+a^2-2a\cos\theta)^2}\right]\dot{\theta}$$

$$+ \frac{a\cos\theta - 1}{(1+a^2-2a\cos\theta)}\ddot{\theta}$$

$$\ddot{\phi} = \frac{a\sin\theta\,(1-a^2)}{(1+a^2-2a\cos\theta)^2}\dot{\theta}^2 + \frac{a\cos\theta - 1}{(1+a^2-2a\cos\theta)}\ddot{\theta} \qquad (1\text{-}8)$$

Lagrange's Equation may now be considered. Since the system under study is not frictionless, Lagrange's Equation for a non-conservative system applies:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_K}\right) - \frac{\partial T}{\partial q_K} = Q_K \qquad (1\text{-}9)$$

where: T = total kinetic energy of the system;

q_K = generalized coordinates necessary to completely describe the system (k 1,2,3,....);

Q_K = q_K component of the generalized force representing all forces acting on the system, excluding inertial forces (included on the left side of 1-9) and constraint forces (drop out of Lagrange's Equation). In particular, Q_K includes the driving torque and the frictional torque; conservative forces are zero (i.e. no potential energy).

The system under consideration has one degree of freedom (θ): therefore, equation 1-9 reduces to

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = Q_\theta$$

To find the generalized torque, Q_θ, consider a virtual displacement $\delta\theta$. Then the virtual work done is

$$\delta W = Q_\theta \delta\theta = \tau\,\delta\theta$$

or $$Q_\theta = \tau$$

where τ is the resultant torque acting on the system; that is, the vector sum of the driving torque and all frictional torques.

There are three frictional torques in the system: the driver bearing friction, the follower bearing friction, and the friction of the pin sliding in the slot. The frictional torque contributed by the driver and follower bearings will be ignored in this analysis. Therefore the generalized torque is

$$Q_\theta = \tau_1 - |\tau_2|$$

where τ_1 is the driving torque, and τ_2 is the frictional torque of the driving pin. The absolute value of τ_2 is used, since the friction torque must always oppose the driving torque, which is taken to be positive. With this generalized torque, Lagrange's equation reduces to

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) - \frac{\partial T}{\partial \theta} = \tau_1 - |\tau_2| \qquad (1\text{-}10)$$

Let μN be the friction between the pin and slot, where μ is the dynamic coefficient of friction, and N is the normal force transmitted from pin to follower (i.e. the driving force). Then figure 5 can be extended as follows:

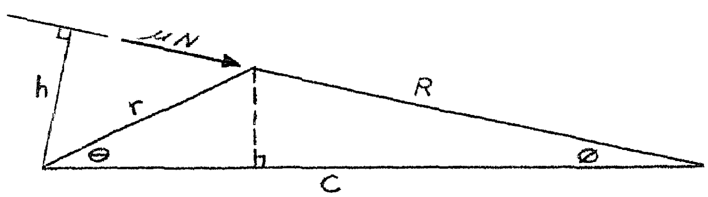


Figure 6. Frictional torque about the driver bearing.

Here the assumption has been made that the pin diameter and the slot width are much less than r, and consequently have been neglected.

The frictional torque about the center of rotation of the driver is

$$\tau_2 = h\mu N = c \sin \phi \mu N$$

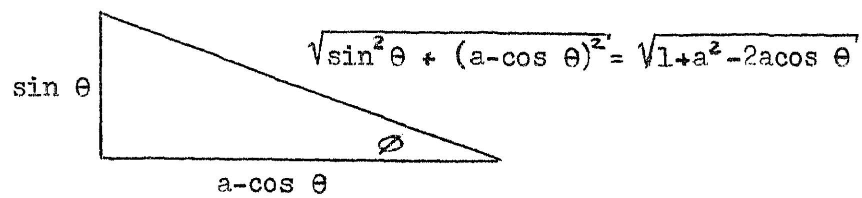
This torque must always oppose the driving torque, which is taken as positive (ccw). Therefore,

$$\tau_2 = -|c \sin \phi \mu N| = -\mu c|N \sin \phi|$$

and from equation 1-1,

$$\tau_2 = -ar\mu|N \sin \phi| \ .$$

From equation 1-6, the following triangle can be drawn.



From this triangle, sin Ø is found to be

$$\sin \emptyset = \frac{\sin \theta}{(1+a^2-2a\cos\theta)^{1/2}} \tag{1-11}$$

and the frictional torque becomes

$$\tau_z = -ar\mu \left| \frac{N \sin\theta}{(1+a^2-2a\cos\theta)^{1/2}} \right| \tag{1-12}$$

To find N, draw the free body diagram of the follower.

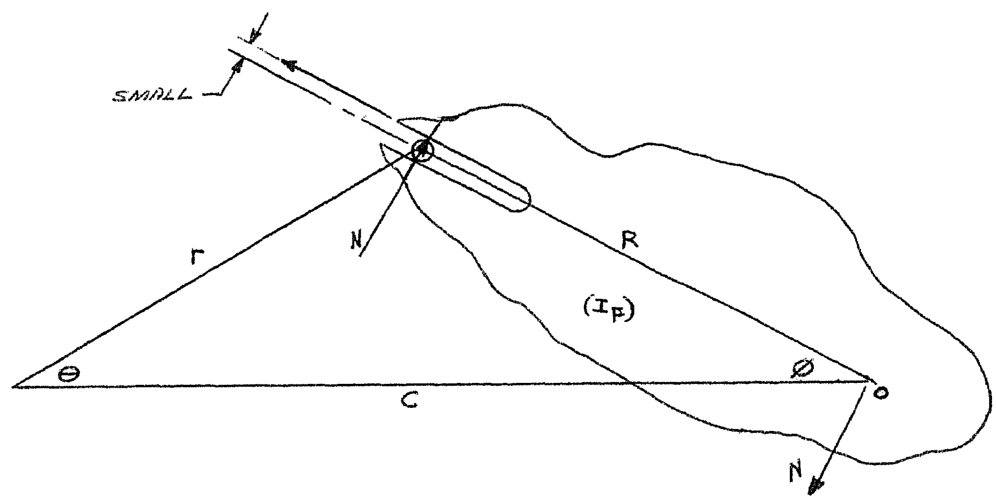


Figure 7. Free body diagram of the follower.

Taking moments about o;

$$NR = I_f \ddot{\emptyset}$$

From equations 1-5 and 1-11,

$$R = \frac{r \sin\theta}{\sin \emptyset} = \frac{r \sin\theta}{(\sin\theta)/(1+a^2-2a\cos\theta)^{1/2}}$$

$$R = r(1+a^2-2a\cos\theta)^{1/2}$$

Substituting this and equation 1-8 into the moment equation,

$$N = \frac{I_f \ddot{\emptyset}}{R}$$

$$N = \frac{I_f\left[\frac{a\sin\theta\ (1-a^2)}{(1+a^2-2a\cos\theta)^2}\dot{\theta}^2 + \frac{a\cos\theta - 1}{(1+a^2-2a\cos\theta)}\ddot{\theta}\right]}{r(1+a^2-2a\cos\theta)^{1/2}}$$

$$N = \frac{I_f}{r}\left[\frac{a\sin\theta\ (1-a^2)}{(1+a^2-2a\cos\theta)^{5/2}}\dot{\theta}^2 + \frac{a\cos\theta - 1}{(1+a^2-2a\cos\theta)^{3/2}}\ddot{\theta}\right] \qquad (1\text{-}13)$$

And finally, substituting into equation 1-12,

$$\tau_2 = -ar\mu\left|\frac{\sin\theta}{(1+a^2-2a\cos\theta)^{1/2}}\ \frac{I_f}{r}\left[\frac{a\sin\theta\ (1-a^2)}{(1+a^2-2a\cos\theta)^{5/2}}\dot{\theta}^2 + \frac{a\cos\theta - 1}{(1+a^2-2a\cos\theta)^{3/2}}\ddot{\theta}\right]\right|$$

$$\tau_2 = -a\mu I_f\left|\frac{a\sin^2\theta\ (1-a^2)}{(1+a^2-2a\cos\theta)^3}\dot{\theta}^2 + \frac{\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^2}\ddot{\theta}\right| \qquad (1\text{-}14)$$

since r and I_f are always positive.

For the rest of the generalized torque term, the mechanism is assumed to be driven by a constant torque negator spring (see Appendix C), such that

$$\tau_1 = K \qquad (1\text{-}15)$$

where K is the torsional spring rate in inch-pounds per radian, and is constant.

