SEPARATIONS VIA SEMICONTINUOUS PARAMETRIC PUMPING

BY

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A THESIS
PRESENTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE
OF
MASTER OF SCIENCE IN CHEMICAL ENGINEERING
AT
NEWARK COLLEGE OF ENGINEERING

Newark, New Jersey
1972

ABSTRACT

A semicontinuous parametric pump with batch operation during one half-cycle and continuous operation in the other half-cycle was experimentally investigated in the model system, toluene-n-heptane on silica gel adsorbent. Comparison was made with analytical results obtained by the equilibrium theory. It was shown that when the penetration distance for the cold cycle was less than or equal to that of the hot cycle and the height of the column, the rate of production of pure solvent in the semicontinuous pump may become quite large relative to that in the continuous pump.

APPROVAL OF THESIS

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FOR
DEPARTMENT OF CHEMICAL ENGINEERING
NEWARK COLLEGE OF ENGINEERING

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APPROVED: ______________________

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NEWARK, NEW JERSEY
JUNE, 1972

ACKNOWLEDGMENTS

I wish to thank Dr. H.T. Chen for the many hours of help as my thesis advisor; and to thank Dr. E.N. Bart and Dr. J.E. McCormick for their reviews of this thesis.

Parts of this thesis were presented at the 73d A.I.Ch.E. National Meeting.

TABLE OF CONTENTS

FIGURE CAPTIONS

FIGURE 1: Internal Flow Rates in Semicontinuous Parametric Pump with Top Feed.

FIGURE 2: Effects of ϕ_B on Product Concentration for Semicontinuous System.

FIGURE 3: Comparison of Semicontinuous vs. Continuous System.

FIGURE 4: Steady-State Concentration Profile.

FIGURE 5: Product Removal per ϕ_B.

FIGURE 6: Parametric Pump Models: (a) batch; (b) continuous with top feed; (c) continuous with bottom feed.

FIGURE 7: Internal Flow Rates in Continuous Parametric Pump with Top Feed.

FIGURE 8: Regions of Parametric Pump Operation

FIGURE 9: Schematic Diagram of Experimental Apparatus.

TABLE CAPTIONS

SCOPE

Pigford, Baker, and Blum (1) developed the equilibrium theory and, based on this theory, derived mathematical expressions for the performance of Wilhelm's batch parametric pump* (2). Two primary assumptions of the theory were those of instantaneous local equilibrium throughout the adsorption column and absence of axial diffusion. The expressions predicted that the batch pump had the unusual capability of separating a two-component mixture into one fraction completely depleted in solute and another fraction enriched in solute by a factor of at most two or three. The data of Wilhelm and Sweed (2) on the removal of toluene from toluene-n-heptane mixtures, in which separation factors as great as 10^5 were readily attained, indicated that the theoretical limits which the Pigford expressions represented might be closely approached in practice. Later, Aris (3) showed that the theory of Pigford et al. (1) was a special case of a more general theory and he derived the general theory.

By extending the equilibrium theory, Chen and Hill (4) derived mathematical expressions for the performance of batch, semicontinuous and continuous parametric pumps.

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*See Appendix 1

They have shown that under certain conditions the batch pump and the continuous pump with feed at the enriched end had the capacity for complete removal of solute from one product stream and at the same time, gave arbitrarily large enrichment of solute in the other product stream. Subsequently, Chen, Hill, Rak, and Stokes (5) verified the models and analytical solutions for the continuous pump in the model system toluene-n-heptane on silica gel adsorbent. The experimental values compared reasonably well with the calculated results.

In a parallel approach based on the equilibrium theory Gregory and Sweed (6) also presented analytical solutions for two types of continuous open systems, a non-symmetric pump involving five distinct flow patterns per cycle and a symmetric pump involving four distinct flow patterns per cycle. Their equations omitted the effect of reservoir dead volume which induces a lag in the transient region. However, for some systems, approach to steady state is very slow, and in this transient range of operation, reservoir dead volume must be taken into account if the mathematical expressions are to accurately predict the separation. Gregory and Sweed pointed out that the ability to achieve infinite separation factors depended on the relative magnitude of the bottom reflux ratio rate and the equilibrium constants, while Chen et al. (5) confirmed experimentally that the conditions necessary to achieve high separation factors were functions of the relative magnitudes of the penetration distances and the height of the column.

In this thesis a semicontinuous pump with top feed was experimentally investigated in the model system, toluene-n-heptane on silica gel adsorbent. This semicontinuous form of operation was characterized by batch operation during downflow (Figure 1). A comparison was made between the experimental results and the predictions of the mathematical expressions based on the equilibrium theory. Emphasis was placed on the operating conditions required to obtain higher production rates of pure solvent for the semicontinuous pump as compared to the continuous pump.

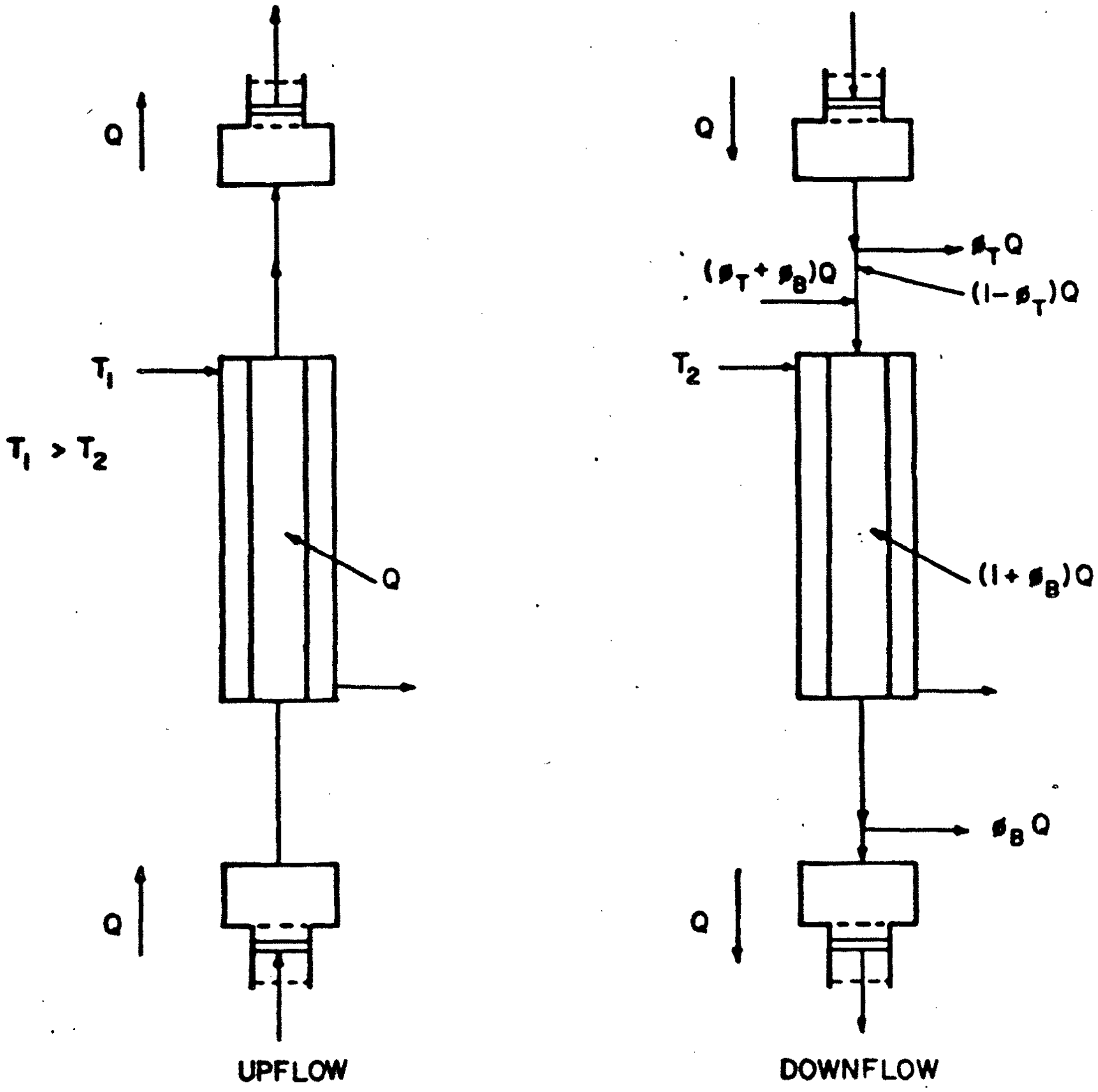

FIG. I

Internal Flow Rates in Semicontinuous Parametric Pump with Top Feed.

CONCLUSIONS AND SIGNIFICANCE

For the continuous and semicontinuous pumps with feed at the top, there were three possible regions of operations (Regions 1, 2, and 3)[*], depending on the relative magnitudes of the penetration distances L_1 and L_2 and the height of the column, h. In a previous paper Chen et al. (5) showed the concentration transients for the continuous pump. Presented here is the mathematical model of concentration transients which corresponded to all three regions for the semicontinuous pump. This pump had unequal reservoir volumes. The feed and product streams flowed steadily during one half of a cycle only and the streams were shut off during the other half of the cycle.

This semicontinuous pump was experimentally investigated in the model system, toluene-n-heptane on silica gel adsorpent. It was shown both theorectically and experimentally that the performance characteristics of both continuous and semicontinuous pumps were similar in nature. Infinite steady-state separation factors were found in Region 1 for both pumps. The main difference between the two pumps was in the switching points (the boundaries between infinite and finite separation) resulting from bottom product flow rate

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[*] See Appendix 2

variation. For the continuous pump these points corresponded to the condition $\phi_B = b^*$, whereas in the case of the semi-continuous pump the condition was $\phi_B = 2b/(1-b)$. This was highly significant because it meant that higher production rates of pure solvent were possible for the semi-continuous pump than for the continuous pump.

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[*] See Appendix 2

EXPERIMENTAL

The apparatus used here was identical to that used by Chen et al.* (5) for the continuous pump. Prior to each run the entire system, including the interstitial column volume, the bottom reservoir, and the feed pump were filled with the feed mixture (10 mole % of toluene in n-heptane) at ambient temperature. At the beginning of each run the feed pump was shut off and the product take-off valves were closed. The reservoir pumps were started and the timer was activated. The bottom reservoir syringe pumped fluid into the bottom of the column and the timer switched the solenoids to supply hot water (343°K) to the jacket. At the end of the hot upflow half-cycle, the microswitch on the pump automatically reversed the action of the reservoir syringes and the timer switched the solenoids to supply cold water (277°) to the jacket. Simultaneously, the feed pump was activated and the product take-off valves were opened and adjusted for the desired product flow rates. This procedure was repeated for each cycle. The time per cycle used for this study was 2400 seconds, that is, 1200 seconds of upflow followed by 1200seconds of downflow. The product streams were analyzed by ultraviolet spectrophotometry.

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*See Appendix 3

MATHEMATICAL MODELS

For the semicontinuous pump, Chen and Hill (4) showed that there were three possible regions of operation (Regions 1, 2, and3) depending on the relative magnitude of the penetration distances L_1 and L_2 and the height of the column, h. As shown in Table 2, the downflow penetration distance, L_2, was the same as for the continuous pump because the feed and product streams were continuous during this half-cycle. During the upflow half-cycle, however, the semicontinuous pump operated batch-wise and the penetration distance L_1 was the same as that for the batch pump.

Chen and Hill (4) discussed the internal and external equations and the steady-state solutions for the semicontinuous pump. The system of internal and external equations were solved by using the method of characteristics. The concentration transients were obtained corresponding to those in Regions 1, 2, and 3. The detailed derivations are available elsewhere (7).

The assumptions made were the same as that of Pigford et al.(1). Local interface equilibrium existed with a linear distributation law having a temperature-dependent distribution coefficient, and there was negligible axial diffusion. In addition, interest was restricted to the situation in which the top product stream during the downflow came only from the top reservoir and not from the column nor from the feed stream, $(0 < \phi_T < 1)$.

