A NEW TECHNIQUE FOR STEADY STATE AND

TRANSIENT ANALYSES OF INCOMPRESSIBLE FLOW NETWORKS

BY

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ABSTRACT

A new technique for the calculation of the transient or the steady-state mass flow rate and pressure distribution in incompressible flow networks is presented. Employing the matrix method of network analysis, the nodal continuity and branch momentum equations are solved simultaneously to obtain explicit relations giving the unknown nodal pressures and branch mass flow rates. In this manner, the transient or the steady-state behavior of incompressible flow networks with arbitrary configuration having nodal sources and sinks as well as branch transducers can be determined. In contrast with the conventional steady-state network analysis methods, the new technique can be extended to the unsteady analysis of compressible flow in networks having an arbitrary configuration with heat transfer and phase change. To ascertain the accuracy of the solution, a numerical stability and convergence analysis is performed which provides an estimate for the upper bound of the time increment needed for a stable and convergent solution. The new technique can be applied to the treatment of transient problems such as flow coastdown studies resulting from loss of pumping power in nuclear water reactors, hydraulic transients of the cooling system for large steam power plants as well as the steady-state analysis of water distribution networks. The latter application is demonstrated in this study.

APPROVAL OF THESIS

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PART ONE

ANALYTICAL FORMULATION AND RESULTS

I. INTRODUCTION

Computation of steady-state or transient mass flow rate and pressure distribution in fluid networks with given geometric configuration and dimensions is a common problem in many branches of engineering. The design of municipal water and gas distribution systems as well as the dynamic analysis of thermal systems in conventional and nuclear power plants are a few typical examples. In the order of their analytical complexity, the problem of fluid flow in networks may be divided into the following categories:

a) Steady-state, isothermal, incompressible flow

b) Transient, isothermal, incompressible flow

c) Transient, compressible flow with heat transfer

d) Transient, compressible flow with heat transfer and phase change.

The solution to the above network problems has been generally obtained on digital computers. Several difficulties have been observed in these numerical solutions:

1) When the number of network nodes and branches become large, the computer storage requirement and running time for the solution become excessive.

2) The iterative procedure, used for the solution of steady-state problems do not always converge or may at times converge very slowly.

3) The time integration used for the solution of transient

problems may become numerically unstable or non-
convergent.

4) The computer programs are not sufficiently general
to treat networks of any arbitrary configuration
with all system components encountered in practice.

These difficulties become more severe as one moves
from problems in the "a" category to "d". This situation
indicates that new techniques for the solution of fluid
networks are needed. To facilitate this development,
the new technique may first be divised for problems in
the "a" or "b" category and later extended to "c" and "d"
categories.

This study presents one such new technique for the
solution of isothermal, incompressible fluid flow in net-
works under steady-state or transient conditions. In
contrast with conventional steady-state network analysis
methods, this new technique can be extended to the unsteady
analysis of compressible flow networks having an arbitrary
configuration with heat transfer and phase change. In
the present work, the matrix method of network analysis
(5)* is applied to the nodal continuity and branch momen-
tum equations. In this manner, explicit relations for
the unknown nodal pressures and branch mass flow rates

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* Underlined numbers in parentheses designate references
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are obtained. The present formulation is set up for
the solution of problems in the "b" category; i.e., the
analysis of isothermal, incompressible fluid flow in net-
works under transient conditions. Employing the present
formulation, problems in the "a" category, i.e., the
analysis of isothermal, incompressible fluid flow in net-
works under steady-state conditions, can be readily
solved by the application of the dynamic relaxation
technique (17) as shown later in this study. The exten-
sion of the present formulation to problems in "c" and "d"
categories require further mathematical development and
is currently in progress by other investigators at the
Newark College of Engineering.

II. REVIEW OF PREVIOUS WORK

The most widely used method for the solution of
problems in the "a" category, i.e., the analysis of
isothermal, incompressible flow in networks under steady-
state conditions, is the well-known Hardy Cross iterative
procedure. This procedure is commonly divided into
1) the nodal method, and 2) the loop method (3). In the
nodal method, the nodal heads are continually corrected
until the mass flow rates at each node is balanced. In
the loop method, the mass flow rates are initially
assumed for each pipe branch so that a mass balance is
maintained at each junction. The head loss between any
two junctions is then calculated for each possible path
between those points. The mass flow rate distribution is
then adjusted until the head loss between any two junc-
tions for each possible path between those points are
sufficiently close. Employing the Hardy Cross relaxation
techniques, Hoag and Weinberg (9), Graves and Branscome (8),
Adams (1), Dillingham (6) and Rosenhan (20) and a number
of other investigators developed digital computer programs
to perform hydraulic network analyses. These programs are
at times hampered by numerical instability and slow rate
of convergence. Various techniques for improving the
convergence problem are discussed in a number of the above
references. However, these techniques cannot guarantee the
convergence of the numerical solution.