Thus the right side of Lagrange's Equation (1-10) has been determined. The left side involves derivatives of the total kinetic energy of the system. This is simply the sum of the rotational energies of the driver and follower.

$$T = \tfrac{1}{2} I_D\dot{\theta}^2 + \tfrac{1}{2} I_f\dot{\phi}^2$$

Substituting equation 1-7 for $\dot{\phi}$,

$$T = \tfrac{1}{2} I_D\dot{\theta}^2 + \tfrac{1}{2} I_f \frac{(a\cos\theta - 1)^2}{(1+a^2-2a\cos\theta)^2}\ \dot{\theta}^2$$

$$T = \tfrac{1}{2}\left[I_D + \frac{(a\cos\theta - 1)^2}{(1+a^2-2a\cos\theta)^2} I_f\right]\dot{\theta}^2 \qquad (1\text{-}16)$$

For Lagrange's Equation,

$$\frac{\partial T}{\partial\dot{\theta}} = \frac{\partial}{\partial\dot{\theta}}\left[\tfrac{1}{2}\left\{I_D\ \frac{(a\cos\theta - 1)^2}{(1+a^2-2a\cos\theta)^2} I_f\right\}\dot{\theta}^2\right]$$

$$\frac{\partial T}{\partial\dot{\theta}} = \left[I_D + \frac{(a\cos\theta - 1)^2}{(1+a^2-2a\cos\theta)^2} I_f\right]\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) = \frac{d}{dt}\left[\ I_D + \frac{(a\cos\theta - 1)^2}{(1+a^2-2a\cos\theta)^2} I_f\right\}\dot{\theta}\ \right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) = \left[I_D + \frac{(a \cos \theta - 1)^2}{(1+a^2-2a\cos \theta)^2} I_f \right] \ddot\theta$$

$$+ \dot\theta \left[0 + I_f \left\{ \frac{(1+a^2-2a\cos\theta)^2 \left[2(a\cos\theta-1)(-a\sin\theta\dot\theta)\right]}{(1+a^2-2a\cos \theta)^4} \right.\right.$$

$$\left.+ \frac{(-a\cos\theta-1)^2(2)(1+a^2-2a\cos\theta)(2a\sin\theta\ \dot\theta)}{(1+a^2-2a\cos \theta)^4}\right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) = \left[I_D + \frac{(a \cos \theta - 1)^2}{(1+a^2-2a\cos \theta)^2} I_f \right] \ddot\theta$$

$$+ I_f \dot\theta \left[\frac{-2(a\cos\theta-1)(a\sin\theta\dot\theta)}{(1+a^2-2a\cos \theta)^2} - \frac{(a\cos\theta-1)^2(2)(2a\sin\theta\dot\theta)}{(1+a^2-2a\cos \theta)^3} \right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) = \left[I_D + \frac{(a \cos \theta - 1)^2}{(1+a^2-2a\cos \theta)^2} I_f \right] \ddot\theta$$

$$-2I_f \dot\theta^2 \left[\frac{a\sin \theta\ (a\cos\theta-1)}{(1+a^2-2a\cos \theta)^2} \left\{ 1 + \frac{2(a \cos \theta - 1)}{(1+a^2-2a\cos \theta)} \right\} \right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) = \left[I_D + \frac{(a\cos\theta-1)^2 I_f}{(1+a^2-2a\cos\theta)^2} \right] \ddot\theta - 2I_f \dot\theta^2 \left[\frac{a(a^2-1)\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos \theta)^3} \right]$$

For the remaining term in Lagrange's Equation,

$$\frac{\partial T}{\partial \theta} = \frac{\partial}{\partial \theta} \left[\tfrac{1}{2} \left\{ I_D + \frac{(a \cos \theta - 1)^2}{(1+a^2-2a\cos \theta)^2} I_f \right\} \dot\theta^2 \right]$$

$$\frac{\partial T}{\partial \theta} = 0 + \tfrac{1}{2} I_f \dot\theta^2 \frac{\partial}{\partial \theta} \left[\frac{(a \cos \theta - 1)^2}{(1+a^2-2a\cos \theta)^2} \right]$$

$$\frac{\partial T}{\partial \theta} = \tfrac{1}{2} I_f \dot\theta^2 \left[\frac{(1+a^2-2a\cos\theta)^2(2)(a\cos\theta-1)(-a\sin\theta)}{(1+a^2-2a\cos \theta)^4} \right.$$

$$\left. - \frac{(a\cos\theta-1)^2(2)(1+a^2-2a\cos\theta)(2a\sin\theta)}{(1+a^2-2a\cos \theta)^4} \right]$$

$$\frac{\partial T}{\partial \theta} = \tfrac{1}{2} I_f \dot\theta^2 \left[\frac{-2(a\cos\theta-1)(a\sin\theta)}{(1+a^2-2a\cos \theta)^2} + \frac{-2(a\cos\theta-1)^2(2a\sin\theta)}{(1+a^2-2a\cos \theta)^3} \right]$$

$$\frac{\partial T}{\partial \theta} = -I_f \dot\theta^2 \left[\frac{(a\cos\theta-1)(a\sin\theta)}{(1+a^2-2a\cos\theta)^2} \left\{ 1 + \frac{(a\cos\theta-1)(2)}{(1+a^2-2a\cos\theta)} \right\} \right]$$

$$\frac{\partial T}{\partial \theta} = -I_f \dot\theta^2 \left[\frac{a(a^2-1)\ \sin \theta\ (a \cos \theta - 1)}{(1+a^2-2a\cos \theta)^3} \right]$$

Now the left side of equation 1-10 can be written;

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right)-\frac{\partial T}{\partial \theta}=\left[\left\{I_p+\frac{(a\cos\theta-1)^2 I_f}{(1+a^2-2a\cos\theta)^2}\right\}\ddot\theta-2I_f\dot\theta^2\left\{\frac{a(a^2-1)\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^3}\right\}\right]$$
$$-\left[-I_f\dot\theta^2\left\{\frac{a(a^2-1)\sin\theta\,(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^3}\right\}\right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right)-\frac{\partial T}{\partial \theta}=\left[I_p+\frac{(a\cos\theta-1)\;I_f}{(1+a^2-2a\cos\theta)^2}\right]\ddot\theta$$
$$-I_f\dot\theta^2\left[\frac{a(a^2-1)\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^3}\right] \tag{1-17}$$

Substituting equations 1-14, 1-15, and 1-17 into equation 1-10,

$$\left[I_p+\frac{(a\cos\theta-1)\;I_f}{(1+a^2-2a\cos\theta)^2}\right]\ddot\theta-\left[\frac{a(a^2-1)\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^3}\right]I_f\,\dot\theta^2$$
$$=K-a\mu I_f\left|\frac{a\sin^2\theta(1-a^2)\dot\theta^2}{(1+a^2-2a\cos\theta)^3}+\frac{\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^2}\;\ddot\theta\right|$$

And the equation of motion is:

$$\left[\frac{I_p}{I_f}+\frac{(a\cos\theta-1)^2}{(1+a^2-2a\cos\theta)^2}\right]\ddot\theta+\left[\frac{a(1-a^2)\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^3}\right]\dot\theta^2$$
$$+a\mu\left|\frac{a\sin^2\theta(1-a^2)}{(1+a^2-2a\cos\theta)^3}\;\dot\theta^2+\frac{\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^2}\;\ddot\theta\right|=\frac{K}{I_f} \tag{1-18}$$

Equation 1-18 has one possible flaw. If the quantity

$$(1+a^2-2a\cos\theta)$$

should ever become zero, the equation would not be valid at that point (or points). Let θ_∞ be the assumed point at which the assumed singularity exists. Then

$$1+a^2-2a\cos\theta_\infty=0$$

$$\theta_\infty=\cos^{-1}\left(\frac{1+a^2}{2a}\right)$$

For θ_∞ to exist,

$$\left(\frac{1+a^2}{2a}\right)\leqslant 1$$

$$1+a^2\leqslant 2a$$

$$a^2-2a+1\leqslant 0$$

$$(a-1)^2\leqslant 0$$

This is only true for $a=1$. But from equation 1-2,

$$a=\frac{1}{\sin(\pi/n)}=1$$

$$\sin\left(\frac{\pi}{n}\right)=1$$

This is only true for $n = 2$ (remember n must be an integer). Since $n = 2$ has been previously ruled out (see equation 1-3), 1-18 contains no singularities.

To simplify equation 1-18, make the following substitutions:

$$A = \left[\frac{I_D}{I_f} + \frac{(a\cos\theta - 1)^2}{(1 + a^2 - 2a\cos\theta)^2}\right]$$

$$B = \left[\frac{a(1-a^2)\sin\theta(a\cos\theta - 1)}{(1+a^2-2a\cos\theta)^3}\right]$$

$$C = \left[\mu\frac{a^2\sin^2\theta(1-a^2)}{(1+a^2-2a\cos\theta)^3}\right]$$

$$D = \left[\mu\frac{a\sin\theta(a\cos\theta-1)}{(1+a^2-2a\cos\theta)^2}\right]$$

$$E = \frac{K}{I_f}$$

Then the equation of motion becomes

$$A\ddot{\theta} + B\dot{\theta}^2 + \left|C\dot{\theta}^2 + D\ddot{\theta}\right| = E \qquad (1\text{-}19)$$

This differential equation will be solved by digital computer (see Appendix B).

CHAPTER II

PARAMETRIC STUDY

The object of this study is to determine the feasibility of the Geneva mechanism as a timing device; optimization is beyond the scope of this paper. The former is demonstrated in this parametric study by showing that for the mechanisms studied, a terminal velocity is attained, and that variations in the parameters of the system have small and predictable effects on the motion of the system.

Having found the equation of motion for the system, six parameters become evident:

- n: the number of slots in the wheel;
- θ_o: the angular position of the pin as it enters the slot;
- μ: the coefficient of friction between the pin and slot;
- K: the torsional spring constant of the driving spring;
- I_F: the polar moment of inertia of the follower;
- I_D: the polar moment of inertia of the driver.

Note that the first two parameters, n and θ_o, are not independent and can be considered as a single parameter (and incidently, the only geometric parameter). Thus there are four parameters for each of the three mechanisms (i.e. the three, four, and six pin mechanisms). For simplicity, the moments of inertia will be fixed. The sizes of the driver and follower are both taken to be approximately 0.5 inch in diameter and 0.1 inch in thickness, and the material is taken to be steel (.283 lb/in^3).

$$I = \tfrac{1}{2}\pi m\, r^4 t\,, \qquad \text{where m is the mass density.}$$

$$I = \tfrac{1}{2}\pi\left(\frac{.283\ \text{lb/in}}{386\text{in/sec}}\right)\left(\frac{0.5\ \text{in}}{2}\right)^4(0.1\ \text{in})$$

$$I \cong 0.5 \times 10^{-6}\ \text{in lb sec}^2$$

This is an approximate value, and the moments of inertia used in this parametric study did vary somewhat from this figure.