The basic equations for the concentration transients for Regions 1 and 3 were identical to those for the continuous pump (5), when appropriate definitions of L_1 and L_2 were used. These equations will not be repeated here. The concentration transients for Region 2 are presented below. It should be noted, that at steady-state, $(n \rightarrow \infty)$, solute was removed completely from the lower stream in Region 1, but was only removed partially in Regions 2 and 3. The expressions for Region 2 are:

$$\frac{\langle y_{TP2}\rangle_n}{y_o} = -\left(\frac{\phi_T+\phi_B}{1-\phi_T}\right) + \left(\frac{1+\phi_B}{1-\phi_T}\right)\left[\frac{C_1+\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}{1+C_1}\right]^{n-1} + \left[\frac{1}{1-\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}\right]\left(\frac{\phi_T+\phi_B}{1-\phi_T}\right)\left\{1-\left[\frac{C_1+\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}{1+C_1}\right]^{n-1}\right\} \tag{1}$$

$$\frac{\langle y_{B2}\rangle_n}{y_o} = (1+C_2)\,\frac{\langle y_{B1}\rangle_{n-1}}{y_o} - C_2\,\frac{\langle y_{B1}\rangle_n}{y_o}$$

where

$$\frac{\langle y_{B1}\rangle_n}{y_o} = \left[\frac{C_2+\alpha_2}{1+C_2}\right]^{n-1} + \frac{(1-b)-\alpha_2(1+b)}{(1-\alpha_2)(1+b)}\left[1-\left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-1}\right]$$

$$\text{if } n \leq p_2 + 1 \tag{2}$$

$$\frac{\langle y_{B1}\rangle_n}{y_o} = \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-1} + \frac{(1-b) - \alpha_2(1+b)}{(1-\alpha_2)(1+b)} \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-p_2-1}$$

$$\left[1 - \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{p}\right] + \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-p_2-2}\left(\frac{1}{1+C_2}\right)$$

$$\left[\left(\frac{1-b}{1+b}\right)\left(\frac{GI}{EF}\right) + \left(\frac{IF}{EF}\right)\right] + \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-p_2-3} \left(\frac{1}{1+C_2}\right)$$

$$\left[\frac{GI}{EF} + \frac{IF}{EF}\left(\frac{C_1+\alpha_1\alpha_2\,\frac{1+b}{1-b}}{1+C_1}\right) + \left(\frac{IF}{EF}\right)\frac{(\emptyset_T+\emptyset_B)}{(1+\emptyset_B)(1+C_1)}\right]$$

$$+\left\{\left[\frac{C_1+\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}{1+C_1} - \frac{C_2+\alpha_2}{1+C_2}\right]^{-1}\left(\frac{1}{1+C_2}\right)\left[\frac{GI}{EF} + \frac{IF}{EF}\left(\frac{C_1+\alpha_1\alpha_2\,\frac{1+b}{1-b}}{1+C_1}\right)\right]\right.$$

$$\left.\left[\frac{C_1+\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}{1+C_1}\right]\left[\left(\frac{C_1+\alpha_1\alpha_2\,\frac{1+b}{1-b}}{1+C_1}\right)^{n-p_2-3} - \left(\frac{C_2+\ 2}{1+C_2}\right)^{n-p_2-3}\right]\right\}$$

$$\left\{1 - \left[\frac{\emptyset_T+\emptyset_B}{1+\emptyset_B}\quad\frac{1}{1-\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}\right]\right\} + \left(\frac{1}{1-\alpha_2}\right)\frac{GF}{EF}\left[\frac{1}{1-\alpha_1\alpha_2\left(\frac{1+b}{1-b}\right)}\right]$$

$$\left(\frac{\emptyset_T+\emptyset_B}{1+\emptyset_B}\right)\left[1 - \left(\frac{C_2+\alpha_2}{1+C_2}\right)^{n-p_2-3}\right]$$

if $n \geq p_2 + 2$ (3)

and

$$\frac{GI}{EF} = \left(1 - \frac{L_1}{L_2}\right) q_2$$

$$\frac{IF}{EF} = \left(1 - \frac{L_1}{L_2}\right)\left(1 - q_2\right)$$

$$\frac{GF}{EF} = \left(1 - \frac{L_1}{L_2}\right)\left(\frac{2}{1+\phi_T}\right)$$

$$\alpha_1 = \frac{1-\phi_T}{1+\phi_T}, \qquad \alpha_2 = \frac{1-\phi_B}{1+\phi_B}$$

$$p_2 + q_2 = \frac{h-L_1}{L_2-L_1}, \qquad p_2 = \text{zero or a positive integer and } 0 \le q_2 < 1 \tag{4}$$

RESULTS AND DISCUSSION

Three experimental runs were made with the conditions set so that operation was in Region 1 in which $L_1 \geq L_2$ (or $\phi_B \leqslant 2b/(1-b)$) and $L_2 \leq h$. The data plotted in Figure 2, and the experimental parameters, are shown in Table 1. One can see that the separation factor, defined as the quotient of top and bottom concentrations, increased as n increased and, as the theory predicted, approached infinity as n became large.

The equations derived by Chen and Hill (4) were used to calculate the concentration transients corresponding to the three experimental runs. A comparison of the results of the computations and the experimental values is presented in Figure 2. The calculated results compare well with the observed values. The value of b used in the computations was 0.22, which was presented earlier in connection with the continuous pump (5). This value of b could be used because it was a function only of the temperatures employed and not of any particular mode of operation.

Figure 3 shows the comparison between the semicontinous and continuous pumps. The performance characteristics of both pumps were similar in nature and infinite steady-state

TABLE 1: EXPERIMENTAL AND MODEL PARAMETERS

$\Pi/\omega = 1200$ sec., $T_c = 277^\circ K$, $T_H = 343^\circ K$

$b = 0.22$ $m_o = 1.88$ $h = 0.90m$

$\emptyset_B$	$\emptyset_T+\emptyset_B$	C_1	C_2	L_1(m)	L_2(m)	
0.15	0.4	0.10	0.11	0.60	0.442	(semicontinuous)
0.20	0.4	0.10	0.05	0.60	0.462	(semicontinuous)
0.24	0.4	0.10	0.10	0.60	0.475	(semicontinuous)
0.15	0.4	0.137	0.15	0.51	0.44	(continuous)
0.30	0.4	0.15	0.25	0.47	0.40	(continuous)

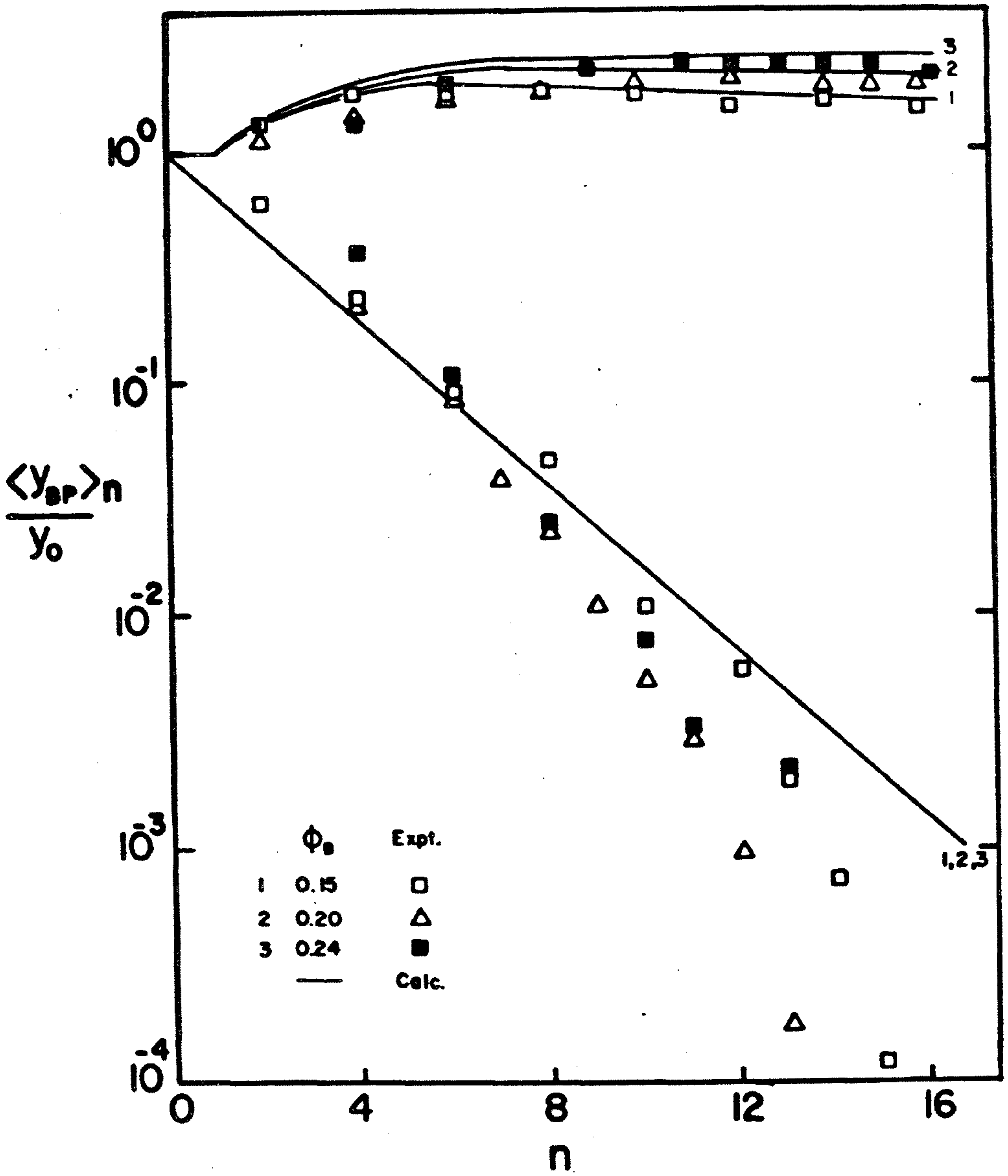

FIG. 2

Effects of ϕ_B on Product Concentration for the Semicontinuous System.

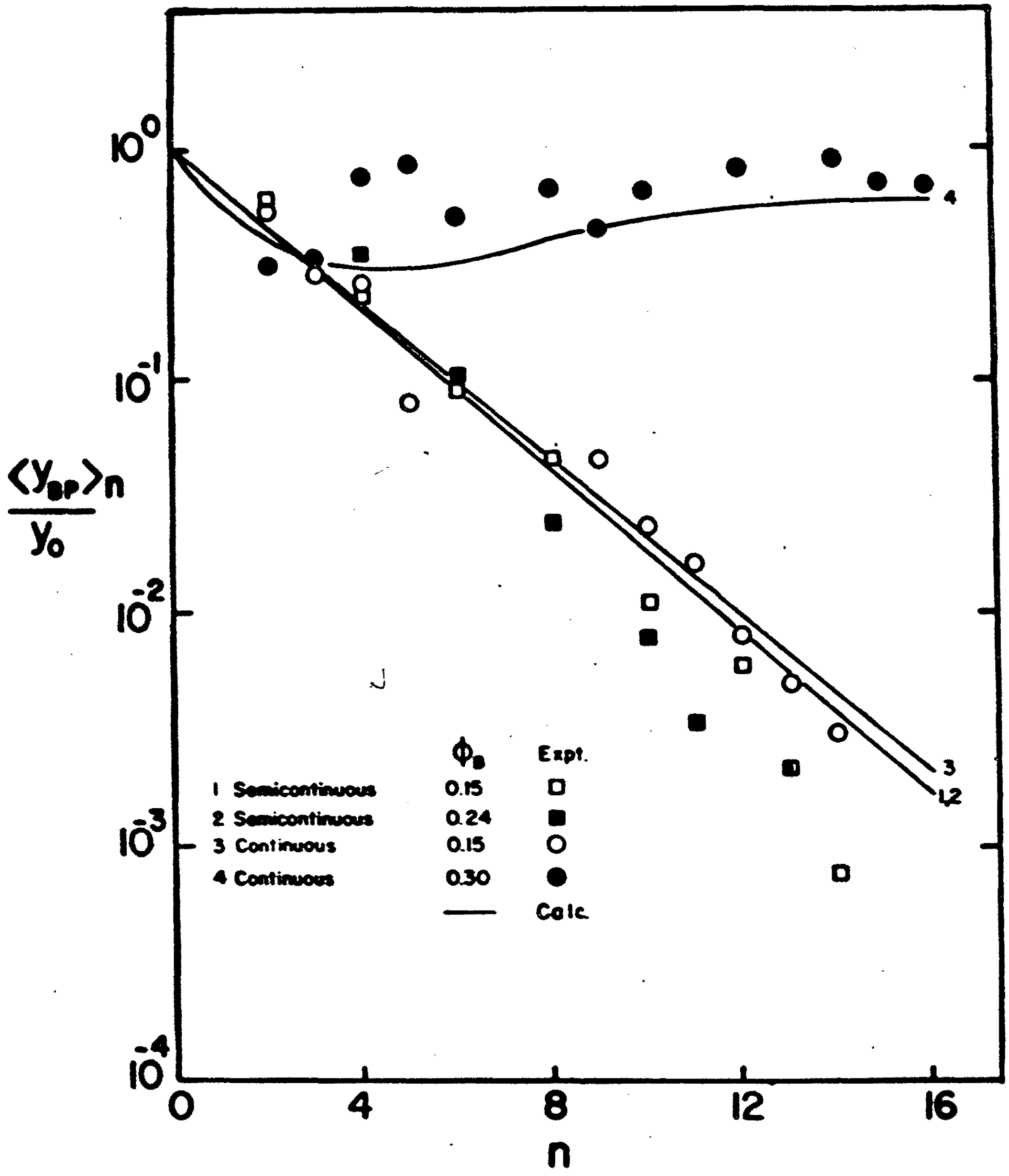

FIG. 3

Comparison of Semicontinuous vs. Continuous System.

separation factors were found in Region 1 for both pumps. The main difference between the two pumps was that of the switching points resulting from variations of the bottom product flow rates (see Table 2). The switching points were the boundaries between Regions 1 and 2 and between Regions 1 and 3. In Region 1 infinite separation factors were obtainable while in Region 2 and 3 the separation factors were always finite. Figure 4 illustrates the steady-state concentrations for both the semicontinuous and continuous pumps. In the interval $0 < \phi_B < 0.22$, corresponding to $0 < \phi_B < b$, no solute appeared in the bottom product stream of either the semicontinuous or the continuous pumps. Beyond the switching point of the continuous pump, i.e., beyond $\phi_B = b$, solute appeared in the bottom product. However, no solute could appear in the bottom product of the semicontinuous pump until the switching point, $\phi_B = 2b/1-b$, i.e., $\phi_B = 0.56$, was reached. This was beyond the present experimental range of $0 < \phi_B < 0.4$ due to the feed limitation $\phi_B + \phi_T = 0.4$. It should be pointed out that the same type of curves would result for the semicontinuous pump as for the continuous pump were the feed rate, $\phi_T + \phi_B$, extended beyond 0.56. Over the interval, $0 < \phi_B <$ "switching point," the top product concentration increased according to $\langle y_{TP} \rangle_\infty / y_0 = 1 + \phi_B / \phi_T$. Beyond the switching point the top concentrations would decrease according to the appropriate expressions by Chen and Hill (4) as previously explained. Note that for large values of b the bottom

TABLE 2: PARAMETRIC PUMP CHARACTERISTICS

Type	L_1	L_2	Region 1 where $\alpha_{\infty} \longrightarrow \infty$	Switching points
Semi-continuous	$\frac{1}{1-b} u_o \left(\frac{\Pi}{\omega}\right)$	$\frac{1+\phi_B}{1+b} u_o \left(\frac{\Pi}{\omega}\right)$	$L_1 \geq L_2 \left(\text{or } \phi_B \leq \frac{2b}{1-b}\right)$ and $L_2 \leq h$	$\phi_B = \frac{2b}{1-b}$
Continuous	$\frac{1-\phi_B}{1-b} u_o \left(\frac{\Pi}{\omega}\right)$	$\frac{1+\phi_B}{1+b} u_o \left(\frac{\Pi}{\omega}\right)$	$L_1 \geq L_2 (\text{or } \phi_B \leq b)$ and $L_2 \leq h$	$\phi_B = b$