The second method used for the analysis of iso-
thermal, incompressible flow in networks under steady-
state conditions is the direct solution method. In
this method, the continuity and momentum equations are
first expressed for every network node and branch
respectively. The momentum equations are then solved
algebraically for the unknown branch flows in terms of
the nodal pressures. Substituting these unknown mass
flow rates into the continuity equations, a system of
nonlinear simultaneous equations for nodal pressures are
obtained. These equations are then solved by using
either the Newton-Raphson method or by the direct solu-
tion of the simultaneous linearized equations. Employing
the direct solutions method, Martin and Peters (12),
Pitchai (18), Shamir and Howard (21), and Marlow et al
(11) developed digital computer programs to perform
hydraulic network analyses. These programs require a
larger computer memory storage, as compared to the programs
based on the Hardy Cross Method (3, 21) and are at times
hampered by the divergence of the numerical solution.
This latter difficulty may be overcome if a good initial
guess for the solution is available (21).

A systematic approach to the analysis and synthesis
of complex networks involving unsteady flow has been reported
by A. Reisman (19). This study presents a generalized

structural and mathematical framework to analyze or synthesize a large class of fluid systems where transient behavior is of primary interest. However, the network analysis and the computer program required for the numerical treatment of complex fluid systems have not been presented in this article. Instead, the need for the development of a computer program with sufficient flexibility to simulate fluid networks with an arbitrary configuration is emphasized.

Analog computer methods, based on nonlinear electrical resistances, have been employed by McIlroy (13), McPherson and Radziul (14), Barker (2), Kiyose (10) and many others. The merit of this approach is that, once the circuits have been prepared, the required solutions are obtained instantaneously. The disadvantages of the analog computer method are that the simulation of large networks require a large number of analog components and considerable time for setting up the machine. For these reasons, digital machines are considered more flexible than analog computers and are generally preferred in this type of study.

The foregoing survey indicates that new techniques for the solution of fluid networks are needed. Recently, a new method suitable for both transient and steady-state analyses of isothermal, incompressible fluid networks was presented by A.N. Nahavandi (16). The main distinctive

features of this method are that: 1) it lends itself to a linkage with the matrix method of network analysis; and 2) the extension of the method to the unsteady analysis of compressible flow in networks having an arbitrary configuration with heat transfer and phase change, seems to be feasible. This method constitutes the foundation for the present study.

III. SCOPE AND OBJECTIVES OF THE PRESENT STUDY

The main objectives of the present study are as follows:

1) To expand the method developed by Nahavandi (16) for the transient and steady-state analyses of isothermal, incompressible flow networks and to link this procedure with the matrix method of network analysis (5) often used in the study of D.C. electrical networks.

2) To develop a digital computer program for the transient and steady-state analyses of complex hydraulic networks based on this new technique.

3) To verify the results of the present study against the results of another computer program based on the Hardy Cross method.

The present study provides the following flexibilities:

a) The hydraulic network considered may consist of any arbitrary number of nodes and branches connected according to any desired configuration. A node is defined as any point in the system at which either three or more flows meet or network geometric dimensions change. A branch is defined as the line connecting any two nodes.

b) Any node may be connected to a number of other nodes.

c) Sources and sinks with given input and output mass flow rates may be introduced at any node.

d) Every branch may include a transducer, such as a
pump or a turbine, whose differential head versus
flow characteristic is known.

The present analysis is based on the following
simplifying assumptions:

a) The fluid flow in the system is considered incom-
pressible, isothermal and turbulent. No mass storage
is allowed at any node or within any branch. Further-
more, the wave propagation effect, heat transfer and
phase change are not considered in the analysis. In
other words, this study presents a procedure for the
solution of problems in categories "a" and "b" dis-
cussed in the introduction.

b) The friction factor for head loss calculations is based
on correlations developed under steady-state conditions.
In other words, the frequency-dependence of friction
is ignored in the present analysis. Two types of
frictional head loss correlations are employed:
1) Darcy-Weisbach frictional head loss equation with
curve-fitted Moody's friction factor correlation (4,
15); and 2) Hazen-Williams frictional head loss equa-
tion (9, 21).

The present analysis can be applied to the treatment
of many engineering problems. A few typical examples of
such applications follows:

In the area of transient analysis, the present
study may be applied to the solution of: 1) flow coast-
down problems resulting from loss of pumping power in
nuclear water reactors (7); and 2) hydraulic transients
of the cooling system for large steam power plants (19).

In the area of steady-state analysis, the present
study may be applied to the solution of water distribution
networks. To understand this particular application, it
should be borne in mind that the hydraulic transient solu-
tion can be made to approach asymptotically to the steady-
state solution if the system boundary conditions are kept
unchanged during the dynamic study. Thus, to perform a
steady-state analysis of a water distribution network, the
initial flow distribution is assumed. The system source
and sink pressures and mass flow rates are then held
constant during the transient analysis. Under these con-
ditions, the nodal pressures and branch mass flow rates
approach asymptotically to steady-state values. This
particular application is commonly known as dynamic
relaxation (17) and is further discussed in this report.

IV. MATHEMATICAL FORMULATION

In the present analysis, first the pressure drop
versus pressure, nodal continuity, and branch momentum
equations are expressed in matrix form. These equations
are then solved to obtain explicit relations for unknown
nodal pressures and branch mass flow rates as follows.
An illustrative example for this procedure is presented
in the Appendix.