Figures 8 through 11 represent a summary of the computer analysis done on the mechanisms. Figure 8 shows the velocity of the mechanism as a function of time for a typical set of parameters. Changing para-

DRIVER AND FOLLOWER ANGULAR VELOCITIES
FOR THE FOUR PIN MECHANISM

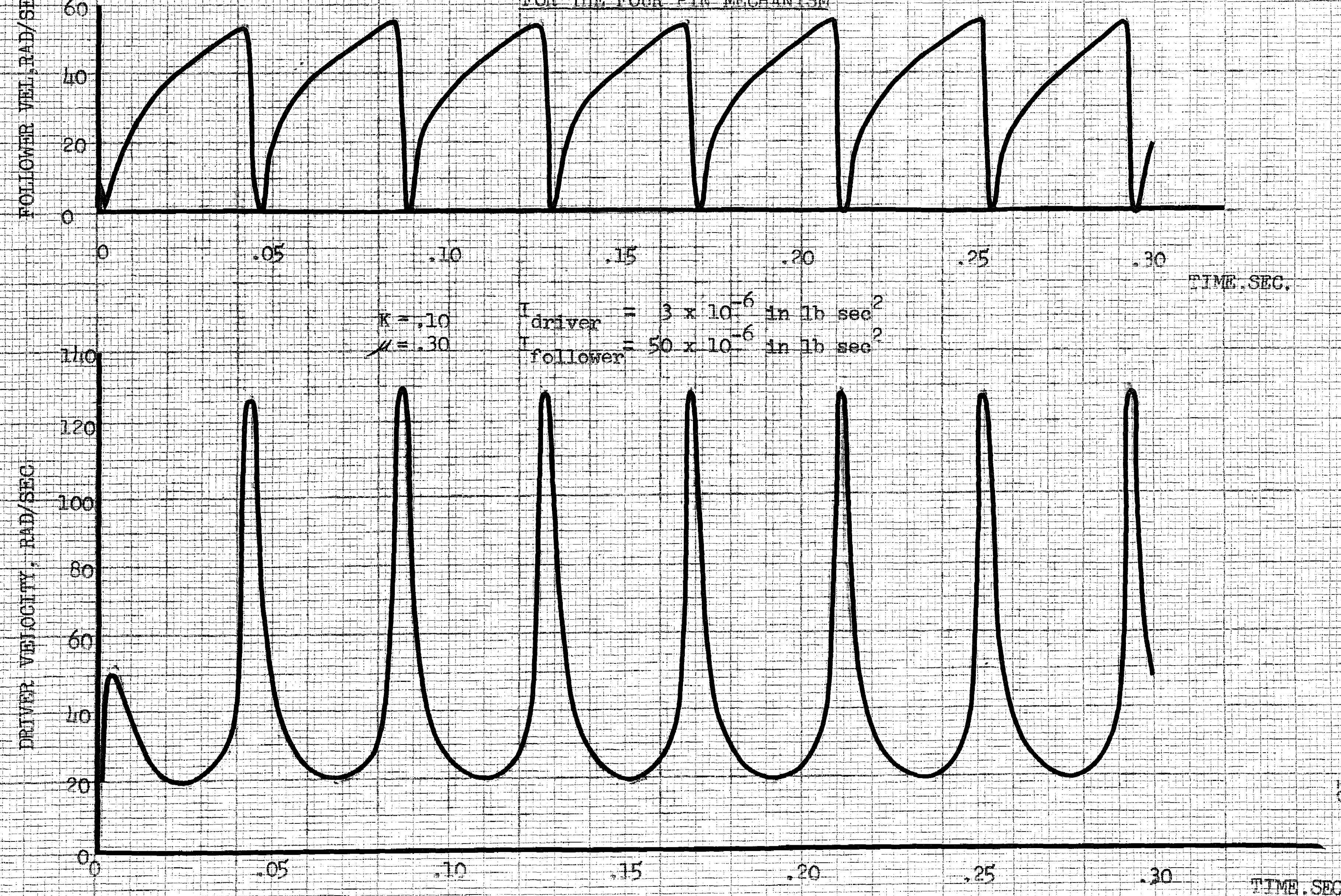


FIGURE 8

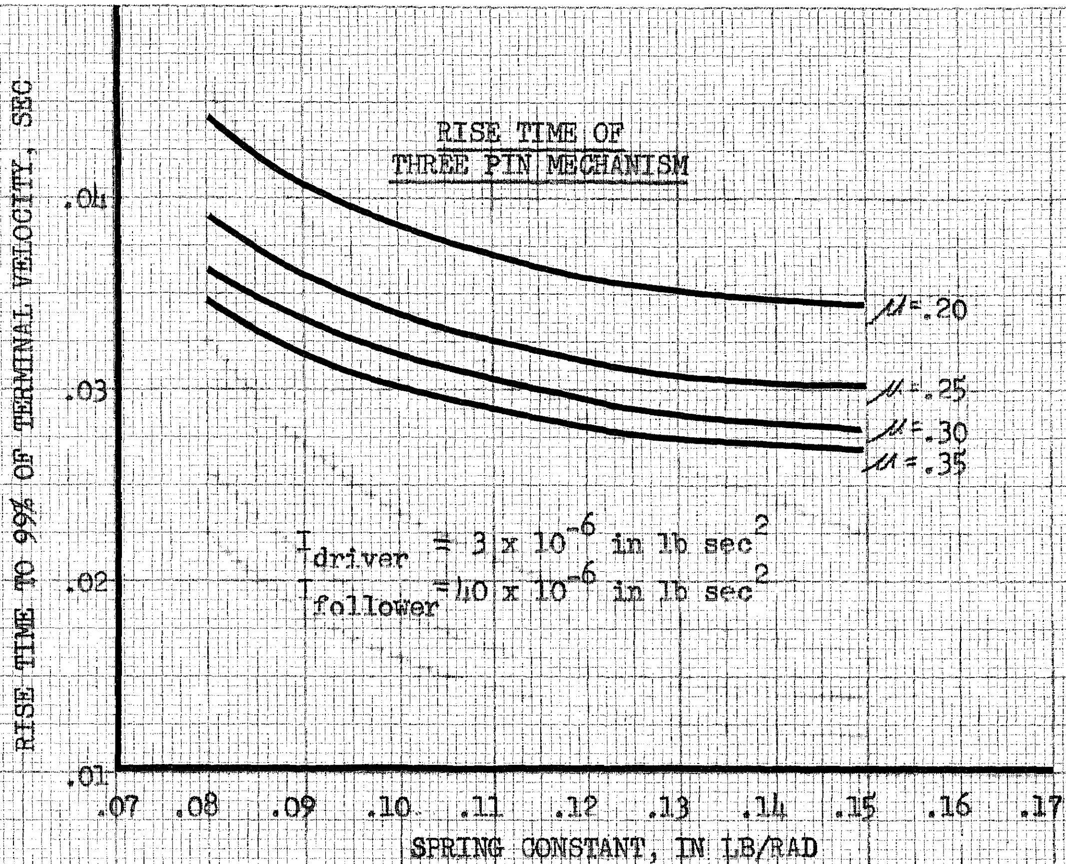
meters will not change the shape of the curve, but will change the magnitude of the velocity, the period of the oscillations, and the rise time (see below). The following discussion applies in general to any of the three mechanisms, but refers in particular to figure 8.

The rise time of the mechanism shown is 0.025 seconds; since by the first minimum in velocity, the driver has achieved its steady state velocity profile. The average terminal velocity, as used here, is simply the arithmetical average of the peak and minimum velocity. The period of oscillation is the time between peaks of velocity, and is a measure of the resolution of the mechanism (for example, a typical watch might have a resolution of one second, if the pallet oscillates once a second).

The peaks in driver velocity represent the point of entry of the pin into the slot (distance between peaks is 90° for the four slot mechanism). In other words, the driver velocity is a maximum at pin entry. Since the trajectory of the pin must be coincident with the centerline of the slot for proper entry (and exit) of the pin, the follower must be instantaneously stationary. This can be seen in the graph, since the peaks in driver velocity coincide with the zero points (minima) in the follower velocity. As the driver continues to turn through the entry point, the follower will pick up speed as the driver, transferring its energy to the follower, slows down. When the pin and slot are on the line between the centers of rotation, the driver velocity is a minimum and the follower velocity is a maximum. Having passed the centerline, the driver velocity increases (and the follower velocity decreases) until it again reaches a maximum as the pin leaves the slot. If terminal velocity has been reached, the velocity of the pin leaving the slot will be the same as when it entered.