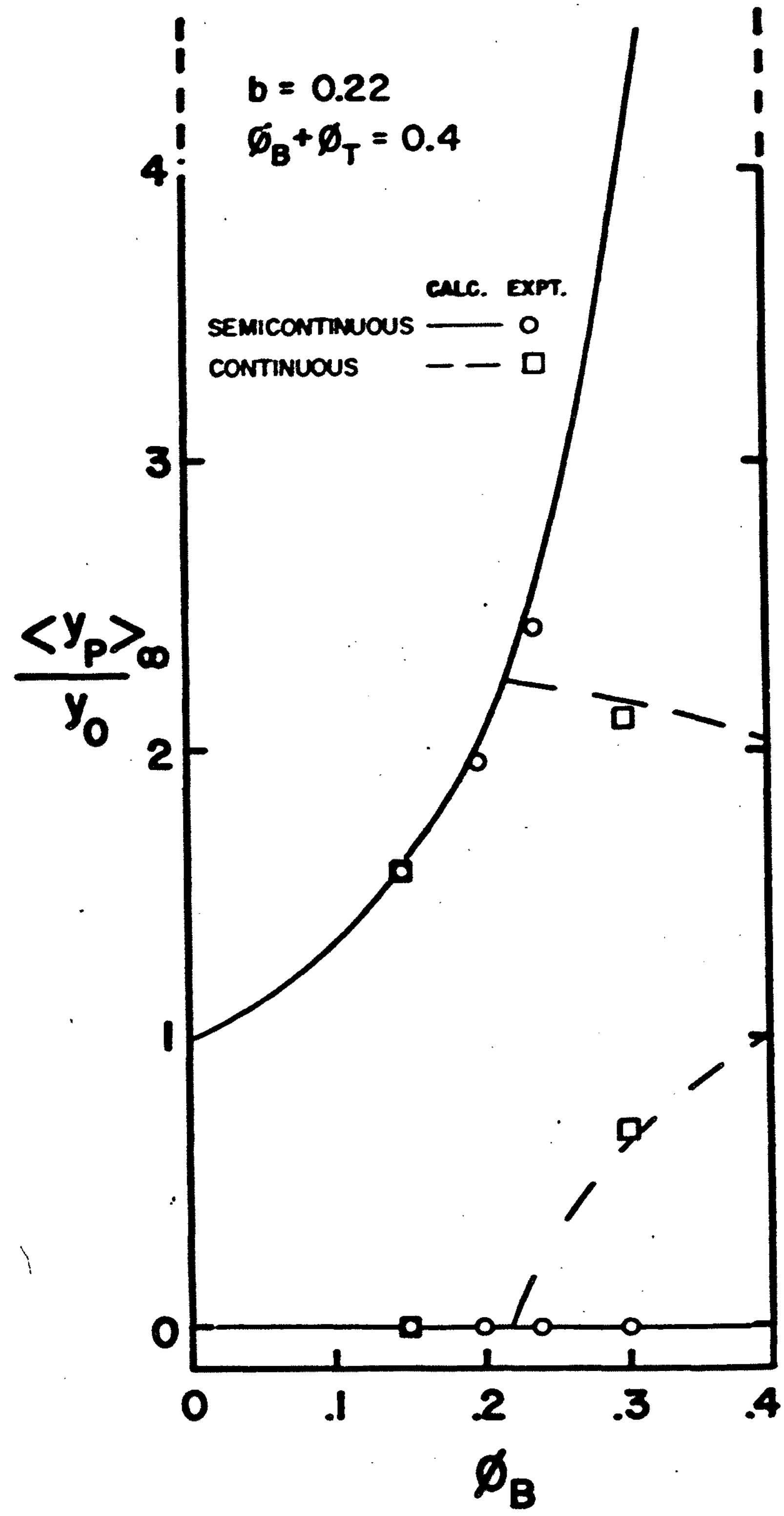

FIG. 4

Steady-State Concentration Profile.

product rate of production of pure solvent in the semicontinuous pump may be quite large relative to that of the continuous pump. This is indicated in Table 2.

Under the assumptions of the equilibrium theory, Gregory and Sweed (6) investigated two continuous open systems and developed analytical solutions for the operations. Their requirement for achieving infinite separation was that

$$R_{BOT} \geq \frac{1+A_H}{A_c-A_H} \tag{5}$$

where R_{BOT} is the bottom reflux ratio, and A_H and A_c are equilibrium constants at the hot and cold temperatures respectively. For the special case of equal reservoir volumes, their pumping operation was essentially the same as that for the semicontinuous pump considered here, i.e.,

$$R_{BOT} = \frac{Q\left(\frac{\Pi}{\omega}\right)}{\emptyset_B\ Q\left(\frac{\Pi}{\omega}\right)} = \frac{1}{\emptyset_B} \tag{6}$$

In terms of the nomenclature used by Pigford, et al. (1),

$$A_H = M_H = m_o - a \tag{7}$$

$$A_c = M_c = m_o + a \tag{8}$$

and

$$b = a/(1+m_o) \tag{9}$$

Then equation (5) becomes

$$\frac{1}{\emptyset_B} \geq \frac{1-b}{2b} \quad \text{or} \quad \emptyset_B \leq \frac{2b}{1-b} \tag{10}$$

Therefore, the pump described by Gregory and Sweed (6), had the necessary condition for infinite separation---that the penetration distance for the hot upflow half-cycle, L_1, be greater than that of the cold downflow half-cycle, L_2. However, Chen and Hill (4) imposed the additional condition that $L_2 \leq h$. This meant that the separation would never be achieved if the concentration front for the cold cycle was pushed beyond the height of the column.

NOTATION

A = equilibrium constant, (see Ref. 6).

a = deviation from m_o, dimensionless.

b = dimensionless equilibrium parameter (see Ref. 1).

$C_1 = \frac{V_T}{Q\frac{\Pi}{\omega}}$, dimensionless.

$C_2 = \frac{V_B}{Q\frac{\Pi}{\omega}}$, dimensionless.

h = column height, m.

L = penetration distances defined by Table 2, m.

M = equilibrium constant.

m_o = equilibrium constant parameter, dimensionless (see Ref. 1).

n = number of cycles of pump operation.

P_2 = defined by Eq. (4).

Q = reservoir displacement, cm^3/sec.

q_2 = defined by Eq. (4).

$u_o = \frac{v_o}{1+m_o}$, m/sec.

v_o = interstitial velocity, m/sec.

V_T = top reservoir volume, cm^3.

V_B = bottom reservoir volume, cm^3.

y = concentration of solute in the liquid phase, kg moles/cm^3.

$\langle\ \rangle$ = average value.

Greek Letters

α_{∞} = steady state separation factor, dimensionless.

ϵ = void fraction in packing, dimensionless.

ϕ = product volumetric flow rate/reservoir displacement rate, dimensionless.

$\frac{\Pi}{\omega}$ = duration of half-cycle, sec.

Subscripts

0 = initial condition.

1 = upflow.

2 = downflow.

B = stream from or to bottom of the column.

BP = bottom product.

H = hot.

c = cold.

T = stream from or to top of the column.

TP = top product.

APPENDIX 1

PARAMETRIC PUMP MODELS

PARAMETRIC PUMP MODELS (Ref. 4)

We will consider the parametric pumps shown in Fig. 6 and certain variations of these pumps. Flow to and from the reservoirs of all pumps during each half-cycle is at the rate Q volume units per unit time. Each half-cycle is π/ω time units in duration so that the displacement volume is $Q\ \pi/\omega$. All pumps have dead volumes of size V_T and V_B volume units associated with the top and bottom reservoirs, respectively.

The pump in Fig. 1a is a batch pump while the others each have a feed stream and top and bottom product streams. The feed flow-rate is $(\varphi_T + \varphi_B)Q$ and the top and bottom product flow rates are $\varphi_T Q$ and $\varphi_B Q$, respectively, where φ_T and φ_B are the ratios of the top and bottom product flow rates to the reservoir displacement rate.

The feed stream is located at the top of the column in Fig. 1b and at the bottom in Fig. 6c. Two modes of operation of these two pumps are treated. In one, the feed and product streams flow steadily both in upflow and downflow, resulting in truly continuous pumps. In the other, a semi-continuous form of operation results from batch operation during one half-cycle and continuous operation in the other. Thus we assume for the pump with feed at the top batch operation during upflow and continuous operation during downflow. The reverse arrangement

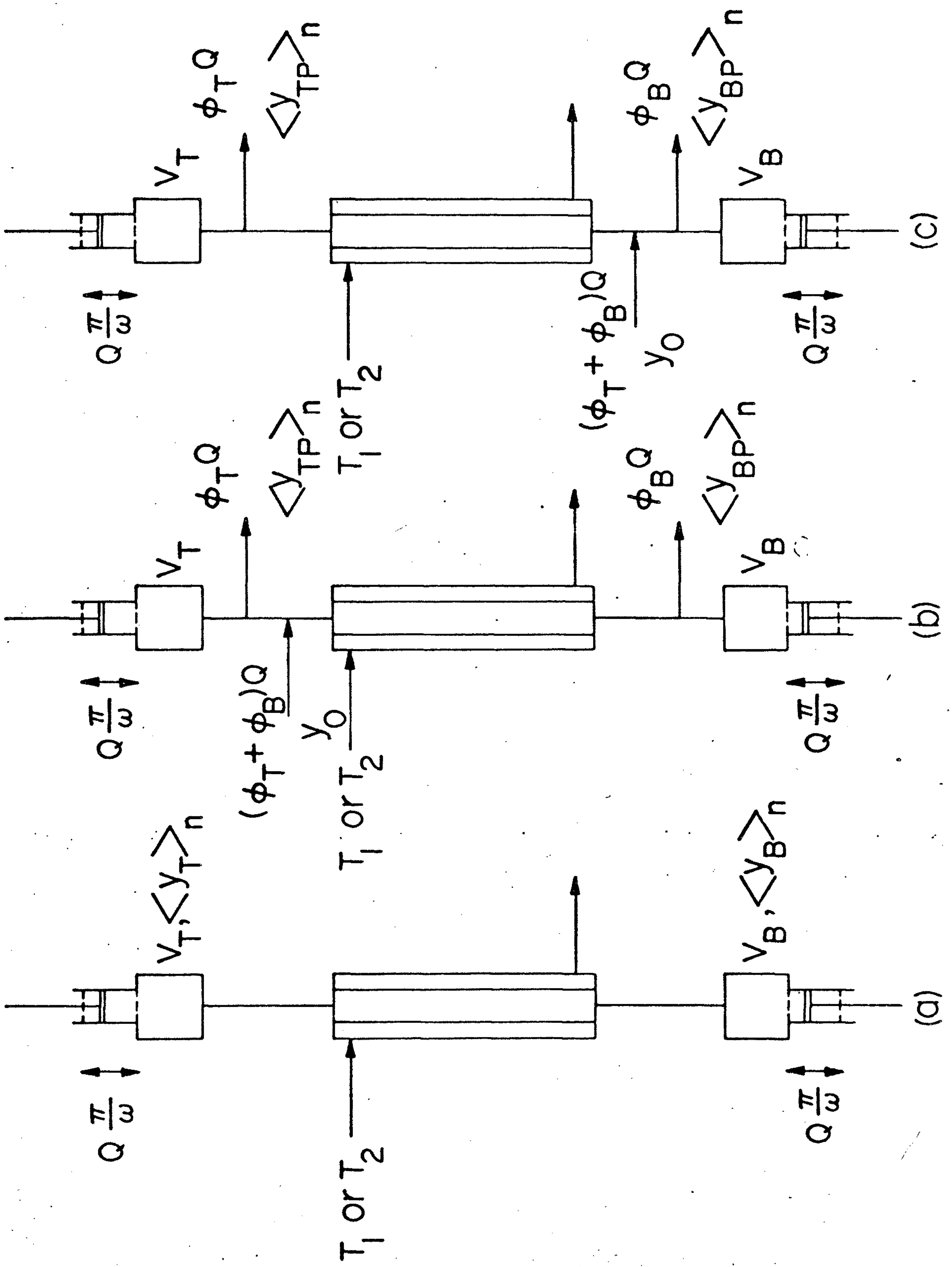

FIGURE 6

Parametric Pump Models: (a) batch; (b) continuous with top feed; (c) continuous with bottom feed.

is assumed for semi-continuous operation of the pump with feed at the bottom.

In the batch pump the flow rates within the column in upflow and downflow are identical and are equal to the reservoir displacement rate Q. The column flow rates in the pumps with feed and product streams may be determined by reference to flow diagrams such as those in Fig. 7 for the continuous pump with top feed. Material balances around the point of bottom product withdrawal show that the column flowrate in upflow must be $(1 - \varphi_B)Q$ and in downflow, $(1 + \varphi_B)Q$. Similarly, for the continuous pump with bottom feed the column flowrate is $(1 + \varphi_T)Q$ in upflow and $(1 - \varphi_T)Q$ in downflow. For the pumps with intermittent feed and product streams, the column flowrates are those of the corresponding continuous pumps during the half-cycle of continuous operation and those of the batch pump during the other half-cycle.

We will restrict our interest in this paper to situations in which a given product stream during discharge of the adjacent reservoir comes only from that reservoir and not also from the column or from the feed stream. This restriction has the effect of limiting the values of φ_T and φ_B.