The branch pressure drop array Δp, designating the
pressure drop in the positive flow direction, is related
to nodal pressures p by

$$\{\Delta p\} = [A] \{p\} \tag{1}$$

where A is the connection matrix defined as follows:
Every branch of the network, under investigation, is first
given an arbitrary positive orientation or direction (see
Fig. 1). The connection matrix for branches and nodes is
then defined as a matrix whose elements are +1, -1, or 0.
only. The rows of the connection matrix correspond to the
network branches such that the maximum number of rows are
equal to the maximum number of system branches. The columns
of the connection matrix correspond to the network nodes
such that the maximum number of columns are equal to the
maximum number of system nodes. If the branch orientation
is "away" from the node, the corresponding element of the
connection matrix is +1. If the branch orientation is
"toward" the node, the corresponding element of the con-

nection matrix is -1. If the branch and the node do not
meet, the corresponding element of the connection matrix
is 0.

The nodal continuity equation, considering both the
branch flow and the nodal source and sink (input and out-
put) flows is given by the following matrix equation:

$$\left[A^T\right] \quad \left\{\dot{m}\right\} \quad + \quad \left\{\dot{m}_{io}\right\} \quad = \quad \left\{0.\right\} \tag{2}$$

where A^T is the transpose of the connection matrix and $\dot{m}$
and $\dot{m}_{io}$ are the branch flow and the nodal source and sink
(input and output) flow arrays respectively.

The branch momentum equation in finite difference form
in time, relating the contributions of inertia, eleva-
tional, frictional and pump head to the branch pressure
drop is given by the following matrix equation:

$$\left\{\frac{\dot{m} - \dot{m}^o}{h}\right\} \quad = \quad \left\{\frac{ag}{L} \quad \left(144 \ \Delta p + \rho \Delta Z + \rho H - F\right)\right\} \tag{3}$$

where
$$\left\{F\right\} \quad = \quad \left\{\frac{1}{2g\rho} \quad \left(\frac{fL'}{D}\right) \quad \frac{\dot{m}^o \ |\dot{m}^o|}{a^2}\right\} \tag{4}$$

and
$$\left\{f\right\} \quad = \quad \left\{.0055 \left[1. + \left(2. \times 10^4 \ \frac{\epsilon}{D} + \frac{10.^6}{R_n}\right)^{.333}\right]\right\} \tag{5}$$

In the above relations, a, D, L and L' represent the
branch flow cross-sectional area, hydraulic diameter,
length and equivalent length respectively. The inertia
term is based on a forward finite difference form in time
as the difference between the updated and the present

values of the mass flow rates ($\dot{m}$ and $\dot{m}^o$) divided by the
time increment h. A criterion for the selection of this
time increment will be presented later. Values of ΔZ, H,
F and R_n represent the branch elevational head, pump head
rise, frictional head loss and Reynolds Number; while $\in$, f
and ρ designate pipe absolute roughness, fluid friction
factor and density.

Equations (4) and (5) above represent Moody's method
(4,15) for the calculation of frictional head loss. In
order to be able to check the results of the present analy-
sis against those of Reference (20), provisions to replace
Moody's method by Hazen-Williams' method (9,21) are incor-
porated in the program. In the context of our present
formulation, the Hazen-Williams' formulation is given by

$$\left\{ F \right\} = \left\{ \frac{3.02}{C^{1.85} \, \rho^{.85}} \left(\frac{L}{D^{1.165}} \right) \frac{\dot{m}^o \, |\dot{m}^o|}{a^{1.85}}^{.85} \right\} \quad (4a)$$

The quantity C, the Hazen-Williams' coefficient, depends on
the type and the condition of the conduit and is tabulated
in Reference (20).

If a network consists of j branches and i nodes, equa-
tions (1), (2) and (3) constitute a set of 2j + i algebraic
equations in 2j + i unknowns. These unknowns are branch
pressure drops, branch mass flow rates and nodal pressures.
To solve the above system of algebraic equations, we eli-
minate the pressure drops and mass flow rates among

equations (1), (2) and (3) to obtain a single matrix equation for the unknown nodal pressures. Solving the matrix equation (3) for $\dot{m}$ array, one obtains

$$\{\dot{m}\} = \{\dot{m}^o\} + \{144 \; \gamma \Delta p + \gamma \rho \Delta Z + \rho \gamma H - \gamma F\} \qquad (6)$$

where

$$\{\gamma\} = \left\{\frac{agh}{L}\right\}$$

Substituting the Δp array from equation (1) into equation (6) and substituting the resulting equation into equation (2) yields

$$[A^T]\{\dot{m}^o\} + [A^T][144\gamma][A]\{p\}$$
$$+ [A^T]\{\rho\gamma\Delta Z + \rho\gamma H - \gamma F\} + \{\dot{m}_{io}\} = \{0.\} \qquad (7)$$

Solving equation (7) for the unknown nodal pressure array gives

$$\{p\} = [M]^{-1}[A^T]\{-\rho\gamma\Delta Z - \rho\gamma H + \gamma F - \dot{m}^o\}$$
$$- [M]^{-1}\{\dot{m}_{io}\} \qquad (8)$$

where

$$[M] = [A^T][144\gamma][A] \qquad (9)$$

Having the nodal pressures from equation (8), one can readily calculate the branch pressure drops and mass flow rates from equations (1) and (6) respectively. It is important to realize that $[144\gamma]$ is a diagonal matrix having branch γ's along its diagonal and zero elsewhere.