Three general statements can be made regarding the parametric graphs for the driver (figures 9, 10, and 11):

1. The greater the driving torque, the lower the rise time, and the greater the terminal velocity. The rise time will, however, reach a certain minimum (a function of the coefficient of friction) beyond which increases in the driving torque will no longer reduce the


RISE TIME OF
THREE PIN MECHANISM
RISE TIME TO 99% OF TERMINAL VELOCITY, SEC
.04
.03
.02
.01
.07
.08
.09
.10
.11
.12
.13
.14
.15
.16
.17
μ=.20
μ=.25
μ=.30
μ=.35
I driver = 3 x 10^-6 in lb sec^2
I follower = 40 x 10^-6 in lb sec^2
SPRING CONSTANT, IN LB/RAD

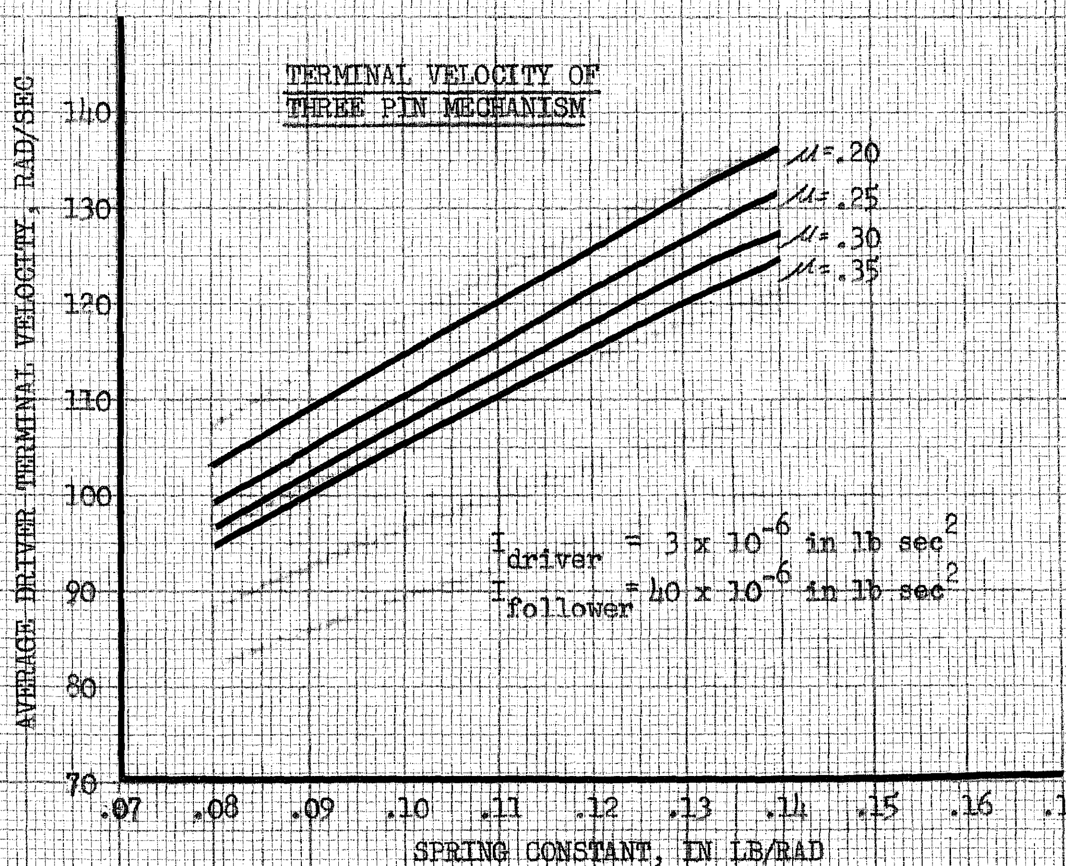
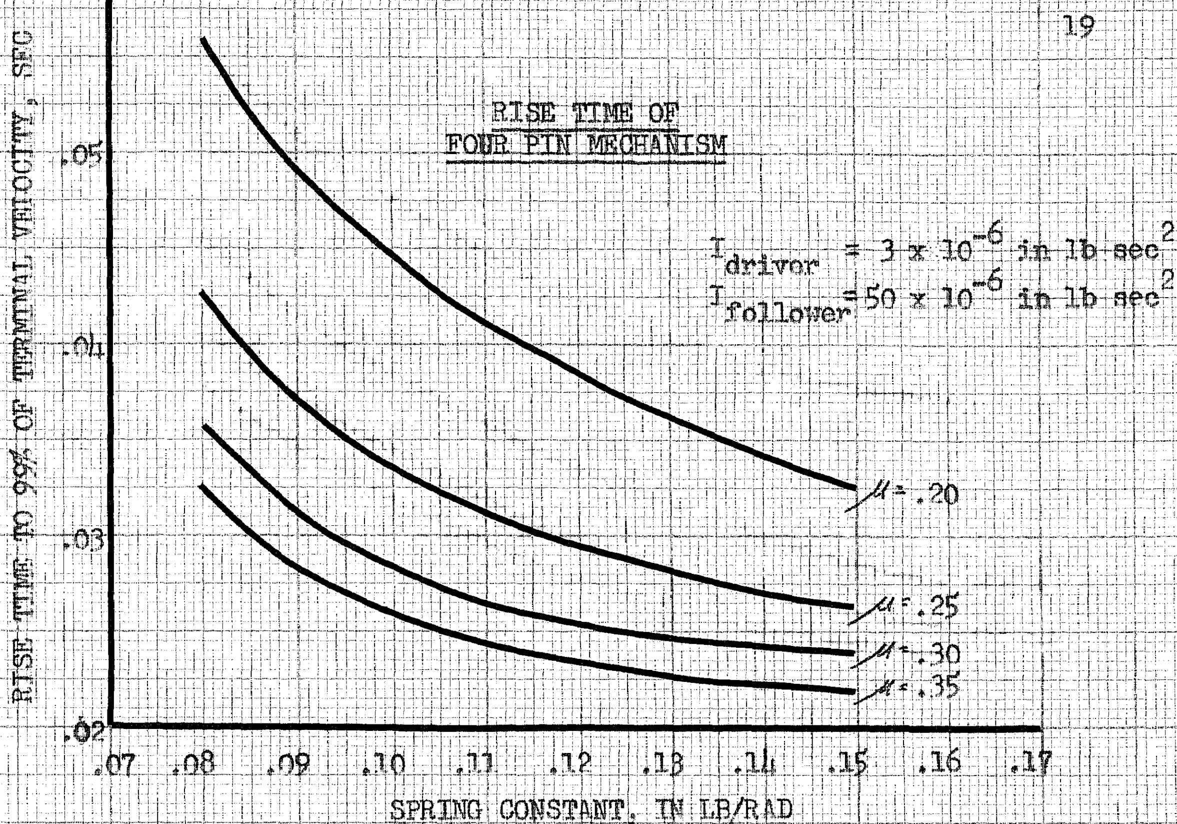

TERMINAL VELOCITY OF
THREE PIN MECHANISM
AVERAGE DRIVER TERMINAL VELOCITY, RAD/SEC
140
130
120
110
100
90
80
70
.07
.08
.09
.10
.11
.12
.13
.14
.15
.16
.17
μ=.20
μ=.25
μ=.30
μ=.35
I driver = 3 x 10^-6 in lb sec^2
I follower = 40 x 10^-6 in lb sec^2
SPRING CONSTANT, IN LB/RAD

FIGURE 9


RISE TIME OF
FOUR PIN MECHANISM
RISE TIME TO 99% OF TERMINAL VELOCITY, SEC
I driver = 3 x 10^-6 in lb sec^2
I follower = 50 x 10^-6 in lb sec^2
μ = .20
μ = .25
μ = .30
μ = .35
.02
.03
.04
.05
.07 .08 .09 .10 .11 .12 .13 .14 .15 .16 .17
SPRING CONSTANT, IN LB/RAD