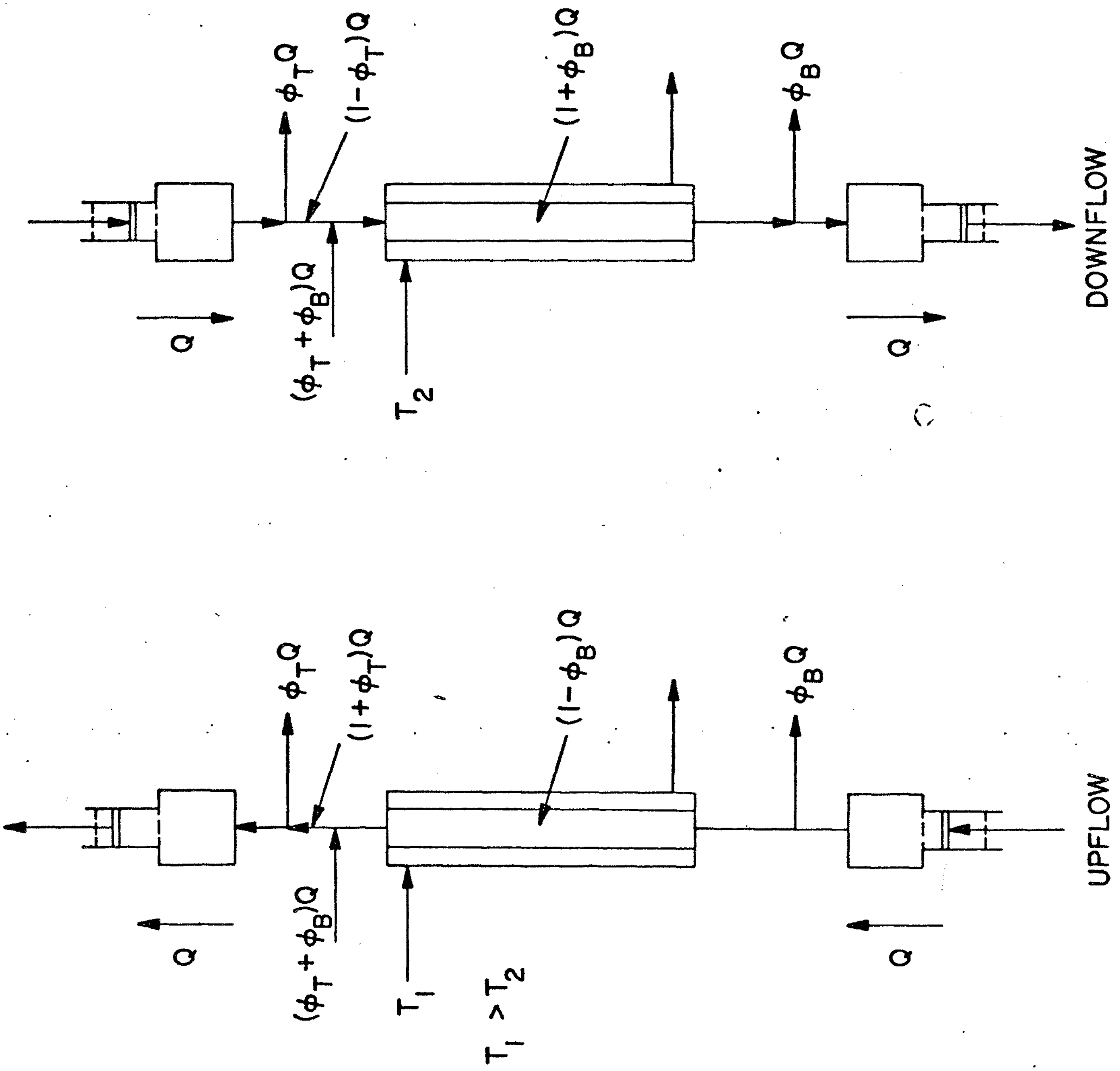

FIGURE 7

Internal Flow Rates in Continuous Parametric Pump with Top Feed.

For all pumps we assume the column is filled with adsorbent particles, and the reservoirs and the voids in the column are filled with a two-component mixture, one component of which distributes between the two phases. Flow is upward during a hot half-cycle and downward during a cold half-cycle. The material in each reservoir is taken to be well mixed prior to flow reversal. The volume of material in the connecting lines is assumed to be included in the dead volume of the adjacent reservoir.

At pump startup the distributing solute fluid phase concentration is equal to the feed concentration y_0 throughout the apparatus and is in equilibrium at the higher temperature with the solute concentration on the adsorbent particles. Flow in the first half cycle is upward.

APPENDIX 2

POSSIBLE REGIONS OF OPERATION

POSSIBLE REGIONS OF OPERATION (REF. 4)

The calculation of pump performance depends on the relative magnitudes of L_1, L_2, and h, the column height. Figure8 indicates three regions in which, depending on these relative magnitudes, different internal equations apply.

When the batch pump and the pumps with feed at the top are operated in Region 1 we will find that at steady-state solute removal from the lower reservoirs and the lower product streams will be complete and the separation factors, defined as the ratio of the top and bottom product or reservoir concentrations, will approach infinity. When these pumps are operated outside Region 1 solute removal from the lower reservoirs and lower product streams will be incomplete and the steady-state separation factors will be modest in size by comparison. No region of infinite separation factor will be found for pumps with feed at the bottom.

The boundaries between Regions 1 and 3 and between Regions 1 and 2 are the loci of so-called "switching points." If in a pump originally operating in Region 1, L_2 is increased until it exceeds h, or L_1 becomes less than L_2, corresponding switching points are encountered and the steady-state behavior of the pump abruptly switches from a mode in

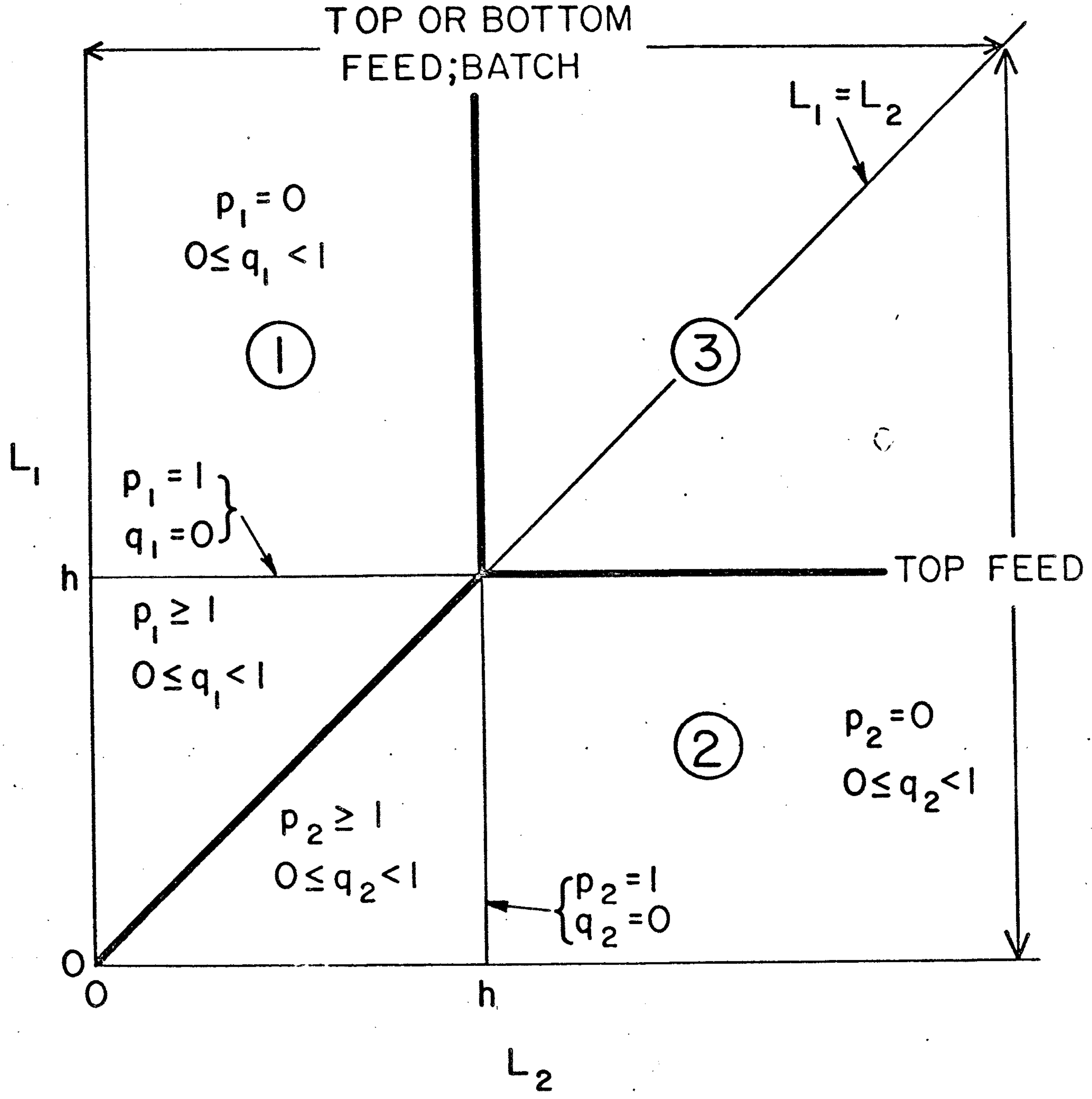

FIGURE 8

Regions of Parametric Pump Operation.

which solute is completely removed from the lower reservoir to one in which solute removal is incomplete. One may visualize the crossing of the boundary $L_2 = h$ as resulting from increasing L_2 by increasing the reservoir displacement volume. Crossing of the boundary $L_1 = L_2$ may be thought of as resulting from increasing the rate of bottom product withdrawal, i.e., increasing ϕ_B so that L_1 becomes less than L_2.

APPENDIX 3

DESCRIPTION OF APPARATUS - CONTINUOUS PUMP

EXPERIMENTAL (Ref. 5)

The experimental apparatus is shown schematically in Figure 9. The equipment consists of a jacketed glass column (0.01 m. inside diameter.) packed with 30 to 60 mesh chromatographic grade silica gel. The reservoirs at the two opposite ends of the column were two 50 cm^3 glass syringes operated by a dual infusion-withdrawal pump manufactured by Harvard Apparatus Company. A micro-switch with stops was wired into the pump circuit to automatically reverse the action of the springe plungers at the end of each half cycle. To fulfill the requirement of perfect mixing in the reservoirs, small magnetic stirrers were placed in the reservoir syringes.

The sources of hot and cold water supply were constant temperature hot and refrigerated baths. The baths were connected to the column and to recycle by solenoid valves wired to a dual timer so that hot water supply was always directed to the column jacket during upflow and cold water during downflow.

The feed was delivered to the top of the column by a second infusion-withdrawal pump with two 50 cm^3 syringes operated in parallel. (Every three cycles, pump operation was interrupted and the feed syringes refilled). The product take-off valves were micrometer capillary valves used both to regulate flow and impose a small back pressure on the system. Rotameters were used in the feed and product lines as a check against the feed and the calibrated product receivers.

Prior to each run the entire system, including the interstitial column volume, the bottom reservoir and the feed pump were filled with the feed mixture at ambient temperature. The reservoir syringes were set to deliver about 40 cm^3 per half cycle with a minimum dead volume of approximately 3 cm^3 in each syringe.

At the beginning of the run the feed and reservoir pumps were started and the timer was activated. The bottom reservoir syringe pumped fluid into the bottom of the column and the timer switched the solenoids to supply hot water (343°K) to the jacket. At the end of the cycle the micro-switch reversed the action of the reservoir pump and simultaneously, the timer switched the solenoids to supply cold water (277°K) to the jacket. This procedure was repeated for each cycle.

Expansion and contraction effects were induced within the column volume bv the periodic temperature changes. Therefore, the top and bottom product flow rates were adjusted so that their average values for the hot and cold cvcles were the desired ones. Samples for analysis were taken from the product streams after each half cycle, and analysed by ultra-violet spectrophotometer.

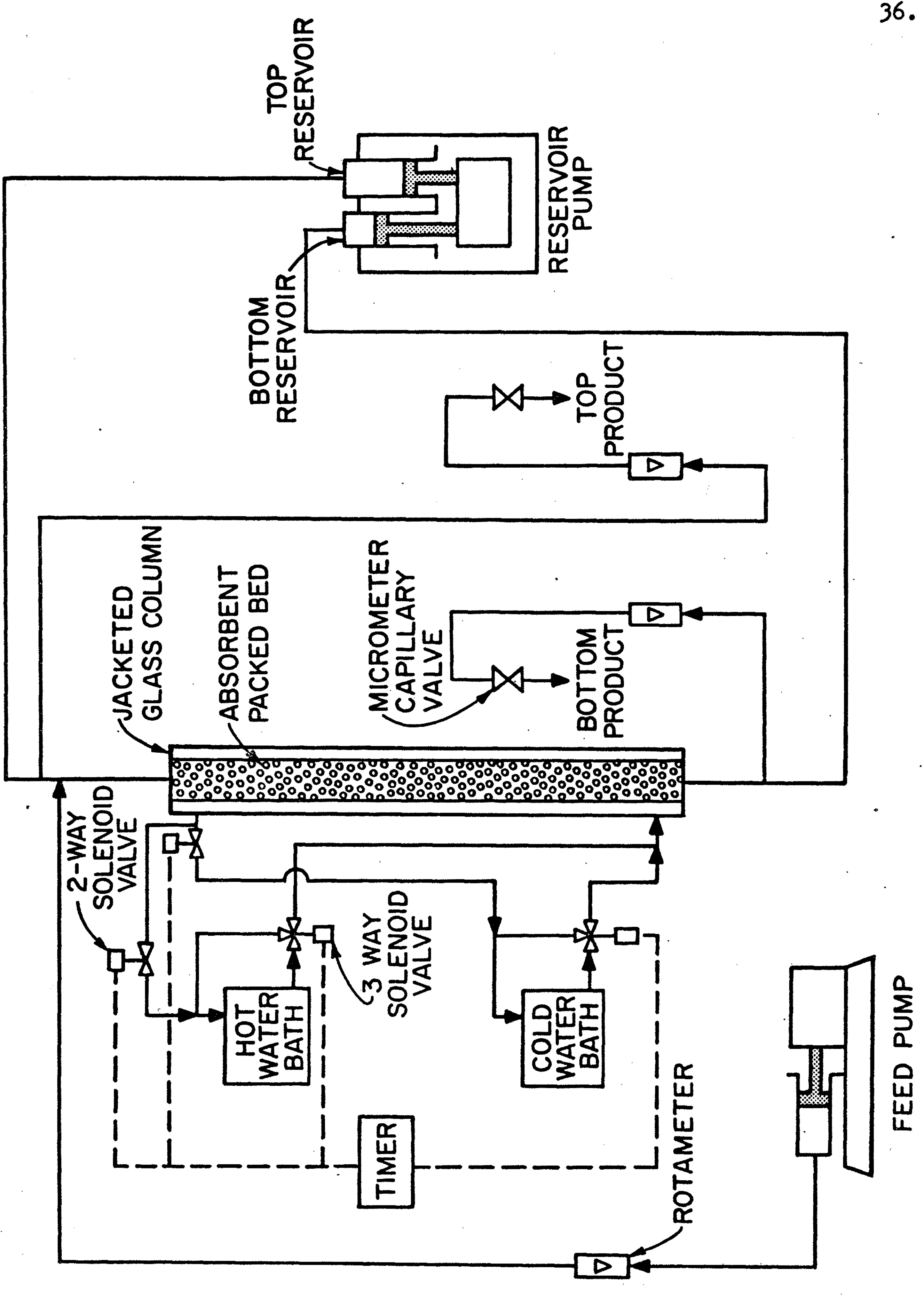

Figure 9

Schematic Diagram of Experimental Apparatus.