The main distinctive feature of the present analysis
is that the steady-state as well as the transient nodal
pressures and branch mass flow rates are calculated by the
time-integration of the network transient equations as
given above. Initial values of the branch mass flow rate
array $\overset{\bullet}{m}{}^{o}$ are assumed such that continuity equations are
satisfied at each node between branch flows $\overset{\bullet}{m}{}^{o}$ and nodal
source and sink flows $\overset{\bullet}{m}_{io}$. Care must be taken not to
confuse values of $\overset{\bullet}{m}{}^{o}$ and $\overset{\bullet}{m}_{io}$. The former quantity repre-
sents the present values of the mass flow rates which vary
during the problem solution; while the latter designates
fixed known flow quantities entering or leaving the system
at various nodes. Following every application of equations
(8), (1) and (6), the updated values of the branch mass
flow rates are computed. The present values of the branch
mass flow rates are then set equal to the updated values
and the application of equations (8), (1) and (6) are con-
tinued. For steady-state analyses, this procedure is
continued until the values of branch mass flow rates con-
verge within a prescribed error.

To perform a transient analysis, the converged steady-
state mass flow rates are entered as the initial values
for $\overset{\bullet}{m}{}^{o}$ array. The problem solution is then restarted
under the new prevailing conditions until a new set of
steady-state mass flow rates is reached. This latter
application can be made more clear by considering an

example such as the flow coast-down problem resulting from
loss of pumping power in a nuclear water reactor. Prior
to the initiation of the pump coast-down transient study,
the present analysis is employed once to bring the system
to steady-state. The loss of pumping power is then ini-
tiated and the present analysis is employed for the second
time to determine the flow behavior during the pump coast-
down.

It should be noted that calculation of nodal pressures
from equation (8) involves the inversion of matrix M. The
elements of this matrix are independent of flow and the
inversion procedure can be performed only once at the
beginning of analysis. Furthermore, multiplications of M
inverse by A transpose and M inverse by $\dot{m}_{io}$ are also inde-
pendent of time and can also be performed once at the
beginning of the analysis. The remaining operations indi-
cated by equation (8) are time dependent and should be
performed once in every time increment during the transient
calculations. For large networks, the order of matrix A
becomes large. This will obviously increase the computer
running time for inversion, multiplications and other opera-
tions described above. This situation, however, will not
create any complication in the analysis.

V. NUMERICAL SOLUTION

The numerical solution to the problem is obtained by solving equations (8), (1) and (6) on a digital computer. The input data for this analysis consists of network topography, geometric dimensions, hydraulic properties, problem initial conditions and control variables. The main input data in this study are as follows:

1) The connection matrix A (only the non-zero terms are entered as input data).

2) The branch actual length L, equivalent length L', hydraulic diameter D, nodal elevation Z arrays.

3) The fluid density ρ, viscosity μ, and pipe absolute roughness ϵ.

4) The pumps locations and their head versus volumetric flow characteristics.

5) Initial branch mass flow rates $\dot{m}^o$.

6) Nodal sink and source mass flow rates $\dot{m}_{io}$.

7) Integration time increment h and steady-state flow convergence error.

The numerical calculations can be conveniently divided into two groups. The first group consists of mathematical operations performed once at the beginning of the program. The assembly of the connection matrix and its transpose, the formation of matrix M defined by equation (9) and its inverse as well as the calculation of all flow-independent terms in equations (8) and (1) are among this group. The

second group consists of iterative operations performed
after each time increment. The calculation of branch
frictional head loss and pump pressure rise as well as
the computation of all flow-dependent terms in equations
(8) and (1) are among this group. Classification of the
numerical calculations into the above groups increases the
computational efficiency of the program and reduces the
computer running time.

To perform the required mathematical computations,
a number of matrix operation subroutines, such as matrix
addition, subtraction, multiplication, inversing and
transposing were needed. These subroutines were obtained
from Reference (22).

VI. NUMERICAL STABILITY ANALYSIS

The present analysis is based on the numerical integration of the branch momentum differential equations (equation 3). To obtain a convergent solution, it is necessary that the selected time increment h for this integration be smaller than the minimum "time constant" associated with these equations by a "good" margin. The momentum equations are nonlinear and an accurate calculation of their "time constants" cannot be readily made prior to their numerical solution. Fortunately, the accurate calculation of these "time constants" are not necessary and a conservative estimate is all that is needed for the purpose of obtaining a convergent solution. This objective can be achieved by making a number of simplifying assumptions. First, since the dependence of the friction factor and the pump head on mass flow rate is generally weak, it is reasonable to assume that these two variables are flow-independent. Second, the stability analysis will be performed on the linearized momentum equations. This assumption is heuristically justified because the linearized equations approximate the nonlinear equations in small time regions. If the linearized solution is convergent for every time increment, the overall nonlinear solution will also be convergent. It should be further emphasized that these simplifying assumptions are made only for the stability analysis and not for the actual analysis presented in this study.