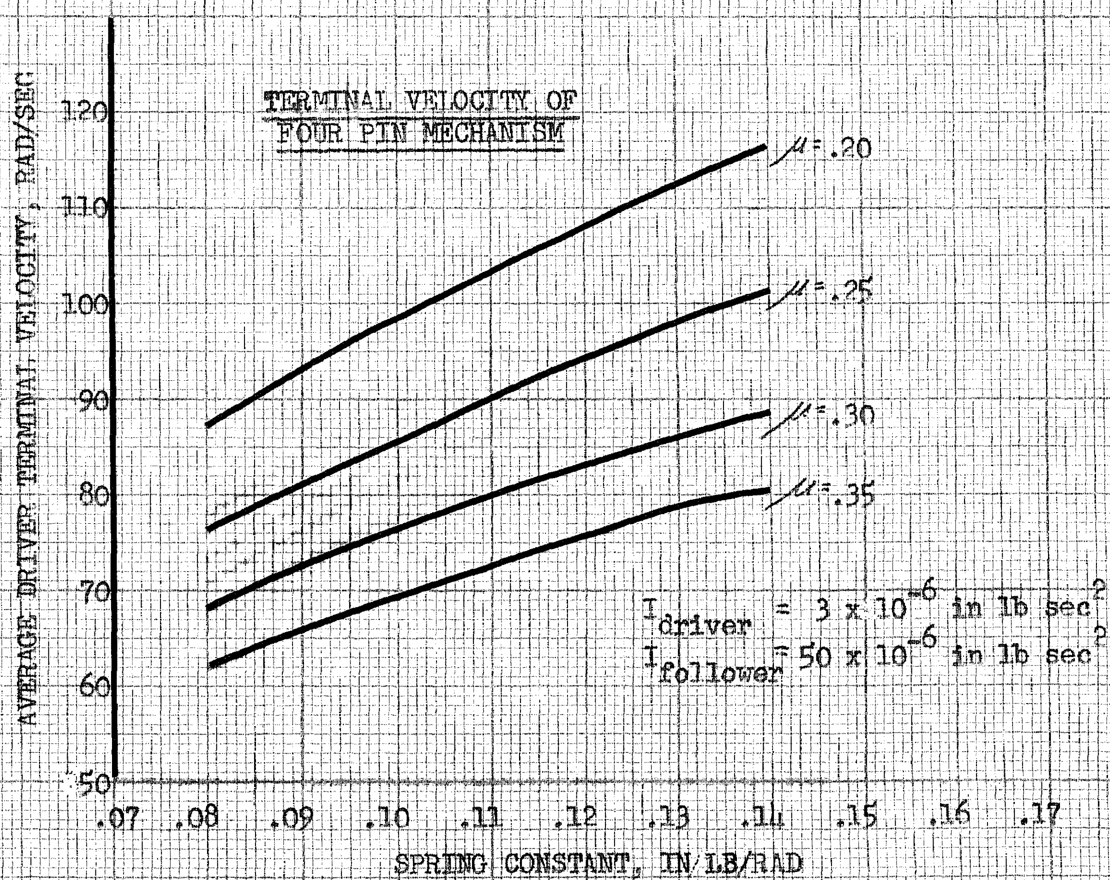
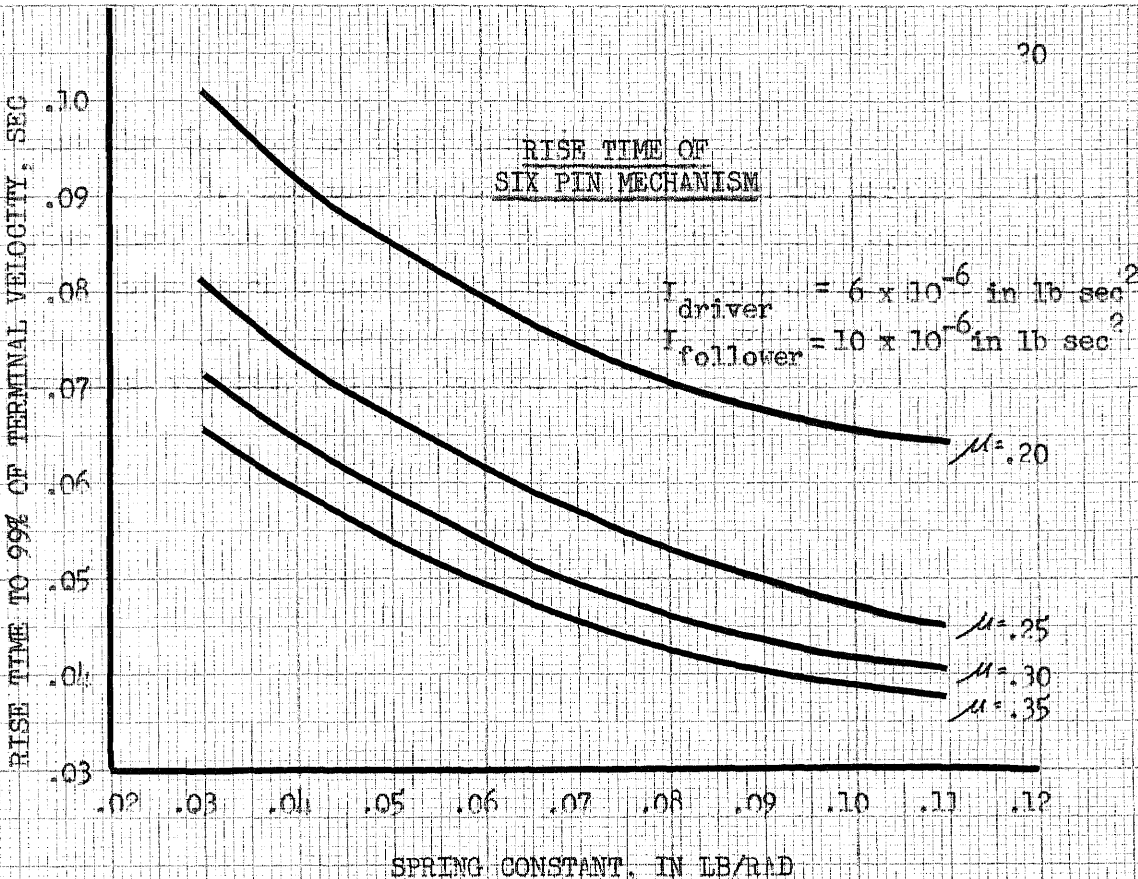

TERMINAL VELOCITY OF
FOUR PIN MECHANISM
AVERAGE DRIVER TERMINAL VELOCITY, RAD/SEC
μ = .20
μ = .25
μ = .30
μ = .35
I driver = 3 x 10^-6 in lb sec^2
I follower = 50 x 10^-6 in lb sec^2
50
60
70
80
90
100
110
120
.07 .08 .09 .10 .11 .12 .13 .14 .15 .16 .17
SPRING CONSTANT, IN LB/RAD

FIGURE 10


RISE TIME OF
SIX PIN MECHANISM
I driver = 6 x 10^-6 in lb sec^2
I follower = 10 x 10^-6 in lb sec^2
μ = .20
μ = .25
μ = .30
μ = .35
RISE TIME TO 99% OF TERMINAL VELOCITY, SEC
.10
.09
.08
.07
.06
.05
.04
.03
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
SPRING CONSTANT, IN LB/RAD

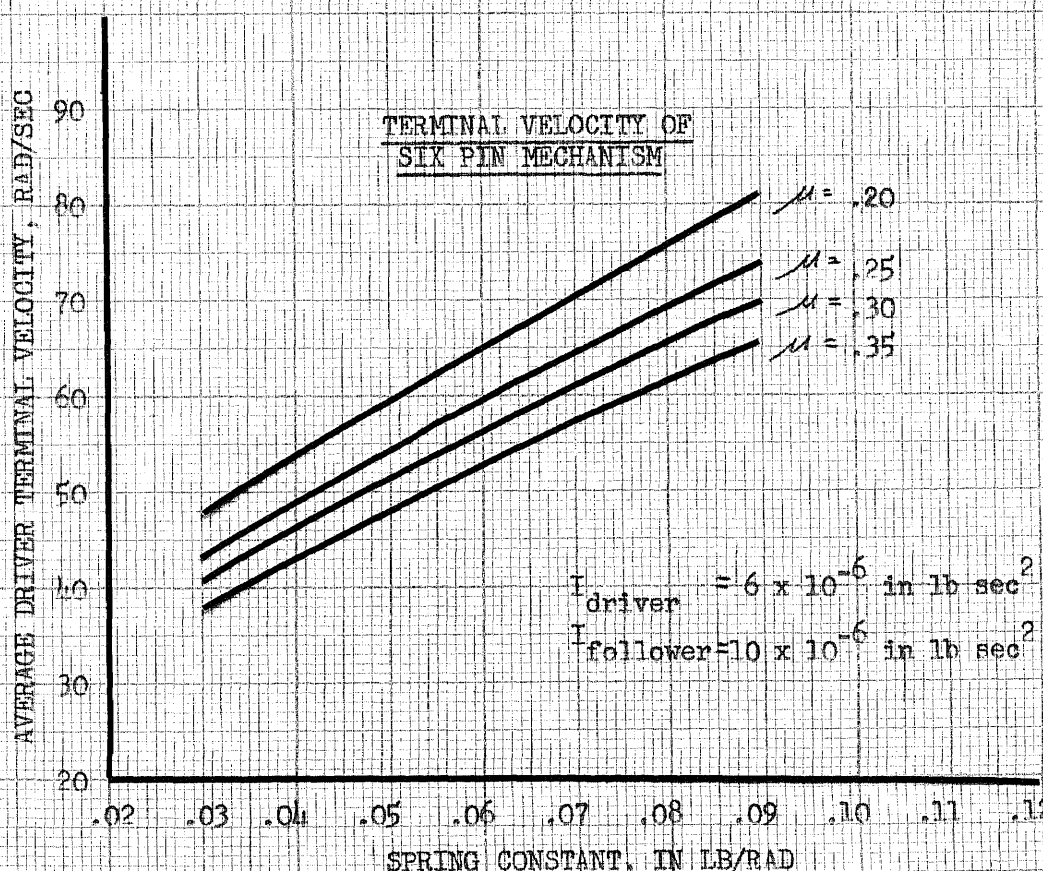

TERMINAL VELOCITY OF
SIX PIN MECHANISM
μ = .20
μ = .25
μ = .30
μ = .35
I driver = 6 x 10^-6 in lb sec^2
I follower = 10 x 10^-6 in lb sec^2
AVERAGE DRIVER TERMINAL VELOCITY, RAD/SEC
90
80
70
60
50
40
30
20
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
SPRING CONSTANT, IN LB/RAD

FIGURE 11

rise time (due to the finite inertia of the system).

2. The more frictionless the system, the greater is the rise time. The limiting case would be a frictionless system, which will never reach a terminal velocity (the velocity will go to infinity).

3. An almost linear relationship exists between the terminal velocity and the driving torque.

CONCLUSIONS

Three important conclusions can be drawn as a result of this study:

1. The Geneva mechanism can be used as a timing device.

2. Small variations in the parameters of the system will not seriously affect the motion.

3. Only three geometrical configurations will work in a timing application: the three pin / six slot mechanism; the four pin / four slot mechanism; and the six pin / three slot mechanism.

RECOMMENDATIONS

Since the Geneva mechanism will work as a timing device, further study is warranted. The following areas should be addressed in any subsequent work.

1. The rise times of the mechanisms examined were not fast enough to allow use of the Geneva mechanism as a short period (i.e. milliseconds) timer. Extensive work is required to optimize the masses to achieve smaller rise times.

2. Excessively high spring rates cause significant cyclic oscillations in the magnitude of the peak velocity of the driver. This phenomenon should be investigated.

3. Finite pin and slot dimensions should be included in the analysis. This would be included in the moment equation used in the derivation of equation 1-14, the friction torque. Also, bearing friction should be included (it is not necessarily constant).

4. A quantitative assessment of the mechanism's accuracy should be made.

5. Angular accelerations of the members seemed high. Any further work should include an investigation of these accelerations and how they might affect the mechanism's accuracy.

6. A prototype should be built and tested to verify the analytical results.

7. In any further studies, the average terminal velocity should redefined (see page 17 for the definition as used here). A more appropriate definition would be based on the time required for the mechanism to complete one (or a part of one) revolution.

APPENDIX A

SCALE DRAWINGS

In order to construct scale drawings of the three possible configurations, the following computations were necessary.