APPENDIX 4

DATA SHEETS

DATA SHEET

Continuous $\phi_B = 0.15$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
Initial	51	51	44	4	--	--
1	43	42	4	44	9.1	12.0
	34	34	48	4	--	11.0
2	27	27	4	44	3.0	11.1
	18	17	44	4	2.0	7.2
Feed	51	50	44	4	--	--
3	45	44	4	41	3.4	15.0
	37	37	44	5	--	5.2
4	30	30	4	44	2.0	9.3
	22	22	44	11	--	16.0
5	14	14	4	49	4.7	11.4
	7	10	44	4	--	8.2
Feed	42	42	44	4	--	--
6	16	26	22	44	5.8	14.4
	8	18	54	4	1.4	16.4
Feed	50	50	54	4	--	--
7	43	43	10	44	2.0	9.8
	35	35	49	5	1.3	3.8
8	27	28	15	42	5.0	16.5
	18	19	44	4	--	18.2
9	11	11	7	44	2.0	10.4
	2	3	48	4	--	6.4

Continuous $\phi_B = 0.15$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
Feed	52	52	46	4	--	--
10	45	45	7	44	2.5	11.2
	36	36	45	5	--	6.8
11	29	29	8	44	2.4	9.5
	21	21	47	4	--	6.0
Feed	50	50	47	4	--	--
12	42	43	8	44	3.1	9.6
	34	35	46	4	--	11.8
13	26	27	8	44	3.8	11.6
	18	19	48	4	--	7.0
Feed	52	52	48	4	--	--
14	42	44	11	44	3.4	10.0
	34	35	45	4	--	9.6
15	26	27	7	44	3.5	9.4
	18	19	43	4	--	9.0
16	10	11	4	44	4.7	9.3
	2	3	44	4	--	9.4

Continuous $\phi_B = 0.3$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
Initial	46	46	45	4	--	--
1	39	39	6	44	10.6	4.1
	31	31	49	4	6.0	2.6
2	23	23	10	44	13.6	3.8
	15	15	52	4	6.4	2.4
Feed	52	52	45	4	--	--
3	44	46	7	44	13.6	3.4
	36	37	47	5	8.8	2.1
4	29	30	9	44	13.2	2.8
	21	22	47	4	11.4	2.9
5	14	14	12	44	12.8	2.5
	5	6	46	4	12.2	2.8
Feed	51	51	46	4	--	--
6	45	45	7	44	4.8	0.6
	37	37	47	4	7.4	2.0
7	27	29	12	43	13.2	2.5
	21	21	50	4	9.8	2.2
8	14	14	13	44	13.6	2.5
	5	5	50	6	8.6	2.0
Feed	51	51	45	6	--	--
9	44	44	8	44	13.6	2.2
	36	36	48	4	5.4	1.3

Continuous $\phi_B = 0.3$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
10	29	29	10	44	16.4	2.6
	21	21	50	4	7.0	1.7
11	14	14	10	44	16.8	2.9
	5	5	51	5	7.0	1.7
Feed	51	51	45	5	--	--
12	45	45	7	44	14.2	2.5
	37	37	47	4	6.9	1.6
13	29	30	9	44	15.0	2.4
	21	21	46	5	11.0	2.4
14	14	14	7	44	13.4	2.6
	5	6	48	4	6.2	1.7
Feed	44	44	44	4	--	--
15	36	36	8	44	10.4	2.5
	29	30	47	4	6.9	2.0
16	22	22	8	44	12.6	3.1
	14	14	50	4	6.4	2.1

Semicontinuous $\phi_B = 0.15$

CYCLE	FEED cc Left	 Right	RESERVOIR cc Bottom	 Top	PRODUCT cc Bottom	 Top
Initial	51	51	44	4	--	--
1	--	--	4	44	--	--
	44	44	46	4	2.4	6.0
2	--	--	7	44	--	--
	36	36	47	4	2.4	7.5
3	--	--	5	44	--	--
	28	28	46	4	5.2	6.0
4	--	--	6	44	--	--
	21	21	46	4	3.2	8.6
5	--	--	7	44	--	--
	13	13	47	4	3.2	9.2
6	--	--	4	44	--	--
	5	5	44	4	3.8	9.4
Feed	50	50	44	4	--	--
7	--	--	4	44	--	--
	42	42	46	4	2.1	5.2
8	--	--	7	44	--	--
	34	35	48	4	2.9	7.7
9	--	--	4	44	--	--
	26	27	44	4	3.9	6.8
10	--	--	4	44	--	--
	18	19	44	4	3.6	9.5

Semicontinuous $\phi_B = 0.15$

CYCLE	FEED cc Left	 Right	RESERVOIR cc Bottom	 Top	PRODUCT cc Bottom	 Top
11	--	--	4	44	--	--
	10	11	44	4	2.8	8.2
Feed	44	45	44	4	--	--
12	--	--	4	44	--	--
	36	37	44	4	2.3	6.0
13	--	--	5	44	--	--
	29	30	44	4	3.9	8.2
14	--	--	4	44	--	--
	21	22	44	4	4.5	8.2
15	--	--	4	44	--	--
	13	14	44	4	4.3	8.4
16	--	--	4	44	--	--
	5	6	44	4	3.1	6.6

Semicontinuous $\phi_B = 0.2$

CYCLE	FEED cc		RESERVOIR cc		PRODUCT cc	
	Left	Right	Bottom	Top	Bottom	Top
Initial	44	43	44	4	--	--
1	--	--	4	44	--	--
	36	35	45	4	4.3	1.9
2	--	--	6	44	--	--
	27	27	44	4	8.4	3.7
3	--	--	5	44	--	--
	20	19	44	4	5.6	1.7
4	--	--	4	44	--	--
	12	11	44	4	5.3	3.5
5	--	--	4	44	--	--
	4	4	44	4	5.8	1.9
Feed	50	50	44	4	--	--
6	--	--	4	44	--	--
	41	42	44	4	3.8	2.6
7	--	--	4	44	--	--
	34	35	44	4	5.2	4.1
8	--	--	5	44	--	--
	26	27	44	4	6.4	3.0
9	--	--	5	44	--	--
	18	19	44	4	6.0	3.0
10	--	--	5	44	--	--
	10	11	44	4	6.6	3.5

Semicontinuous $\emptyset_B = 0.2$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
Feed	49	50	4	44	--	--
11	--	--	4	44	--	--
	41	42	44	4	5.0	3.8
Feed	41	42	44	4	--	--
12	--	--	4	44	--	--
	33	34	44	4	6.6	3.5
13	--	--	5	44	--	--
	25	26	44	4	6.4	4.2
14	--	--	4	44	--	--
	17	18	44	4	7.0	3.7
15	--	--	4	44	--	--
	9	10	44	4	4.8	3.7
16	--	--	4	44	--	--
	1	2	44	4	6.4	3.6

Semicontinuous $\phi_B = 0.24$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
Initial	46	45	44	4	--	--
1	--	--	7	44	--	--
	39	38	45	4	7.4	2.3
2	--	--	7	44	--	--
	32	31	44	4	6.5	8.6
3	--	--	6	44	--	--
	24	23	42	4	6.6	8.4
4	--	--	4	44	--	--
	16	15	44	4	6.5	3.5
FEED	50	--	44	4	--	--
5	--	--	8	40	--	--
	35	--	44	4	6.8	11.0
6	--	--	7	44	--	--
	19	--	44	4	6.5	9.4
7	--	--	7	44	--	--
	3	--	42	4	6.7	9.2
FEED	50	--	42	4	--	--
8	--	--	5	44	--	--
	35	--	41	4	6.6	1.2
9	--	--	4	44	--	--
	20	--	44	4	6.7	8.2

Semicontinuous $\phi_B = 0.24$

CYCLE	FEED cc Left	Right	RESERVOIR cc Bottom	Top	PRODUCT cc Bottom	Top
10	--	--	7	44	--	--
	5	--	43	4	6.3	1.8
FEED	50	--	43	4	--	--
11	--	--	7	44	--	--
	35	--	44	4	6.5	13.2
12	--	--	5	44	--	--
	19	--	44	4	6.5	5.0
13	--	--	6	44	--	--
	4	--	45	4	6.5	10.0
FEED	50	--	45	4	--	--
14	--	--	5	44	--	--
	35	--	44	4	6.3	--
15	--	--	7.5	44	--	--
	20	--	44	4	6.6	14.8
16	--	--	6	44	--	--
	4	--	44	4	6.6	4.1

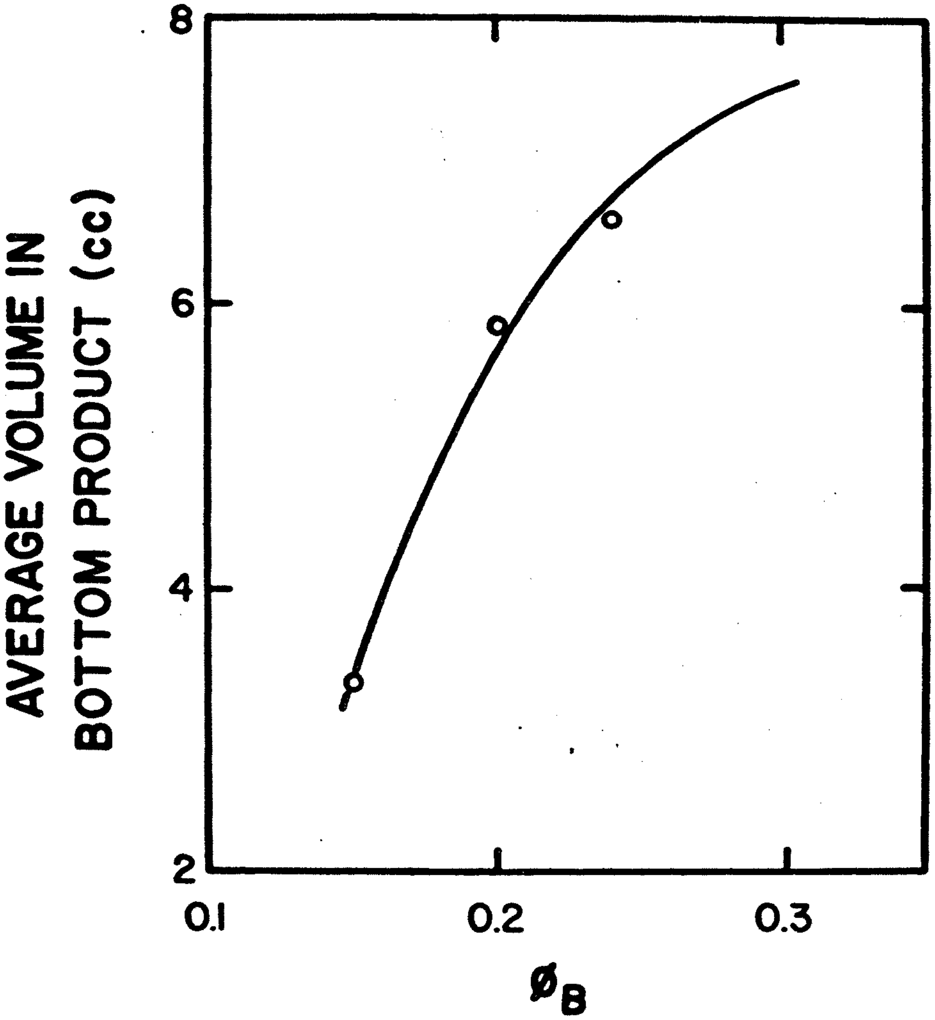

FIG. 5

Product Removal per ϕ_B.

APPENDIX 5

CONCENTRATION ANALYSIS

CONCENTRATION ANALYSIS

The absorption is a linear function of the concentration for solutions of the same compound in the same solvent, when the intensity and wave length of the light passing through each solution is the same and the thickness of each sample is the same. This follows Beer-Lambert's law. Mathematically, this function is $A = cK$ where A is the absorbance, c is the concentration of absorbing solute, and K is a constant equal to the length of the light path in the cell times the molar absorbancy index of the absorbing solute. Thus $A_1/c_1 = A_2/c_2 = K$. Therefore if the ratio of the absorbances of the two samples is known, the ratio of the concentrations is known. The ratio of the concentrations is $\langle y_B \rangle / y_o$ or $\langle y_T \rangle / y_o$.

A Beckman Instruments Spectrophotometer, Model DBG, was used in the ultraviolet range to measure the absorbance. The absorbance was measured at a wave length of 262 millimicrons.

The feed solution, prepared to have a concentration of 10 mole per cent toluene in n-heptane, was the first sample to be analyzed. At this concentration the absorbance goes to infinity and can not be read accurately. Therefore, the feed solution was dilutedto a concentration that gave an absorbance accurately readable on-scale. A mixture of 1cc of feed solution in 215cc of n-heptane diluted the feed by a factor of 216. At this dilution a readable measurement was obtained.