Based on the above assumptions, the branch momentum equation may be expressed by

$$\frac{L}{ag} \frac{d\dot{m}}{dt} + K \dot{m}^2 = c \tag{10}$$

where

$$K = \frac{1}{2g\rho} \left(\frac{fL'}{D} \right) \frac{1}{a^2} \tag{11}$$

and c represents the sum of the pressure drop, the elevational head and the pump head rise. Since the linearized equation will be used only within a small time increment, we can treat both K and c as constants and find the variation of equation (10)

$$\frac{L}{ag} \frac{d(\delta\dot{m})}{dt} + 2K \dot{m}^o \delta\dot{m} = 0 \tag{12}$$

Equation (12) defines a linear differential equation in $\delta\dot{m}$ at each value of $\dot{m}^o$ for each time increment h. The time constant for this linear differential equation is

$$\tau_o = L \Big/ 2Kag \dot{m}^o \tag{13}$$

Substituting equation (11) into (13), the final expression for branch time constants become

$$\tau_o = \left(\pi D^3 \rho L \right) \Big/ 4fL' \dot{m}^o \tag{14}$$

Experience with the operation of the present program has indicated that if for each integration step, the time increment is smaller than the minimum time constant by a margin of 20. i.e. if:

$$h < \frac{\tau_{o\ min}}{20.} \tag{15}$$

the numerical integration is convergent. Thus, to obtain
a convergent solution, the branch time constants are com-
puted from equation (14) prior to each integration step
and their minimum value is selected. The time increment
for the next time interval is then chosen to be one twen-
tieth of the minimum time constant. This margin of "20"
has proved to be satisfactory for the present analysis and
has resulted in a convergent solution. In the event that
approximate values of the steady-state flows are known
from a previous run, it is more economical to compute the
time increment on the basis of these flows. The time incre-
ment can then be held constant throughout the dynamic ana-
lysis. This latter method is used in the computer program
presented in this study.

The reason for using the minimum time constant can be
made clear by the following argument. Any arbitrary net-
work consists of a number of parallel and series branches.
For parallel branches, the line with minimum time constant
has the fastest time response and should therefore be used
for numerical stability purposes. For a j number of series
branches, the equivalent time constant can be easily shown
to be

$$\mathcal{T} = \sum_{n=1}^{j} \left(\frac{L}{ag} \right)_n \Bigg/ \left[2\dot{m}^0 \sum_{n=1}^{j} (K)_n \right] \qquad (16)$$

This equivalent time constant, can be readily shown to be

larger than the smallest time constant among these branches.
Consequently, the minimum time constant branch provides a
conservative estimate for usage in the numerical stability
analysis.

When the transient solution is of no interest and only
the final steady-state values are desired, the time incre-
ment criterion is not as stringent as indicated by equation
(15). The time increment can be increased to the order of
the system minimum time constant or even higher leading to
a large reduction in computer running time. The increase
in the time increment reduces the actual branch time
constants without adversely affecting the steady-state
results.

It should be further emphasized that numerical in-
stability and lack of convergence constitute two major
hazards of any numerical integration scheme. Numerical
oscillations are caused by computational round-off errors
and the lack of convergence in the result of integration
truncation error. Both the numerical instability and the
lack of convergence problems can be overcome by the intro-
duction of an appropriate upper bound for the integration
time step h as indicated by equation (15).

VII. PRESENTATION OF RESULTS

The new computational technique, developed in this
study, is verified by employing the present digital pro-
gram to determine the flow and pressure distribution in
the hydraulic network shown on Figure 1. A steady-state
numerical solution to this problem, based on the Hardy
Cross method is available (20) and is used to check the
correctness of the present dynamic analysis. The hydraulic
network, shown on Figure 1, represents a typical city water
distribution system. All system nodes and branches are
numbered independently and positive flow directions are
shown by arrows. Water enters the system through branch 1,
at a rate of 2825 gpm with a constant pressure of 60 psi,
and leaves the system through branches 30, 31, 32, 34 and
35 at 450, 400, 500, 650 and 825 gpm respectively. The
length, internal diameter, and elevational difference for
every system branch is specified in Table 1. In order to
be able to match the present results against those of Ref-
erence (20), no booster pump is placed in any branch and
the equivalent length of each branch is taken equal to its
actual length. It should be pointed out that the present
analysis is not restricted by these assumptions. These
restrictions are introduced to obtain a meaningful compari-
son between the results of the present study and those of
Reference (20).

Typical transient results are demonstrated on Figures

2 and 3. The final steady-state results are shown on Tables 2 and 3 for all system branches.