Let R= 1.000 for simplicity.

m = 4; n = 4

$$\theta_o = \frac{\pi}{m} \quad \frac{\pi}{4} = 45^\circ \qquad c = \frac{r}{\sin(\pi/n)} = \frac{1.}{\sin(\pi/4)} = 1.414$$

$$\phi_o = \frac{\pi}{n} \quad \frac{\pi}{4} = 45^\circ \qquad R = \frac{r \sin \theta_o}{\sin \phi_o} = \frac{1 \sin 45^\circ}{\sin 45^\circ} = 1.000$$

m = 6; n = 3

$$\theta_o = \frac{\pi}{m} = \frac{\pi}{6} = 30^\circ \qquad c = \frac{r}{\sin(\pi/n)} = \frac{1.}{\sin(\pi/3)} = 1.156$$

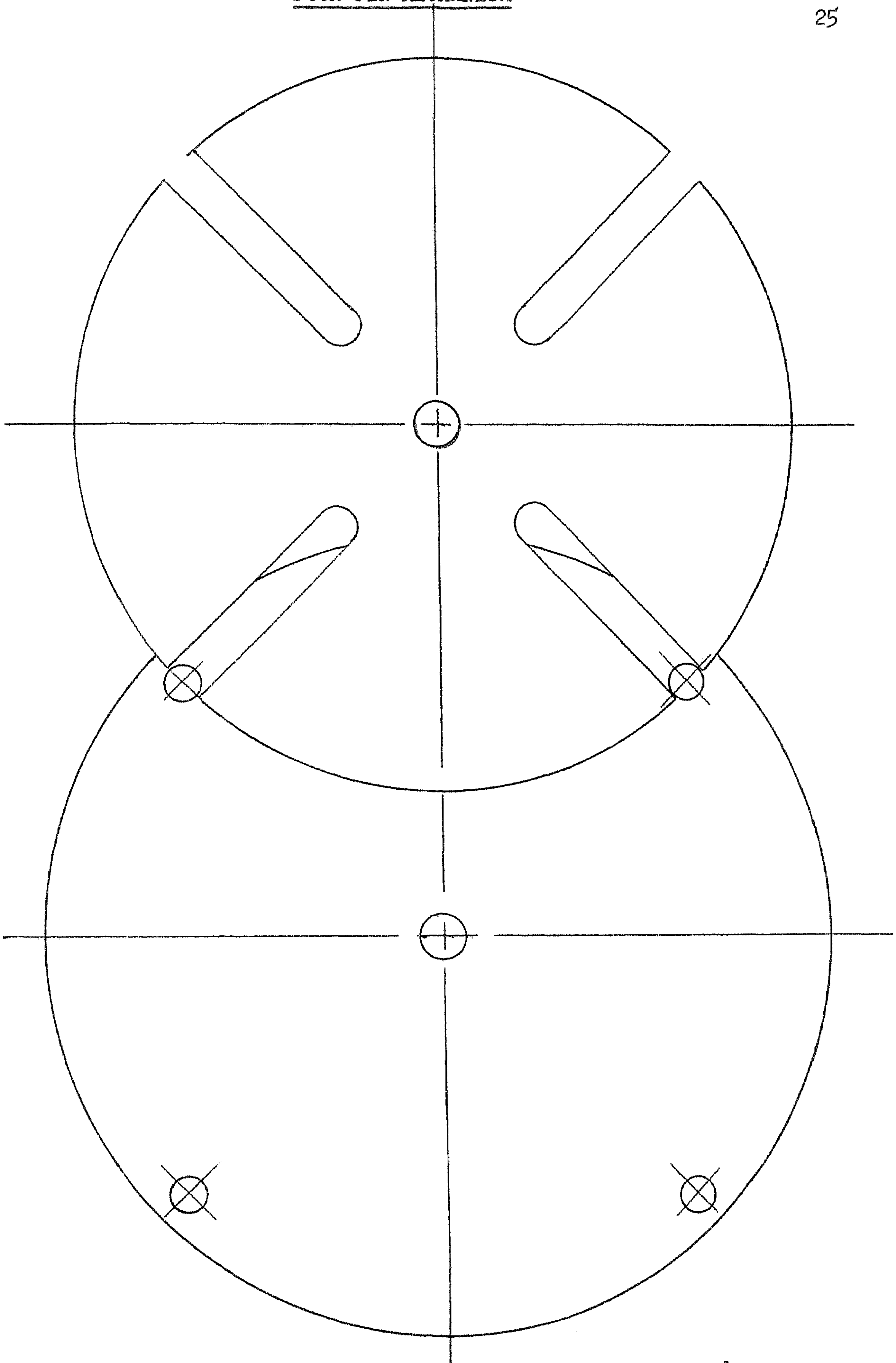
$$\phi_o = \frac{\pi}{n} = \frac{\pi}{3} = 60^\circ \qquad R = \frac{r \sin \theta_o}{\sin \phi_o} = \frac{1 \sin 30^\circ}{\sin 60^\circ} = 0.557$$

m = 3; n = 6

$$\theta_o = \frac{\pi}{m} = \frac{\pi}{3} = 60^\circ \qquad c = \frac{r}{\sin(\pi/n)} = \frac{1.}{\sin(\pi/6)} = 2.000$$

$$\phi_o = \frac{\pi}{n} = \frac{\pi}{6} = 30^\circ \qquad R = \frac{r \sin \theta_o}{\sin \phi_o} = \frac{1 \sin 60^\circ}{\sin 30^\circ} = 1.732$$