The bottom product samples were analyzed next, beginning with the 16th cycle and proceeding backwards to the first cycle. Approximately 3cc of the sample were placed in the test cell and analyzed. If the absorbance was off-scale to the left (toward infinity) 1cc of the original sample was diluted with 5cc of n-heptane. This gave a dilution factor of 6. Again 3cc of the solution were placed in the test cell and analyzed. This procedure was continued until the solution was dilute enough to give a reading on-scale. The dilution factor was then multiplied by the absorbance of the diluted sample to give the absorbance of the original solution.

The top samples were analyzed in similar manner. However, the the first sample analyzed was diluted to approximate the concentration of the diluted feed sample before being placed in the test cell for measurement.

Pipets were used for all volumetric measurements. The spectrophotometer was calibrated before each set of measurements. Spectroquality n-heptane was used to calibrate the instrument and to dilute the test samples.

The tabulated results for each experiment follow where the "reading" is equal to the absorbance of the diluted sample times 100.

CONCENTRATION ANALYSIS

Continuous $\phi_B = 0.15$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	726	21	1.00
Bottom Up			
2	1056	9	0.62
3	216	41	0.58
4	216	23	0.32
5	66	62	0.27
6	66	40	0.17
8	15	84	0.088
9	36	35	0.082
10	11	69.5	0.050
12	6	47	0.018
13	2	62	0.0081
14	3	31.5	0.0062
15	2	25.5	0.0033
16	1	11.2	0.00073

Continuous $\phi_B = 0.15$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	726	21	1.00
Top Down			
3	1056	18	1.25
5	1936	9	1.14
9	1936	14.5	1.84
11	1056	27	1.87
12	1056	23.5	1.62
14	1936	13	1.65
15	1936	13	1.65
16	1056	23.2	1.61

Continuous $\phi_B = 0.3$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	216	68	1.00
Bottom Down			
2	216	21	0.31
3	216	23	0.34
4	216	51	0.75
5	216	57	0.84
6	216	33	0.48
8	216	43.5	0.64
9	216	28	0.41
10	216	43	0.62
11	216	35	0.51
12	216	55	0.81
13	216	68	1.00
14	216	53	0.78
15	216	48	0.70
16	216	46	0.68

Continuous $\phi_B = 0.3$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y\rangle_n}{y_o}$
Feed	216	68	1.00
Top Down			
2	216	91	1.34
4	216	103	1.51
6	216	100	1.47
8	216	88	1.30
10	648	38.5	1.70
12	216	100	1.47
13	648	42	1.80
14	216	101	1.48
15	648	42.5	1.80
16	648	24	1.06

Semicontinuous $\phi_B = 0.15$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	216	78.5	1.00
Bottom Down			
2	216	47	0.60
4	216	20	0.24
6	36	43	0.091
8	36	22	0.046
10	6	33	0.011
12	3	37	0.0060
13	1	35	0.0021
14	1	13	0.00077
15	1	2	0.00012
16	1	0	0.00
Top Down			
2	216	103	1.31
4	396	76	1.77
6	726	43	1.84
8	726	43	1.84
10	726	41	1.75
12	726	37	1.58
14	726	38	1.64
16	726	35	1.50

Semicontinuous $\phi_B = 0.2$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	216	78	1.00
Bottom Down			
4	216	19	0.24
6	36	41	0.088
7	36	19	0.040
8	6	65	0.023
9	6	31.5	0.011
10	6	15	0.0053
11	1	50	0.0030
12	1	16.5	0.00098
13	1	1	0.00018
14	1	0	0.00
Top Down			
2	216	90	1.15
4	396	63	1.48
6	396	74	1.74
8	396	80	1.88
10	396	82	1.93
12	396	86	2.02
14	396	81	1.90
15	396	80	1.88
16	396	83	1.95

Semicontinuous $\phi_B = 0.24$

CYCLE	DILUTION FACTOR	READING	$\frac{\langle y \rangle_n}{y_o}$
Feed	396	34	1.00
Bottom Down			
4	216	23	0.368
6	36	40.5	0.108
8	6	56	0.0249
10	6	18	0.00802
11	1	46	0.00341
13	1	29	0.00215
Top Down			
3	396	55.5	1.31
4	576	40	1.37
6	576	57.5	1.97
9	576	66	2.26
11	576	73	2.49
12	396	79	2.32
13	396	80	2.35
15	396	79.5	2.33
16	396	70	2.063

APPENDIX 6

SAMPLE CALCULATION

SAMPLE CALCULATION

Data for this sample calculation is from the semi-continuous system where $\phi_B = 0.15$.

A. $$\frac{\langle y_T \rangle_n}{y_o} = \frac{(\text{Dilution Factor})_{Tn}(\text{Reading})_{Tn}}{(\text{Dilution Factor})_F(\text{Reading})_F}$$

Let $n = 2$

$$\frac{\langle y_T \rangle_2}{y_o} = \frac{(216)(103)}{(216)(78.5)} = 1.31$$

B. $$\frac{\langle y_B \rangle_n}{y_o} = \frac{(\text{Dilution Factor})_{Bn}(\text{Reading})_{Bn}}{(\text{Dilution Factor})_F(\text{Reading})_F}$$

Let $n = 2$

$$\frac{\langle y_B \rangle_n}{y_o} = \frac{216(47)}{216(78.5)} = 0.60$$

C. $$L_2 = \frac{1 + \phi_B}{1 + b} \cdot u_o \cdot \frac{\Pi}{\omega}$$

where $$u_o = \frac{v_o}{1 + m_o} \cdot \frac{1}{\epsilon} \cdot \frac{\omega}{\Pi}$$

$$L_2 = \frac{1 + 0.15}{1 + 0.22} \cdot \frac{40}{1 + 1.88} \cdot \frac{4}{\pi} \cdot \frac{1}{0.38} = 44.29$$

APPENDIX 7

COMPUTER PROGRAM

COMPUTER PROGRAM

Due to the complexity of the transient solution equations, two computer programs, written in Fortran IV language by Dr. H. T. Chen, were used to solve for the transient concentration ratios.

The two programs, one for the continuous system and the second for the semicontinuous system, have the flexibility to solve the equations for one set of parameters, to print the results, and then to change one or more of the parameters and resolve the equations for the new set of parameters. The final print out contains the solutions for all the combinations possible for the parameters given.

The parameters fed into the computer are: the height of the column; values of "b", C_1, C_2, L_2; feed rate; ratio of bottom product flow to feed rate. Additional data needed are the number of different values of C_1, C_2, L_2, feed rate, ratio of bottom product flow rate to feed rate, plus the beginning cycle calculation number and the ending cycle calculation number.

These programs were run on the Spectra 70 Computer at Newark College of Engineering.