The above results are obtained by the application of the digital computer program developed for this study, designated as HYTRAN, which stands for Hydraulic Transient Analysis. The convergence criterion for reaching steady state was initially based on the relative mass flow rate error. This value was computed by subtracting the present values of the branch mass flow rates $\dot{m}^o$ from the updated values of the branch mass flow rates $\dot{m}$ and dividing the result by the updated values of the branch mass flow rates $\dot{m}$. The absolute values of these relative errors were compared with a quantity designated as steady-state flow convergence error. When the relative mass flow rate errors for all branches are smaller than the steady-state flow convergence error, steady-state conditions were considered to be reached.

Experience with the usage of this convergence criterion during the debugging of the HYTRAN program showed that this criterion is not desirable. A better criterion based on a fluid acceleration, defined by $(\dot{m}-\dot{m}^o)/h$, proved to be more satisfactory.
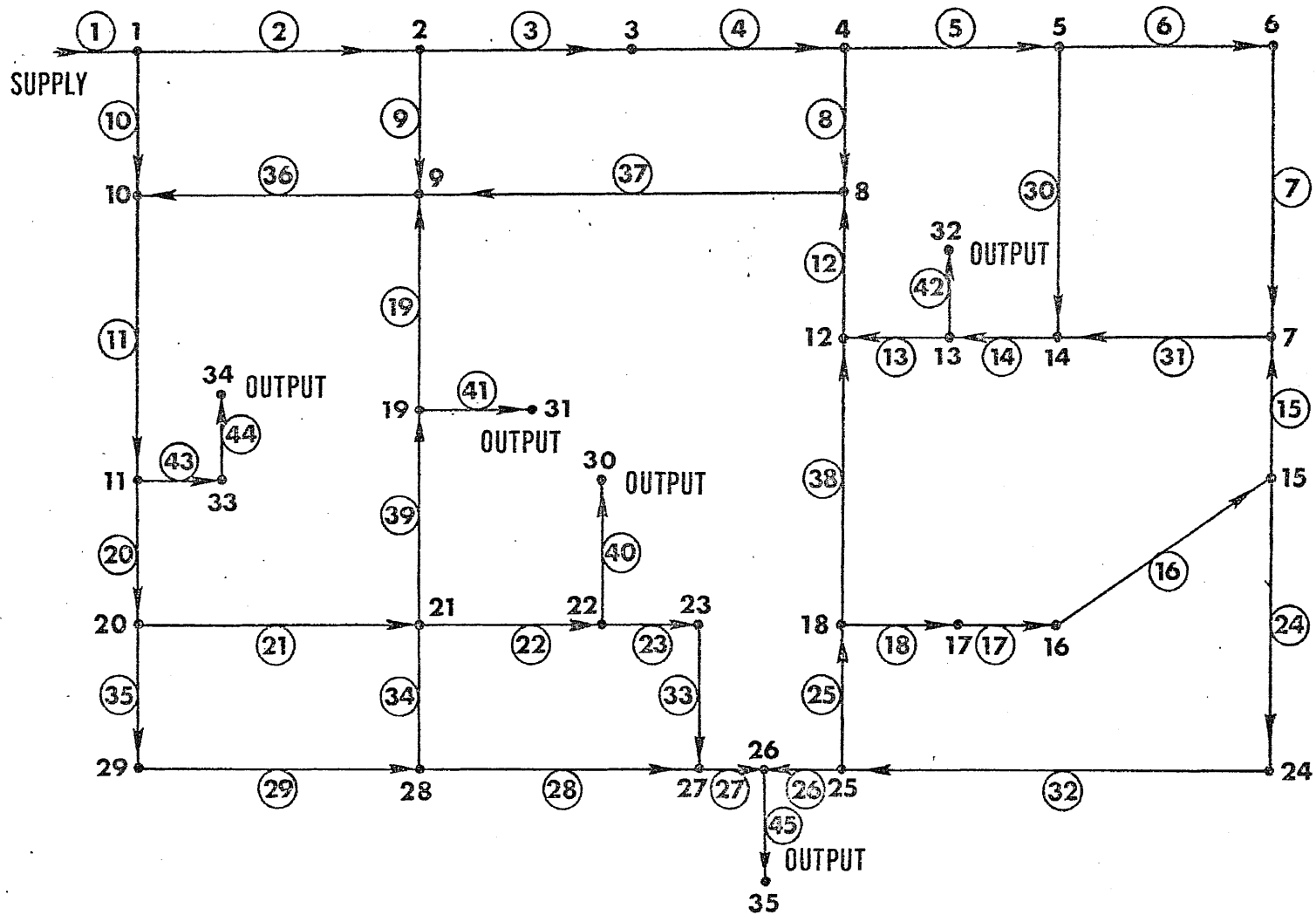
Figure 1
A Typical City Water Distribution System Employed to Verify
the New Computational Technique Presented in This Study