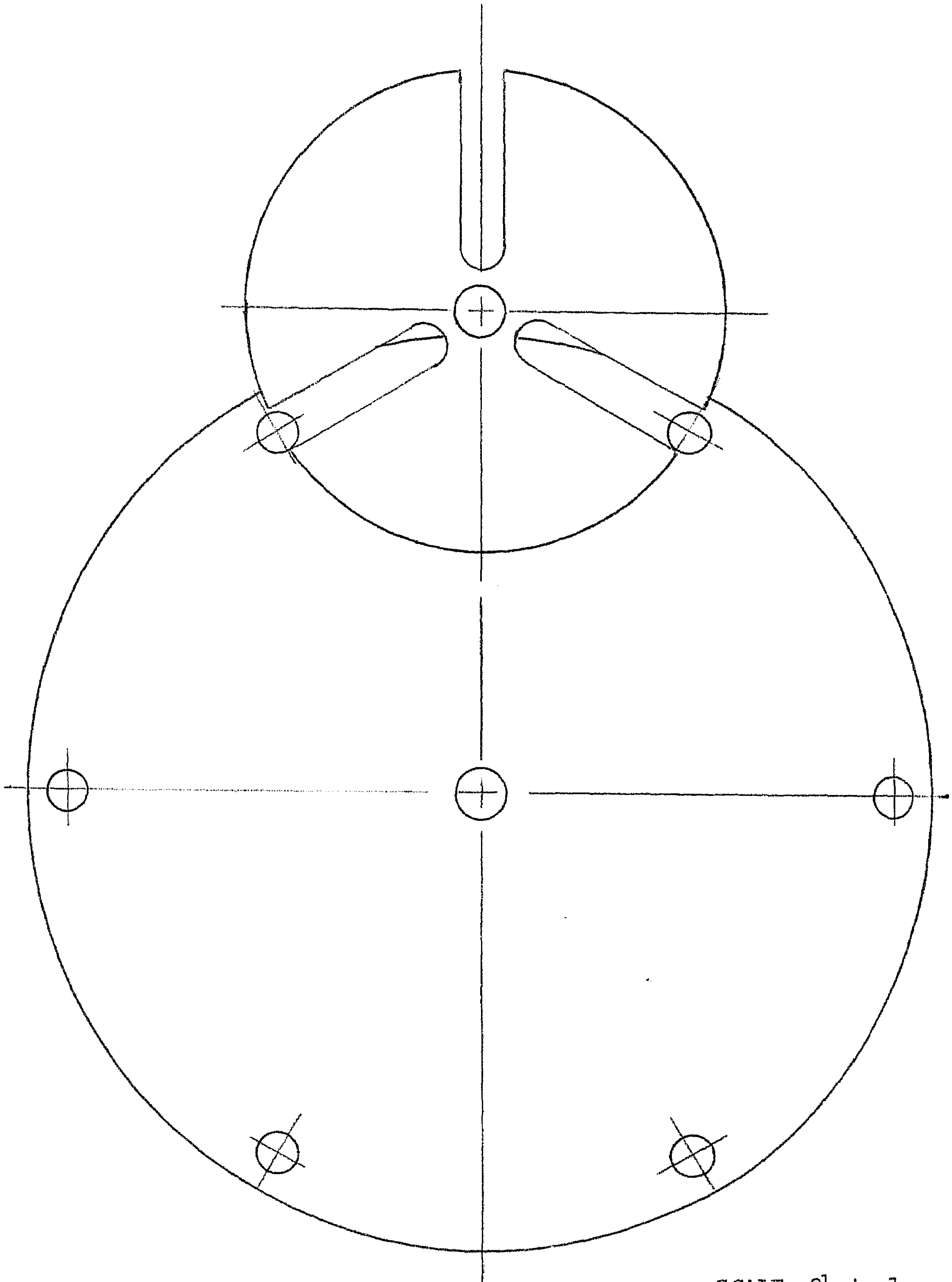
FOUR PIN MECHANISM



SCALE: $2\frac{1}{2}$ to 1

FIGURE 12

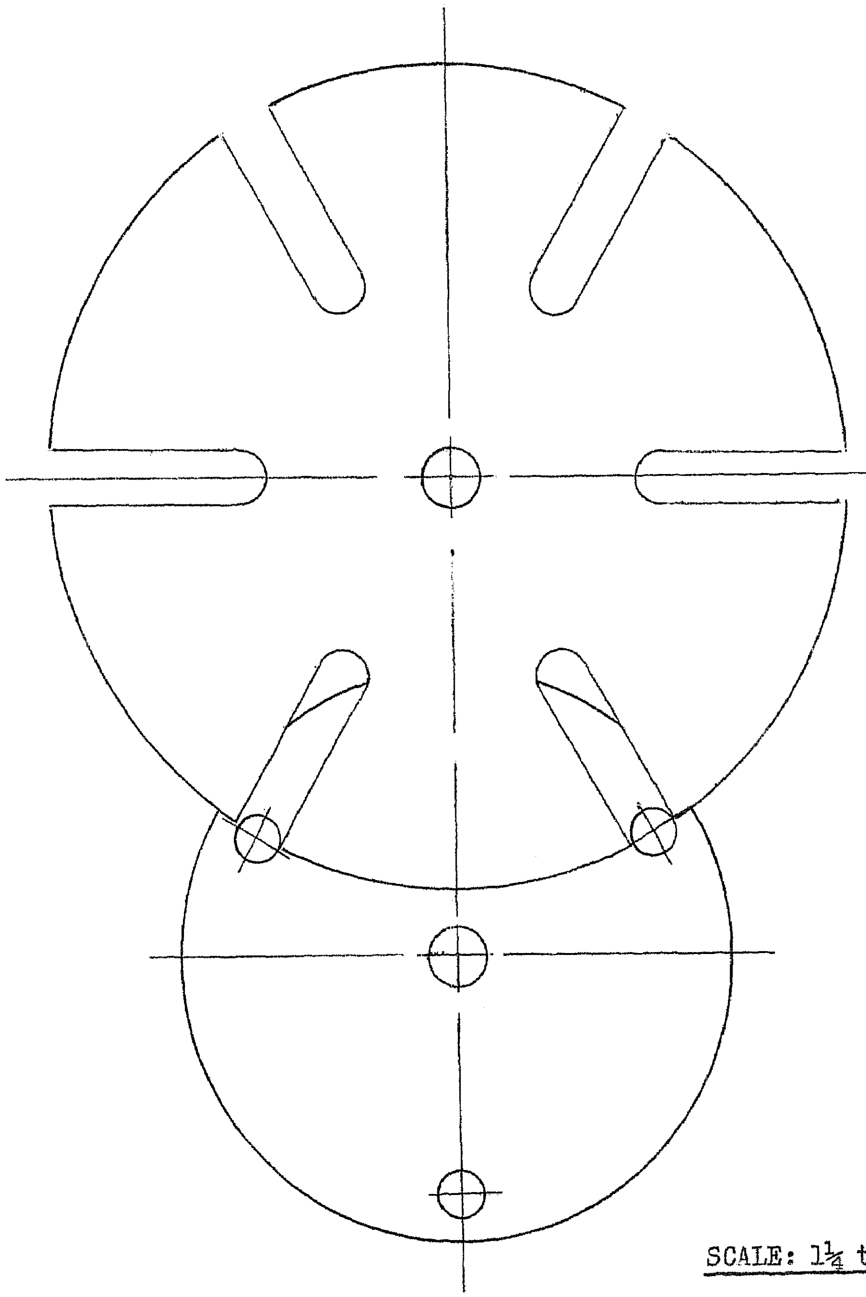
SIX PIN MECHANISM



SCALE: $2\frac{1}{2}$ to 1

FIGURE 13

THREE PIN MECHANISM



SCALE: 1¼ to 1

FIGURE 14

APPENDIX B

COMPUTER PROGRAM

The equation of motion was solved on a CDC 6500 digital computer. The program utilized is called MIMIC, a program used to solve systems of ordinary differential equations. MIMIC is simply a series of FORTRAN subroutines which have been designed to aid the user in the solution of his problem. To demonstrate the use of MIMIC, consider the following differential equation:

$$m\ddot{x} + c\dot{x} + kx = 0$$

The first step in the solution is to solve the equation for the highest order derivative,

$$\ddot{x} = -(c\dot{x} + kx)/m$$

The mathematical portion of the program would then be

```
2DX = -(C*1DX + K*X)/M
1DX = INT(2DX, A)
  X = INT(1DX, B)
```

In these equations, 1DX and 2DX represent the first and second derivatives respectively, and A and B are the initial ($t = 0$) values of $\dot{x}$ and x respectively. INT is the command to the computer to use the integration subroutine.

In solving these equations, the computer will use the values of A and B to calculate the initial value of 2DX. Then with the integration subroutine, this value is used to calculate the second ($t = 0+\delta t$) value of 1DX, which is in turn used to calculate the second value of x. These steps keep repeating until the computer is told to stop. The error incurred by the use of the value of 2DX at t_n to compute the values of 1DX and x at t_{n+1} can be made negligible by proper choice of the increment of t, which is automatically taken care of in the MIMIC program.

The heart of the MIMIC program is the integration subroutine. This subroutine makes use of a fourth order, variable step, Runge-

Kutta method. The equation to be integrated is of the form

$$y'(x) = f(x,y)$$

In this method, the function to be integrated is divided into a finite number of intervals (x_n, x_{n+1}) of width h. Each interval is in turn divided into four subintervals of widths h/6, h/3, h/3, and h/6. The integral of f(x,y) over the whole interval h is computed as the sum of the integrals over the four subintervals. the function f(x,y) is taken to be constant over each of the subintervals.

To solve the equation

$$y'(x) = f(x,y)$$

given that $y = y_0$ at $x = x_0$; $y = y_n$ at $x = x_n$; $y = y_{n+1}$ at $x = (x_n+h)$, the following equation can be solved.

$$y_{n+1} = y_n + \frac{h}{6}\ (f_1 + 2f_2 + 2f_3 + f_4)$$

where

$$f_1 = f(x_n; y_n)$$

$$f_2 = f(x_n + \frac{h}{2};\ y_n + \frac{f_1}{2})$$

$$f_3 = f(x_n + \frac{h}{2};\ y_n + \frac{f_2}{2})$$

$$f_4 = f(x_n + h;\ y_n + f_3)$$

There are two main advantages to this approach. First, it requires only an initial point to start the integration. Knowing the values of y_0 and x_0, y_1 may be computed, and the integral over the subinterval computed. Knowing y_1 and $x_1 = x_0 + h$, y_2 may be computed, and so on. The other advantage of the Runge - Kutta method is small truncation errors, of the order of h^5. The step size h is automatically adjusted by the integration subroutine to keep the relative error (determined by comparing the values of y_{n+1} obtained by computing first with two half steps and then with one full step) at less than 5×10^{-6}.

In order to solve equation 1-19 with MIMIC, it must first be solved for the highest order derivative. In its present form,

$$A\ddot{\theta} + B\dot{\theta}^2 + |C\dot{\theta}^2 + D\ddot{\theta}| = E \qquad (1\text{-}19)$$

the equation cannot be solved for $\ddot{\theta}$. To eliminate the problem, let

$$F = (C\dot{\theta}^2 + D\ddot{\theta})$$

Then define S such that

$$S = +1 \qquad \text{if} \quad F \geq 0$$

$$S = -1 \qquad \text{if} \quad F < 0$$

Equation 1-19 then becomes

$$A\ddot{\theta} + B\dot{\theta}^2 + S(C\dot{\theta}^2 + D\ddot{\theta}) = E$$

$$A\ddot{\theta} + B\dot{\theta}^2 + SC\dot{\theta}^2 + SD\ddot{\theta} = E$$

$$\ddot{\theta}(A+SD) + \dot{\theta}^2(B+SC) = E$$

$$\ddot{\theta} = \frac{E-(B+SC)\ \dot{\theta}^2}{A+SD}$$

Which is of the form required by the computer.

A printout of the actual program used is given below. The first seven cards are control cards. The PAR card lists the parameters which will be found on data cards at the end of the program. Any number of values for the parameters can be read in, and the program will run separately for each individual set of parameters. In this way, the effect of varying a parameter (spring constant, coefficient of friction, etc) can be observed. The next card, DT, simply sets the time increment between printout data points.

G,H,I,J, and L are equations for the factors of equation 1-18. A,B,C,D, and E represent the factors of equation 1-19. F is as defined previously in this appendix, except that in place of the second derivative (2DX), the symbol OLD2DX is used. The second derivative cannot be used to compute F, since the second derivative itself depends on F (the computer cannot solve simultaneous equations). To circumvent this problem, the equation for OLD2DX tells the computer to use the previous value of 2DX (i.e. the value at t-DT/10) to compute the current value of F. Knowing F, the computer can then assign a value for S according to the above criteria.