The two programs are listed below along with a sample data sheet.

```
C     PARAMETRIC PUMP- CONTINUOUSLY FEEDING FROM TOP
      DIMENSION B(20),FEED(20),RATO(20),C1(20),C2(20),
     1DIST2(20),N(200),YT(200),YB(200),SF(200)
      READ20,H
      READ10,NB,NDIST2,NFEED,NRATO,NDET,NFINAL,NC1,NC2
   10 FORMAT(7I10)
      READ20,(B(I),I=1,NB)
      READ20,(C1(I),I=1,NC1)
      READ20,(C2(I),I=1,NC2)
      READ20,(FEED(I),I=1,NFEED)
      READ20,(RATO(I),I=1,NRATO)
      READ20,(DIST2(I),I=1,NDIST2)
   20 FORMAT(7F10.3)
      DO 100 I=1,NB
      DO 100 II=1,NC1
      DO 100 III=1,NC2
      DO 100 J=1,NFEED
      DO 100 JJ=1,NRATO
      PHOB=FEED(J)*RATO(JJ)
      PHOT=FEED(J)-PHOB
      DO 100 K=1,NDIST2
      DIST1=((1.-PHOB)/(1.+PHOB))*(1.+B(I))/(1.-B(I))*DIST2(K)
      L=1
      N(L)=1
   96 IF(DIST1-DIST2(K))40,40,30
   30 IF(DIST2(K)-H)50,50,60
   50 CALL FT01C(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
      GO TO 90
   40 IF(DIST1-H)70,70,60
   70 CALL FT02C(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
      GO TO 90
   60 CALL FT03C(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
   90 YT(L)=YTP2
      YB(L)=YBP2
      SF(L)=YT(L)/YB(L)
      IF(N(L)-NFINAL)95,300,300
   95 L=L+1
      N(L)=(1+NDET)*(L-1)
      GO TO 96
  300 PRINT301,H,B(I),PHOT,PHOB,DIST1,DIST2(K),
     1YTINF,YBINF,C1(II),C2(III),
     2RATO(JJ),FEED(J)
  301 FORMAT(1H1,15H***************/2X,2HH=,F10.3,5X,2HB=,F10.3/
     12X,11H(PHO)UPPER=,F10.3,5X,11H(PHO)LOWER=,F10.3/
     22X,6HDIST1=,F10.3,5X,6HDIST2=,F10.3/
     32X,11H(YT/YO)INF=,E20.5,5X,11H(YB/YO)INF=,E20.5/
     42X,3HC1=,F20.3,10X,3HC2=,F20.3/
     52X,16H(PHO)LOWER/FEED=,F10.3,5X,5HFEED=,F10.3)
  350 PRINT302,(N(LL),YT(LL),YB(LL),SF(LL),LL=1,L)
  302 FORMAT(//6X,
     11HN,14X,5HYT/YO,20X,5HYB/YO,20X,2HSF//(I5,3E25.5))
  100 CONTINUE
      STOP
```

```
      SUBROUTINE  FT01C(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      IP1=(H-DIST2)/(DIST1-DIST2)
      P1=IP1
      Q1=(H-DIST2)/(DIST1-DIST2)-P1
      GAMA=P1+1.
      NGAMA=GAMA
      AB=DIST2/DIST1
      BC=Q1*(1.-AB)
      CD=(1.-AB)*(1.-Q1)
      ALPA1=(1.-PHOT)/(1.+PHOT)
      ALPA2=(1.-PHOB)/(1.+PHOB)
      W=((1.-B)/(1.+B)+C2)/(1.+C2)
      FN=N
      CC=C1/(1.+C1)
      FACT=(C1+ALPA1)/(1.+C1)
      YBP2=((1.-B)/(1.+B))*(W**(N-1))
      IF(N-NGAMA)10,10,50
   10 IF(PHOT)200,15,20
   15 YTP2=1.+(FN-1.)*((1.+PHOB)/(1.+C1))*(2.*B/(1.+B))
      GO TO 100
   20 IF(PHOT-1.)30,40,200
   30 YTP2=-((PHOT+PHOB)/(1.-PHOT))+((1.+PHOB)/(1.-PHOT))*
     1(((C1+ALPA1)/(1.+C1))**(N-1)+(1./(1.-ALPA1))*
     2(1.-ALPA1*(1.-B)/(1.+B))*(1.-FACT**(N-1)))
      GO TO 100
   40 YTP2=(C1/(1.+C1))**(N-1)+(1.+PHOB+(B-PHOB)/(1.+B))*
     1(1.-(C1/(1.+C1))**(N-1))
      GO TO 100
   50 IF(PHOT)200,60,70
   60 YTP2=-PHOB+(1.+PHOB)*(1.+(GAMA-1.)*(2.*B/(1.+B))/(1.+C1)
     1+(FN-GAMA)*(1.-ALPA2)/(1.+C1)+(ALPA2/(1.+C1))*
     2(BC+W*CD)*(1.-W**(N-NGAMA))/(1.-W))
      GO TO 100
   70 IF(PHOT-1.)80,90,200
   80 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(FACT**(N-1)+(1./(1.-ALPA1))*(1.-ALPA1*(1.-B)/(1.+B))*
     2(1.-FACT**(NGAMA-1))*(FACT**(N-NGAMA))+
     3(FACT**(N-NGAMA-1))*(1./(1.+C1))*(1.-ALPA1*ALPA2+
     4ALPA1*ALPA2*BC+ALPA1*ALPA2*W*CD)+
     5((1.-ALPA1*ALPA2)/(1.-ALPA1))*(1.-FACT**(N-NGAMA-1))+
     6(ALPA1*ALPA2/(W*(1.+C1)-C1-ALPA1))*W*(BC+W*CD)*(W**(N-NGAMA-1)-
     7FACT**(N-NGAMA-1)))
      GO TO 100
   90 IF(N-1)200,92,91
   92 YTP2=1.
      GO TO 100
   91 YTP2=CC**(N-1)+(1.+PHOB)*(CC**(N-NGAMA))*
     1(1.+0.5*ALPA2*(BC+CD))*(1.-CC**(NGAMA-1))+
     2(CC**(N-NGAMA-1))*(1./(1.+C1))*(1.+PHOB+0.5*(1.-PHOB)*
     3(BC+W*CD))+(1.+PHOB)*(1.-CC**(N-NGAMA-1))+
     40.5*((1.-PHOB)/(W*(1.+C1)-C1))*(BC+W*CD)*W*
     5(W**(N-NGAMA-1)-CC**(N-NGAMA-1))
  100 YBINF=0.
      IF(PHOT)200,120,110
  110 YTINF=(PHOT+PHOB)/PHOT
```

```
      GO TO 200
  120 IF(PHOB)200,140,130
  140 YTINF=1.+((GAMA-1.)/(1.+C1))*(2.*B/(1.+B))+
     1(1./(1.+C1))*(BC+W*CD)*(1./(1.-W))
      GO TO 200
  130 YTINF=10.**49
  200 RETURN
      END
```

```
      SUBROUTINE FT02C(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      IP2=(H-DIST1)/(DIST2-DIST1)
      P2=IP2
      Q2=(H-DIST1)/(DIST2-DIST1)-P2
      GAMA=P2+1.
      NGAMA=GAMA
      EG=DIST1/DIST2
      GI=Q2*(1.-EG)
      FI=(1.-EG)*(1.-Q2)
      ALPA1=(1.-PHOT)/(1.+PHOT)
      ALPA2=(1.-PHOB)/(1.+PHOB)
      FN=N
      IF(PHOT)200,10,30
   10 IF(PHOB-B)200,20,30
   20 YTP2=1.+(FN-1.)*2.*B/((1.+C1)*(1.+B))
      GO TO 150
   30 IF(PHOT-1.)40,50,200
   40 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(((C1+ALPA1*ALPA2*(1.+B)/(1.-B))/(1.+C1))**(N-1))+(
     2(PHOT+PHOB)/(1.-ALPA1*ALPA2*(1.+B)/(1.-B)))*
     3(2./((1.+PHOT)*(1.-PHOT)))*(1.-((C1+ALPA1*ALPA2*(1.+B)/(1.-B))
     4/(1.+C1))**(N-1))
      GO TO 150
   50 IF(PHOB-1.)60,70,200
   60 IF(N-1)200,62,61
   62 YTP2=1.
      GO TO 150
   61 YTP2=(C1/(1.+C1))**(N-1)-0.5*(1.+PHOB+(1.+B)*(1.-PHOB)/(1.-B))*
     1(1.-(C1/(1.+C1))**(N-1))
      GO TO 150
   70 YTP2=1.
  150 FACT=(C2+ALPA2)/(1.+C2)
      BETA=(C1+ALPA1*ALPA2*(1.+B)/(1.-B))/
     1(1.+C1)
      I=N
  160 IF(I-NGAMA)163,163,180
  163 YB1=FACT**(I-1)+((1.-B-ALPA2*(1.+B))/((1.-ALPA2)*
     1(1.+B)))*(1.-FACT**(N-1))
      GO TO 175
  180 IF(PHOT-1.)171,172,200
  171 YB1=FACT**(I-1)+((1.-B-ALPA2*(1.+B))/((1.-ALPA2)*(1.+B)))*(FACT**
     1(I-NGAMA))*(1.-FACT**(NGAMA-1))+(FACT**(I-NGAMA-1))*(GI*((1.-B)/
     2(1.+B))+FI)/(1.+C2)+(FACT**(I-NGAMA-2))*(GI+BETA*FI+FI*2.*
     3(PHOT+PHOB)/((1.+PHOT)*(1.+PHOB)*(1.+C1)))/(1.+C2)+BETA*
     4(GI+BETA*FI)*(BETA**(I-NGAMA-2)-FACT**(I-NGAMA-2))/((1.+C2)*
     5(BETA-FACT))+(GI+FI)*2.*(PHOT+PHOB)*(1.-FACT**(I-NGAMA-2))/
     6((1.-ALPA2)*(1.-BETA)*(1.+C1)*(1.+PHOB)*(1.+PHOT))-
     7(GI+BETA*FI)*2.*BETA*(PHOT+PHOB)*(BETA**(I-NGAMA-2)-
     8FACT**(I-NGAMA-2))/((BETA-FACT)*(1.+C2)*(1.-BETA)*(1.+PHOB)*
     9(1.+PHOT)*(1.+C1))
      GO TO 175
  172 YB1=FACT**(I-1)+(FACT**(I-NGAMA))*(1.-B-ALPA2*(1.+B))*
     1(1.-FACT**(NGAMA-1))/((1.-ALPA2)*(1.+B))+
     2(FACT**(I-NGAMA-1))*(1.-ALPA2*(1.+B)/(1.-B))/(1.+C2)+
     3(1.-ALPA2*(1.+B)/(1.-B))*(1.-FACT**(I-NGAMA-1))/(1.-ALPA2)
```

```
  175 IF(I-N)161,161,165
  161 YB1N=YB1
  162 I=I+1
      GO TO 160
  165 YB1NN=YB1
  190 YBP2=(C2+1.)*YB1NN-C2*YB1N
      IF(PHOB-B)200,291,192
  291 IF(PHOT)200,193,192
  193 YTINF=10.**49
      GO TO 198
  192 IF(PHOT-1.)194,195,200
  194 YTINF=(PHOT+PHOB)*(1.-B*PHOB)/(PHOT+PHOB-B*(1.+PHOT*PHOB))
      GO TO 198
  195 IF(PHOB-1.)196,197,200
  196 YTINF=(1.-PHOB*B)/(1.-B)
      GO TO 198
  197 YTINF=1.
  198 IF(PHOT-1.)199,201,200
  199 YBINF=(PHOB-B)*(PHOT+PHOB)/(PHOB*(PHOT+PHOB-B*(1.+PHOT*PHOB)))
      GO TO 200
  201 YBINF=(PHOB-B)/(PHOB*(1.-B))
  200 RETURN
      END
```

```
      SUBROUTINE FT03C(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      AC=H/DIST1
      CD=1.-AC
      EF=H/DIST2
      FG=1.-EF
      ALPA1=(1.-PHOT)/(1.+PHOT)
      ALPA2=(1.-PHOB)/(1.+PHOB)
      A1=AC*(1.+B)/(1.-B)
      A2=CD
      A3=FG/(1.+C2)
      A4=(C2+EF*(1.-B)/(1.+B))/(1.+C2)
      W1=(A1+A2*A3)*ALPA1*ALPA2/(1.+C1)+A4+C1/(1.+C1)
      W2=A1*A4*ALPA1*ALPA2/(1.+C1)+A4*C1/(1.+C1)
      W3=(1.-ALPA1*ALPA2)/(1.+C1)
      W4=(C1+ALPA1*ALPA2*(A1+A2*A3+A2*A4))/(1.+C1)+W3
      W5=A3+A4
      W6=A3*W4+A4*(A3+A4)
      THTA1=0.5*(W1+(W1**2-4.*W2)**0.5)
      THTA2=0.5*(W1-(W1**2-4.*W2)**0.5)
      IF(PHOT)200,10,50
   10 IF(PHOB)200,20,50
   20 ETA=0.
      GO TO 30
   50 ETA=W3/(1.-W1+W2)
   30 G2=(W4-THTA1+(1.-A4)*(THTA1-1.)*ETA)/(THTA2*(THTA2-THTA1))
      G1=(1.-G2*THTA2-(1.-A4)*ETA)/THTA1
      G4=(W6-THTA1*W5+A3*(THTA1-1.)*ETA)/(THTA2*(THTA2-THTA1))
      G3=(W5-G4*THTA2-A3*ETA)/THTA1
      IF(PHOT-1.)60,70,200
   60 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(G1*(THTA1**N)+G2*(THTA2**N)+(1.-A4)*ETA)
      YBP2=(C2+1.)*(G3*(THTA1**N)+G4*(THTA2**N)+A3*ETA)-
     1C2*(G3*(THTA1**(N-1))+G4*(THTA2**(N-1))+A3*ETA)
      GO TO 150
   70 YBP2=(C2+1)*  (W5*(A4**(N-1))+A3*(1.-A4**(N-1))/(1.-A4))-
     1C2*(W5*(A4**(N-2))+A3*(1.-A4**(N-2))/(1.-A4))
      YTP2=(C1/(1.+C1))**(N-1)+0.5*(1.+PHOB+(1.-PHOB)*(A1+A2*A3/(1.-A4))
     1)*(1.-(C1/(1.+C1))**(N-1))+0.5*(1.-PHOB)*(A2*W5-A2*A3/(1.-A4))*
     2(A4**(N-1)-(C1/(1.+C1))**(N-1))/(A4*(1.+C1)-C1)
  150 IF(PHOT)200,160,180
  160 IF(PHOB)200,170,180
  170 YTINF=1.+H*(1./DIST2-1./DIST1)/((1.+C1)*(1.+C2)/(C2+H/DIST1)-
     1(C1+H/DIST2))
      YBINF=1.-H*(1.-DIST2/DIST1)/(DIST2*(1.+C2))+
     1(1.+C1)*(DIST1-DIST2)*(H/(DIST1*DIST2))*
     2(1.-(1.-H/DIST1)/(1.+C2)-(1.-H/DIST2)/(1.+C1)+(1.-H/DIST2)*
     3(1.-H/DIST1)/((1.+C1)*(1.+C2)))/((C1+H/DIST2)*
     4(C2+H/DIST1)-(1.+C1)*(1.+C2))
      GO TO 200
  180 IF(PHOT-1.)190,195,200
  190 YTINF=((PHOT+PHOB)/(1.-PHOT))*(-1.+(1.+PHOB-H*(1.-PHOB)/DIST1)/
     1(PHOT*(1.-H/DIST1+H*PHOB/DIST1)+PHOB*(1.-H/DIST2+PHOT*H/DIST2)))
      YBINF=(PHOT+PHOB)*(1.-H/DIST2)/
     1(PHOT*(1.-H/DIST1+H*PHOB/DIST1)+PHOB*(1.-H/DIST2+PHOT*H/DIST2))
      GO TO 200
```

```
  195 YBINF=(1.-H/DIST2)/(1.-ALPA2*H/DIST1)
      YTINF=0.5*((1.+PHOB)*(1.+H/DIST2)+(1.-PHOB)*(1.-H/DIST1)*
     1(1.-H/DIST2)/(1.-ALPA2*H/DIST1))
  200 RETURN
      END
```

```
C     PARAMETRIC PUMP- SEMICONTINUOUSLY FEEDING FROM THE TOP
      DIMENSION B(20),FEED(20),RATO(20),C1(20),C2(20),
     1DIST2(20),N(200),YT(200),YB(200),SF(200)
      READ20,H
      READ10,NB,NDIST2,NFEED,NRATO,NDET,NFINAL,NC1,NC2
   10 FORMAT(7I10)
      READ20,(B(I),I=1,NB)
      READ20,(C1(I),I=1,NC1)
      READ20,(C2(I),I=1,NC2)
      READ20,(FEED(I),I=1,NFEED)
      READ20,(RATO(I),I=1,NRATO)
      READ20,(DIST2(I),I=1,NDIST2)
   20 FORMAT(7F10.3)
      DO 100 I=1,NB
      DO 100 II=1,NC1
      DO 100 III=1,NC2
      DO 100 J=1,NFEED
      DO 100 JJ=1,NRATO
      PHOB=FEED(J)*RATO(JJ)
      PHOT=FEED(J)-PHOB
      DO 100 K=1,NDIST2
      DIST1=(1.        /(1.+PHOB))*(1.+B(I))/(1.-B(I))*DIST2(K)
      L=1
      N(L)=1
   96 IF(DIST1-DIST2(K))40,40,30
   30 IF(DIST2(K)-H)50,50,60
   50 CALL FT01D(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
      GO TO 90
   40 IF(DIST1-H)70,70,60
   70 CALL FT02D(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
      GO TO 90
   60 CALL FT03D(DIST1,DIST2(K),H,C1(II),C2(III),PHOT,PHOB,B(I),YTP2,
     1YBP2,YTINF,YBINF,N(L))
   90 YT(L)=YTP2
      YB(L)=YBP2
      SF(L)=YT(L)/YB(L)
      IF(N(L)-NFINAL)95,300,300
   95 L=L+1
      N(L)=(1+NDET)*(L-1)
      GO TO 96
  300 PRINT301,H,B(I),PHOT,PHOB,DIST1,DIST2(K),
     1YTINF,YBINF,C1(II),C2(III),
     2RATO(JJ),FEED(J)
  301 FORMAT(1H1,15H***************/2X,2HH=,F10.3,5X,2HB=,F10.3/
     12X,11H(PHO)UPPER=,F10.3,5X,11H(PHO)LOWER=,F10.3/
     22X,6HDIST1=,F10.3,5X,6HDIST2=,F10.3/
     32X,11H(YT/YO)INF=,E20.5,5X,11H(YB/YO)INF=,E20.5/
     42X,3HC1=,F20.3,10X,3HC2=,F20.3/
     52X,16H(PHO)LOWER/FEED=,F10.3,5X,5HFEED=,F10.3)
  350 PRINT302,(N(LL),YT(LL),YB(LL),SF(LL),LL=1,L)
  302 FORMAT(//6X,
     11HN,14X,5HYT/YO,20X,5HYB/YO,20X,2HSF//(I5,3E25.5))
  100 CONTINUE
      STOP
      END
```

```
      SUBROUTINE  FT01D(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      IP1=(H-DIST2)/(DIST1-DIST2)
      P1=IP1
      Q1=(H-DIST2)/(DIST1-DIST2)-P1