TABLE I – GEOMETRIC SPECIFICATIONS FOR THE HYDRAULIC
NETWORK SHOWN ON FIG. 1

Branch No.	Diameter D, In.	Length L, Ft.	Elevation Difference ΔZ, Ft.
1	20	1	0
2	20	200	0
3	20	150	− 10
4	20	120	+ 5
5	20	130	+ 5
6	20	100	− 10
7	10	200	− 10
8	10	50	− 5
9	10	100	− 40
10	10	100	− 40
11	8	80	− 10
12	8	50	+ 5
13	8	200	0
14	6	70	0
15	6	80	+ 10
16	5	72	0
17	5	16	0
18	5	130	− 10
19	8	120	− 10
20	6	85	+ 10
21	6	100	+ 20
22	6	70	+ 5
23	5	30	+ 5
24	5	205	− 10
25	8	90	0
26	5	20	0
27	5	30	− 5
28	5	205	0
29	5	100	+ 15
30	6	200	− 15
31	8	75	+ 5
32	5	300	+ 20
33	6	100	− 5
34	6	100	− 5
35	5	100	+ 10
36	10	200	0
37	10	300	− 30
38	5	200	+ 5
39	6	100	− 10
40	6	100	− 5
41	5	5	− 5
42	5	5	− 5
43	5	50	0
44	5	5	− 5
45	5	6	− 5

BRANCH 11

------- STEADY STATE RESULT OF REFERENCE (20)
BASED ON HAZEN-WILLIAMS HEAD LOSS EQUATION

------- PRESENT ANALYSIS BASED ON MOODY'S
HEAD LOSS EQUATION

——— PRESENT ANALYSIS BASED ON HAZEN-WILLIAMS
HEAD LOSS EQUATION

TIME, SEC.
Figure 2
Typical Time Variation of Mass Flow Rate and Frictional Head Loss for Branch 11

28

FRICTION HEAD LOSS, FT.

1.0

.5

0

BRANCH 23

40

MASS FLOW RATE, LBS/SEC.