SMT ("small time") is a logical control variable which is defined as true when T (time) is less than or equal to zero, and false when T is greater than zero. Thus the computer will see only those statements controlled by SMT when T equals zero (it is never negative in this problem). The SMT statements, then, are used to compute the

MIMIC COMPUTER PROGRAM

```
C           N=NUMBER OF SLOTS
C           U=COEFFICIENT OF FRICTION
C           K=SPRING RATE
C           IW=MOMENT OF INERTIA OF FOLLOWER
C           IC=MOMENT OF INERTIA OF DRIVER
                         PAR(N,X0,U,K,IW,IC)
            DT           .0005
            G            1./SIN(3.1416/N)
            H            (G*COS(X))-1.
            I            G*SIN(X)
            J            1.-(G*G)
            L            1.+(G*G)-(2.*G*COS(X))
            A            IC/IW+((H*H)/(L*L))
            B            (I*J*H)/(L*L*L)
            C            (I*I*U*J)/(L*L*L)
            D            (I*U*H)/(L*L)
            E            K/IW
            F            (C*1DX*1DX)+(D*OLD2DX)
            S            FSW(F,-1.,1.,1.)
            OLD2DX       TDL(2DX,DT/10.,100.)
            SMT          FSW(T,TRUE,TRUE,FALSE)
      SMT   P            1.+(G*G)-(2.*G*COS(X0))
      SMT   Q            (G*COS(X0))-1.
      SMT   R            (IC/IW)+((Q*Q)/(P*P))
      SMT   V            ABS((U*G*Q*SIN(X0))/(P*P))
      SMT   2DX          E/(R+V)
            LGT          NOT(SMT)
      LGT   2DX          (E-(B+(S*C))*(1DX*1DX))/(A+(S*D))
            1DX          INT(2DX,0.)
            THETA        INT(1DX,0.)
            THDEG        57.296*THETA
            X            (MOD(THETA,1.5708))+X0
            XDEG         57.296*X
            PHIDOT       (H*1DX)/L
                         FIN(T,.2)
                         HDR(TIME,DISPL,VEL,ACCEL,PHIDOT)
                         OUT(T,THETA,1DX,2DX,PHIDOT)
                         PLO(T,1DX)
                         SCA(.002,2.5)
                         ZER(0.,0.,)
                         PLO(T,PHIDOT)
                         SCA(.002,2.5)
                         ZER(0.,0.)
                         END
```

initial value of the second derivative, 2DX. These statements are simply equation 1-18 with the initial value of θ ($-30°$, $-45°$, or $-60°$) and $\dot{\theta}$ (0 rad/sec) substituted, and solved for $\ddot{\theta}$ (see Appendix D).

The equation for 2DX controlled by LGT ("large time"-this simply shuts off the equation at $t = 0$) is the primary equation in this program. It is identical to the equation derived in this appendix. 1DX, the angular velocity of the driver, is the integral of 2DX, with a value of zero at $t = 0$.

THETA, the angular displacement of the driver, is equal to the integral of 1DX. It will start from zero and increase for as long as the program runs. However, the equation of motion was written only for that part of the motion where the pin engages the slot ($\pm 30°$, $\pm 45°$, or $\pm 60°$). X is therefore the true variable which must be used in the equations H through L, since it will vary only between those points where the pin engages the slot (literally, x is the modulo of THETA and $90°$, minus either $30°$, $45°$, or $60°$). This method is valid because the problem can be considered to be a series of problems of changing initial conditions. When each pin in its turn enters a slot, it takes for initial values of velocity and acceleration the computed values from the preceding pin (which is simultaneously leaving its slot). The continuous motion of the mechanism is therefore a series of discrete, single pin engagements.

Continuing, PHIDOT is the equation for the velocity of the follower (equation 1-7). The FIN statement shuts off the program after the motion has proceeded for 0.2 seconds. The remainder of the cards are for the output commands (print and plot statements).

APPENDIX C

CONSTANT TORQUE SPRING

The neg'ator spring is a constant force spring. It consists of a long strip of spring steel (or other spring material) which is wound onto a spool, much as the steel measuring tape used by carpenters. A constant force is required to unwind the strip from the spool; that is, a constant linear pull which causes the spool to rotate on its axis. Conversely, a spring which is unwound from its spool will tend to wind itself up on the spool, just as any spring tends to return to its equilibrium position. Therefore, if the free end of the spring is wound around a second spool, the spring will exert an almost constant torque about the second spool as it rewinds itself about the original spool. This device is called the Neg'ator B Motor, and can develop as many as fifty turns of output. A schematic drawing is shown below.

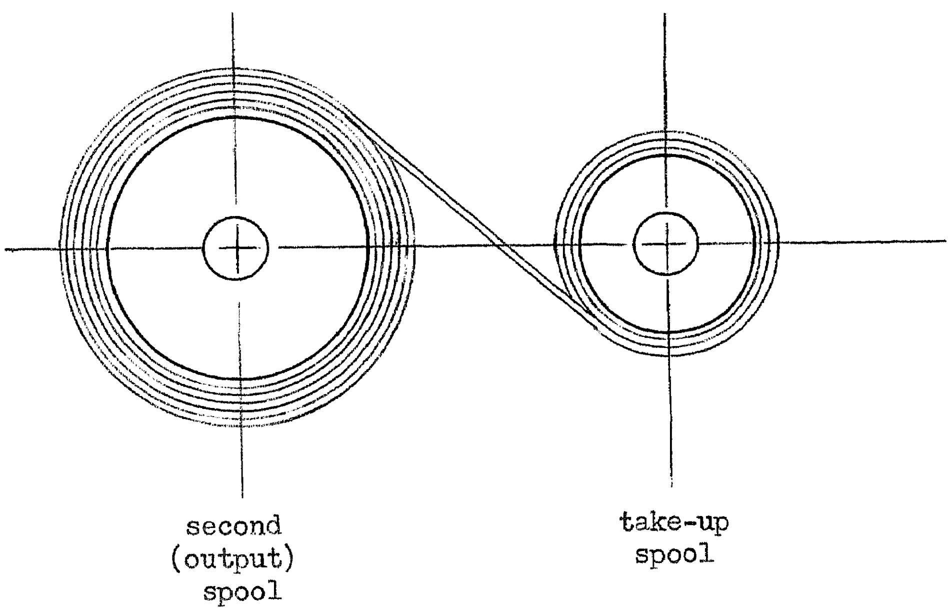


Figure 15. Neg'ator B Motor.

APPENDIX D

COMPUTATION OF ACCELERATION AT T=0

The computer requires a value for the second derivative at $t=0$ in order to initiate the S/2DX loop as described in Appendix B. θ and $\dot{\theta}$ are known at $t=0$:

$$\dot{\theta}_o = 0 \qquad @\ t=0$$

$$\theta_o = -30^\circ, -45^\circ, -60^\circ \qquad @\ t=0$$

θ_o depends on the configuration of the mechanism being analyzed (i.e. three, four, or six pins).

Equation 1-19, with $\dot{\theta}=0$, is

$$A\ddot{\theta} + |D\ddot{\theta}| = E$$

Let R be the value of A at $t=0$, and V be the value of D at $t=0$. E remains the same (i.e. it is a constant of the motion). Then the equation of motion at $t=0$ is

$$R\ddot{\theta} + |V\ddot{\theta}| = E$$

Now $\ddot{\theta}_o$ (acceleration at $t=0$ only) must always be positive, since the mechanism starts from rest and must always begin its motion in the direction of the positive torque of the driving spring. Therefore, the equation of motion for $t=0$ can be written:

$$R\ddot{\theta} + |V|\ddot{\theta} = E$$

$$\ddot{\theta}_o = \frac{E}{R+|V|}$$

which is of the form to be used in the computer program. Note that R is always positive, and thus the requirement that $\ddot{\theta}_o$ be in the direction of the torque K is satisfied.

APPENDIX E

SUMMARY OF COMPUTER RUNS

Six Pin Mechanism ($I_{driver} = 10 \times 10^{-6}$ in lb sec^2 ;
$I_{follower} = 6 \times 10^{-6}$ in lb sec^2)

μ	K in lb/rad	$\dot{\theta}_{max}$ rad/sec	$\dot{\theta}_{min}$ rad/sec	Rise Time sec	Period sec
.20	.03	83	10	.101	.046
.20	.05	107	13	.078	.036
.20	.07	127	15	.074	.030
.20	.09	144	17	.051	.027
.25	.03	76	9	.092	.050
.25	.05	98	12	.084	.039
.25	.07	116	14	.061	.033
.25	.09	130	16	.047	.029
.30	.03	71	8	.073	.054
.30	.05	91	11	.057	.043
.30	.07	108	13	.036	.036
.30	.09	123	14	.034	.031
.35	.03	67	8	.070	.058
.35	.05	86	10	.046	.046
.35	.07	102	12	.047	.039
.35	.09	116	14	.044	.034

Four Pin Mechanism ($I_{driver} = 3 \times 10^{-6}$ in lb sec^2 ;
$I_{follower} = 50 \times 10^{-6}$ in lb sec^2)

μ	K	$\dot{\theta}_{max}$	$\dot{\theta}_{min}$	Rise Time	Period
	in lb/rad	rad/sec	rad/sec	sec	sec
.20	.08	155	19	.056	.040
.20	.10	174	21	.050	.035
.20	.12	191	23	.046	.032
.20	.14	206	25	.034	.030
.25	.08	134	17	.037	.043
.25	.10	150	19	.033	.039
.25	.12	164	21	.030	.035
.25	.14	178	23	.027	.033
.30	.08	118	17	.037	.047
.30	.10	128	18	.030	.042
.30	.12	144	20	.029	.038
.30	.14	154	22	.027	.035
.35	.09	111	17	.028	.046
.35	.10	117	18	.026	.044
.35	.11	123	19	.024	.042
.35	.12	129	20	.023	.040
.35	.20	163	25	.028	.031

Three Pin Mechanism ($I_{driver} = 3 \times 10^{-6}$ in lb sec^2 ;

$I_{follower} = 40 \times 10^{-6}$ in lb sec^2)

μ	K in lb/rad	$\dot{\theta}_{max}$ rad/sec	$\dot{\theta}_{min}$ rad/sec	Rise Time sec	Period sec
.20	.08	144	61	.044	.018
.20	.10	161	68	.039	.017
.20	.12	176	74	.036	.015
.20	.14	191	80	.030	.014
.25	.08	140	58	.036	.020
.25	.10	156	65	.036	.018
.25	.12	171	71	.029	.015
.25	.14	184	77	.028	.015
.30	.08	136	56	.037	.020
.30	.10	152	63	.033	.018
.30	.12	167	69	.030	.016
.30	.14	180	74	.028	.015
.35	.08	133	54	.038	.021
.35	.10	148	61	.038	.018
.35	.12	163	67	.031	.017
.35	.14	176	72	.028	.015

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