      GAMA=P1+1.
      NGAMA=GAMA
      AB=DIST2/DIST1
      BC=Q1*(1.-AB)
      CD=(1.-AB)*(1.-Q1)
      ALPA1=1.-PHOT
      ALPA2=1./(1.+PHOB)
      W=((1.-B)/(1.+B)+C2)/(1.+C2)
      FN=N
      CC=C1/(1.+C1)
      FACT=(C1+ALPA1)/(1.+C1)
      YBP2=((1.-B)/(1.+B))*(W**(N-1))
      IF(N-NGAMA)10,10,50
   10 IF(PHOT)200,15,20
   15 YTP2=1.+(FN-1.)*((1.+PHOB)/(1.+C1))*(2.*B/(1.+B))
      GO TO 100
   20 IF(PHOT-1.)30,40,200
   30 YTP2=-((PHOT+PHOB)/(1.-PHOT))+((1.+PHOB)/(1.-PHOT))*
     1(((C1+ALPA1)/(1.+C1))**(N-1)+(1./(1.-ALPA1))*
     2(1.-ALPA1*(1.-B)/(1.+B))*(1.-FACT**(N-1)))
      GO TO 100
   40 YTP2=CC**(N-1)+(1.+PHOB+(B*(2.+PHOB)-PHOB)/(1.+B))*(1.-CC**(N-1))
      GO TO 100
   50 IF(PHOT)200,60,70
   60 YTP2=-PHOB+(1.+PHOB)*(1.+(GAMA-1.)*(2.*B/(1.+B))/(1.+C1)
     1+(FN-GAMA)*(1.-ALPA2)/(1.+C1)+(ALPA2/(1.+C1))*
     2(BC+W*CD)*(1.-W**(N-NGAMA))/(1.-W))
      GO TO 100
   70 IF(PHOT-1.)80,90,200
   80 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(FACT**(N-1)+(1./(1.-ALPA1))*(1.-ALPA1*(1.-B)/(1.+B))*
     2(1.-FACT**(NGAMA-1))*(FACT**(N-NGAMA))+
     3(FACT**(N-NGAMA-1))*(1./(1.+C1))*(1.-ALPA1*ALPA2+
     4ALPA1*ALPA2*BC+ALPA1*ALPA2*W*CD)+
     5((1.-ALPA1*ALPA2)/(1.-ALPA1))*(1.-FACT**(N-NGAMA-1))+
     6(ALPA1*ALPA2/(W*(1.+C1)-C1-ALPA1))*W*(BC+W*CD)*(W**(N-NGAMA-1)-
     7FACT**(N-NGAMA-1)))
      GO TO 100
   91 YTP2=CC**(N-1)+(1.+PHOB)*(CC**(N-NGAMA))*
     1(1.+    ALPA2*(BC+CD))*(1.-CC**(NGAMA-1))+
     2(CC**(N-NGAMA-1))*(1./(1.+C1))*(1.+PHOB+
     3(BC+W*CD))+(1.+PHOB)*(1.-CC**(N-NGAMA-1))+
     4(1.            /(W*(1.+C1)-C1))*(BC+W*CD)*W*
     5(W**(N-NGAMA-1)-CC**(N-NGAMA-1))
  100 YBINF=0.
      IF(PHOT)200,120,110
  110 YTINF=(PHOT+PHOB)/PHOT
      GO TO 200
  120 IF(PHOB)200,140,130
  140 YTINF=1.+((GAMA-1.)/(1.+C1))*(2.*B/(1.+B))+
     1(1./(1.+C1))*(BC+W*CD)*(1./(1.-W))
```

```
      GO TO 200
  130 YTINF=10.**49
  200 RETURN
      END
```

```
      SUBROUTINE FT02D(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      IP2=(H-DIST1)/(DIST2-DIST1)
      P2=IP2
      Q2=(H-DIST1)/(DIST2-DIST1)-P2
      GAMA=P2+1.
      NGAMA=GAMA
      EG=DIST1/DIST2
      G1=Q2*(1.-EG)
      FI=(1.-EG)*(1.-Q2)
      ALPA1=(1.-PHOT)
      ALPA2=1./(1.+PHOB)
      FN=N
      IF(PHOT)200,10,30
   10 IF(PHOB-2.*B/(1.-B))200,20,30
   20 YTP2=1.+(FN-1.)*2.*B/((1.+C1)*(1.+B))
      GO TO 150
   30 IF(PHOT-1.)40,60,200
   40 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(((C1+ALPA1*ALPA2*(1.+B)/(1.-B))/(1.+C1))**(N-1))+(
     2(PHOT+PHOB)/(1.-ALPA1*ALPA2*(1.+B)/(1.-B)))*
     3(1./(1.-PHOT))              *(1.-((C1+ALPA1*ALPA2*(1.+B)/(1.-B))
     4/(1.+C1))**(N-1))
      GO TO 150
   60 YTP2=(C1/(1.+C1))**(N-1)+((1.+B)/(1.-B))*
     1(1.-(C1/(1.+C1))**(N-1))
      GO TO 150
  150 FACT=(C2+ALPA2)/(1.+C2)
      BETA=(C1+ALPA1*ALPA2*(1.+B)/(1.-B))/
     1(1.+C1)
      I=N
  160 IF(I-NGAMA)163,163,180
  163 YB1=FACT**(I-1)+((1.-B-ALPA2*(1.+B))/((1.-ALPA2)*
     1(1.+B)))*(1.-FACT**(N-1))
      GO TO 175
  180 IF(PHOT-1.)171,172,200
  171 YB1=FACT**(I-1)+((1.-B-ALPA2*(1.+B))/((1.-ALPA2)*(1.+B)))*(FACT**
     1(I-NGAMA))*(1.-FACT**(NGAMA-1))+(FACT**(I-NGAMA-1))*(GI*((1.-B)/
     2(1.+B))+FI)/(1.+C2)+(FACT**(I-NGAMA-2))*(GI+BETA*FI+FI*
     3(PHOT+PHOB)/((          1.+PHOB)*(1.+C1)))/(1.+C2)+BETA*
     4(GI+BETA*FI)*(BETA**(I-NGAMA-2)-FACT**(I-NGAMA-2))/((1.+C2)*
     5(BETA-FACT))+(GI+FI)*    (PHOT+PHOB)*(1.-FACT**(I-NGAMA-2))/
     6((1.-ALPA2)*(1.-BETA)*(1.+C1)*(1.+PHOB))-
     7(GI+BETA*FI)*    BETA*(PHOT+PHOB)*(BETA**(I-NGAMA-2)-
     8FACT**(I-NGAMA-2))/((BETA-FACT)*(1.+C2)*(1.-BETA)*(1.+PHOB)*
     9(1.+C1))
      GO TO 175
  172 YB1=FACT**(I-1)+(FACT**(I-NGAMA))*(1.-B-ALPA2*(1.+B))*
     1(1.-FACT**(NGAMA-1))/((1.-ALPA2)*(1.+B))+
     2(FACT**(I-NGAMA-1))*(1.-ALPA2*(1.+B)/(1.-B))/(1.+C2)+
     3(1.-ALPA2*(1.+B)/(1.-B))*(1.-FACT**(I-NGAMA-1))/(1.-ALPA2)
  175 IF(I-N)161,161,165
  161 YB1N=YB1
  162 I=I+1
      GO TO 160
  165 YB1NN=YB1
```

```
      GO TO 190
  190 YBP2=(C2+1.)*YB1NN-C2*YB1N
      IF(PHOB-2.*B/(1.-B))200,291,192
  291 IF(PHOT)200,193,192
  193 YTINF=10.**49
      GO TO 198
  192 IF(PHOT-1.)194,195,200
  194 YTINF=(PHOT+PHOB)*(1.+B)/(PHOT+PHOB-B*(2.+PHOB-PHOT))
      GO TO 198
  195 YTINF=(1.+B)/(1.-B)
  198 IF(PHOT-1.)199,201,200
  199 YBINF=(PHOT+PHOB)*(PHOB-B*(2.+PHOB))/(PHOB*(PHOT+PHOB-B*(2.+
     1PHOB-PHOT)))
      GO TO 200
  201 YBINF=(1.+PHOB-(1.+B)/(1.-B))/PHOB
  200 RETURN
      END
```

```
      SUBROUTINE FT03D(DIST1,DIST2,H,C1,C2,PHOT,PHOB,B,YTP2,YBP2,
     1YTINF,YBINF,N)
      AC=H/DIST1
      CD=1.-AC
      EF=H/DIST2
      FG=1.-EF
      ALPA1=(1.-PHOT)
      ALPA2=1./(1.+PHOB)
      A1=AC*(1.+B)/(1.-B)
      A2=CD
      A3=FG/(1.+C2)
      A4=(C2+EF*(1.-B)/(1.+B))/(1.+C2)
      W1=(A1+A2*A3)*ALPA1*ALPA2/(1.+C1)+A4+C1/(1.+C1)
      W2=A1*A4*ALPA1*ALPA2/(1.+C1)+A4*C1/(1.+C1)
      W3=(1.-ALPA1*ALPA2)/(1.+C1)
      W4=(C1+ALPA1*ALPA2*(A1+A2*A3+A2*A4))/(1.+C1)+W3
      W5=A3+A4
      W6=A3*W4+A4*(A3+A4)
      THTA1=0.5*(W1+(W1**2-4.*W2)**0.5)
      THTA2=0.5*(W1-(W1**2-4.*W2)**0.5)
      IF(PHOT)200,10,50
   10 IF(PHOB)200,20,50
   20 ETA=0.
      GO TO 30
   50 ETA=W3/(1.-W1+W2)
   30 G2=(W4-THTA1+(1.-A4)*(THTA1-1.)*ETA)/(THTA2*(THTA2-THTA1))
      G1=(1.-G2*THTA2-(1.-A4)*ETA)/THTA1
      G4=(W6-THTA1*W5+A3*(THTA1-1.)*ETA)/(THTA2*(THTA2-THTA1))
      G3=(W5-G4*THTA2-A3*ETA)/THTA1
      IF(PHOT-1.)60,70,200
   60 YTP2=-(PHOT+PHOB)/(1.-PHOT)+((1.+PHOB)/(1.-PHOT))*
     1(G1*(THTA1**N)+G2*(THTA2**N)+(1.-A4)*ETA)
      YBP2=(C2+1.)*(G3*(THTA1**N)+G4*(THTA2**N)+A3*ETA)-
     1C2*(G3*(THTA1**(N-1))+G4*(THTA2**(N-1))+A3*ETA)
      GO TO 150
   70 YBP2=(C2+1)* (W5*(A4**(N-1))+A3*(1.-A4**(N-1))/(1.-A4))-
     1C2*(W5*(A4**(N-2))+A3*(1.-A4**(N-2))/(1.-A4))
      YTP2=(C1/(1.+C1))**(N-1)+                     (A1+A2*A3/(1.-A4)
     1)*(1.-(C1/(1.+C1))**(N-1))+              (A2*W5-A2*A3/(1.-A4))*
     2(A4**(N-1)-(C1/(1.+C1))**(N-1))/(A4*(1.+C1)-C1)
  150 IF(PHOT)200,160,180
  160 IF(PHOB)200,170,180
  170 YTINF=1.+H*(1./DIST2-1./DIST1)/((1.+C1)*(1.+C2)/(C2+H/DIST1)-
     1(C1+H/DIST2))
      YBINF=1.-H*(1.-DIST2/DIST1)/(DIST2*(1.+C2))+
     1(1.+C1)*(DIST1-DIST2)* ((H/(DIST1*DIST2))**2)*
     2(DIST1+(H-DIST1)/(1.+C2))/((C1+H/DIST2)*(C2+H/DIST1)-(1.+C1)*
     3 (1.+C2))
      GO TO 200
  180 IF(PHOT-1.)190,195,200
  190 YTINF=((PHOT+PHOB)/(1.-PHOT))*(-1.+(1.+PHOB-H/DIST1)/
     1(PHOT*(1.-H/DIST1)+PHOB*(1.-H/DIST2+PHOT*H/DIST2)))
      YBINF=(PHOT+PHOB)*(1.-H/DIST2)/
     1(PHOT*(1.-H/DIST1)              +PHOB*(1.-H/DIST2+PHOT*H/DIST2))
      GO TO 200
  195 YBINF=(1.-H/DIST2)/(1.-ALPA2*H/DIST1)
```

```
      YTINF=H/(ALPA2*DIST2)+(1.-H/DIST1)*(1.-H/DIST2)/
     |(1.-ALPA2*H/DIST1)
  200 RETURN
      END
```

```
 ***************
  H=     90.000       B=       0.220
  (PHO)UPPER=      0.255      (PHO)LOWER=      0.145
  DIST1=    60.499      DIST2=    44.296
  (YT/YO)INF=          0.15699E 01      (YB/YO)INF=           0.00000E 00
  C1=                 0.100          C2=                  0.110
  (PHO)LOWER/FEED=     0.363      FEED=      0.400


       N               YT/YO                      YB/YO                     SF

       1           0.10000E 01               0.63934E 00               0.15641E 01
       2           0.13755E 01               0.43161E 00               0.31868E 01
       4           0.18715E 01               0.19670E 00               0.95142E 01
       6           0.19714E 01               0.89645E-01               0.21991E 02
       8           0.19087E 01               0.40855E-01               0.46719E 02
      10           0.18163E 01               0.18619E-01               0.97551E 02
      12           0.17365E 01               0.84856E-02               0.20464E 03
      14           0.16779E 01               0.38672E-02               0.43388E 03
      16           0.16380E 01               0.17624E-02               0.92941E 03
      18           0.16121E 01               0.80322E-03               0.20071E 04
      20           0.15957E 01               0.36606E-03               0.43592E 04
```

```
***************
 H=    90.000      B=      0.220
 (PHO)UPPER=      0.165      (PHO)LOWER=      0.235
 DIST1=    60.247      DIST2=    47.563
 (YT/YO)INF=            0.24213E 01      (YB/YO)INF=           0.00000E 00
 C1=                  0.100            C2=                    0.100
 (PHO)LOWER/FEED=      0.587      FEED=      0.400

        N                YT/YO                          YB/YO                          SF

        1             0.10000E 01                    0.63934E 00                    0.15641E 01
        2             0.14048E 01                    0.42972E 00                    0.32692E 01
        4             0.20413E 01                    0.19413E 00                    0.10515E 02
        6             0.23756E 01                    0.87701E-01                    0.27088E 02
        8             0.24917E 01                    0.39620E-01                    0.62890E 02
       10             0.25188E 01                    0.17899E-01                    0.14073E 03
       12             0.25128E 01                    0.80860E-02                    0.31076E 03
       14             0.24969E 01                    0.36529E-02                    0.68354E 03
       16             0.24802E 01                    0.16503E-02                    0.15029E 04
       18             0.24658E 01                    0.74552E-03                    0.33075E 04
       20             0.24543E 01                    0.33680E-03                    0.72872E 04
```

```
***************
H=     90.000       B=       0.220
(PHO)UPPER=       0.200     (PHO)LOWER=       0.200
DIST1=     60.247       DIST2=     46.222
(YT/YO)INF=             0.20000E 01       (YB/YO)INF=              0.00000E 00
C1=                   0.100             C2=                     0.050
(PHO)LOWER/FEED=        0.500       FEED=       0.400


     N                YT/YO                          YB/YO                           SF

     1             0.10000E 01                    0.63934E 00                    0.15641E 01
     2             0.13934E 01                    0.41974E 00                    0.33198E 01
     4             0.19787E 01                    0.18092E 00                    0.10937E 02
     6             0.22037E 01                    0.77978E-01                    0.28260E 02
     8             0.22303E 01                    0.33610E-01                    0.66358E 02
    10             0.21946E 01                    0.14486E-01                    0.15150E 03
    12             0.21477E 01                    0.62439E-02                    0.34397E 03
    14             0.21064E 01                    0.26912E-02                    0.78270E 03
    16             0.20745E 01                    0.11600E-02                    0.17884E 04
    18             0.20513E 01                    0.49996E-03                    0.41028E 04
    20             0.20349E 01                    0.21549E-03                    0.94430E 04
```

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