30

20

10

0

— — — — STEADY STATE RESULT OF REFERENCE [20]
BASED ON HAZEN-WILLIAMS HEAD LOSS EQUATION

— — — — — PRESENT ANALYSIS BASED ON MOODY'S
HEAD LOSS EQUATION

————— PRESENT ANALYSIS BASED ON HAZEN-WILLIAMS
HEAD LOSS EQUATION

0   5   10   15   20   25   30   35   40

TIME, SEC.
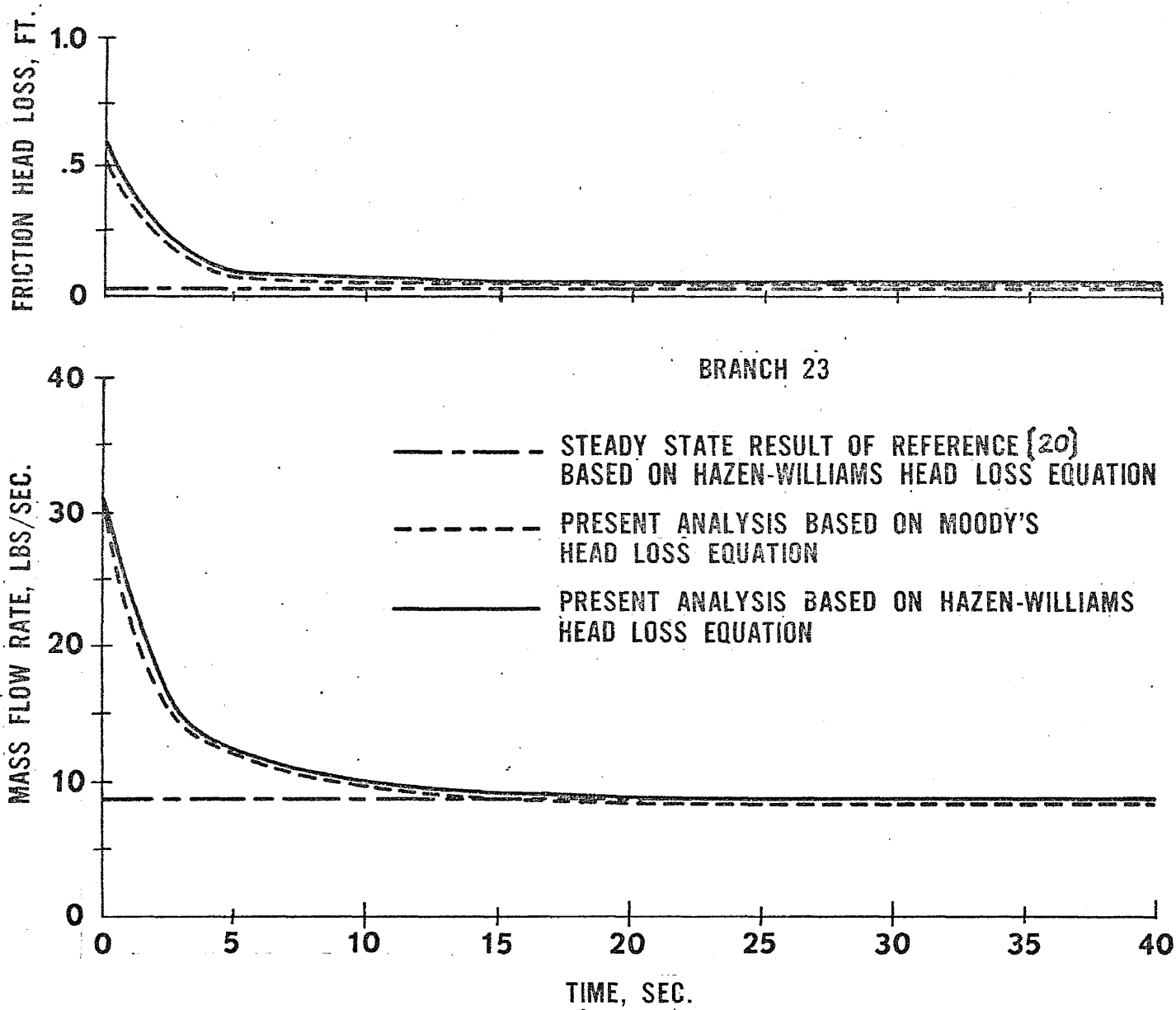
Figure 3

Typical Time Variation of Mass Flow Rate and Frictional Head Loss for Branch 23

TABLE 2 — STEADY- STATE MASS FLOW RATES (lbs./sec.) FOR
THE HYDRAULIC NETWORK SHOWN ON FIG. 1

Branch No.	Present Analysis Using Moody's Head Loss Equation	Present Analysis Using Hazen-Williams' Head Loss Equation	Reference (20)
1	392.7	392.7	392.7
2	274.0	271.6	271.1
3	176.8	174.5	173.7
4	176.8	174.5	173.7
5	74.9	74.1	73.9
6	59.3	58.5	58.3
7	59.3	58.5	58.3
8	101.9	100.4	99.8
9	97.1	97.1	97.5
10	118.7	121.1	121.3
11	134.2	133.9	133.8
12	75.3	76.7	76.7
13	42.1	42.9	42.9
14	27.4	26.6	26.7
15	47.5	47.5	47.5
16	27.9	28.0	28.0
17	27.9	28.0	28.0
18	27.9	28.0	28.0
19	108.2	108.0	107.9
20	43.8	43.6	43.5
21	31.7	31.7	31.7
22	71.1	71.4	71.3
23	8.5	8.8	8.8
24	19.7	19.5	19.5
25	61.1	61.8	61.7
26	80.8	81.3	81.2
27	33.9	33.4	33.4
28	25.3	24.6	24.6
29	12.1	11.9	11.9
30	15.6	15.6	15.7
31	11.8	11.0	10.8
32	19.7	19.5	19.5
33	8.5	8.8	8.8
34	13.2	12.7	12.7
35	12.1	11.9	11.9
36	15.5	12.8	12.5
37	26.5	23.7	23.1
38	33.2	33.8	33.7
39	52.6	52.4	52.4
40	62.6	62.6	62.5
41	55.6	55.6	55.6
42	69.5	69.5	69.5
43	90.3	90.3	90.3
44	90.3	90.3	90.3
45	114.7	114.7	114.7

TABLE 3 — STEADY-STATE FRICTIONAL HEAD LOSS (ft.) FOR THE HYDRAULIC NETWORK SHOWN ON FIG. 1

Branch No.	Present Analysis Using Moody's Head Loss Equation	Present Analysis Using Hazen-Williams' Head Loss Equation	Reference (20)
1	.0	.0	.0
2	.163	.239	.23
3	.052	.079	.08
4	.042	.063	.06
5	.008	.014	.01
6	.004	.007	.01
7	.294	.409	.40
8	.213	.278	.27
9	.387	.523	.52
10	.575	.787	.77
11	1.884	2.249	2.20
12	.375	.500	.49
13	.048	.068	.07
14	.321	.402	.39
15	1.085	1.341	1.31
16	.881	1.105	1.08
17	.196	.245	.24
18	1.591	1.996	1.96
19	1.843	2.267	2.22
20	.982	1.214	1.19
21	.609	.792	.78
22	2.106	2.493	2.44
23	.036	.054	.05
24	1.264	1.604	1.57
25	.447	.605	.59
26	2.022	2.201	2.16
27	.539	.638	.63
28	2.079	2.474	2.43
29	.479	.627	.61
30	.305	.429	.43
31	.015	.021	.02
32	1.850	2.348	2.30
33	.047	.074	.07
34	.109	.146	.14
35	.239	.313	.31
36	.022	.024	.02
37	.093	.115	.11
38	3.468	4.338	4.25
39	1.655	2.012	1.97
40	2.335	2.791	2.74
41	.240	.272	.27
42	.374	.412	.40
43	6.312	6.696	6.56
44	.631	.669	.66
45	1.218	1.249	1.22

Figures 2 and 3 show the typical time-variation of mass flow rate and frictional head loss for branches 11 and 23. An examination of these curves and Tables 2 and 3 reveals the following features:

1) Application of Moody's and Hazen-Williams' frictional head loss correlations affect the shape of the system response curves. This is to be expected since the coefficients and powers appearing in equations (5) and (4a) are somewhat different. The difference between values obtained from the application of Moody and Hazen-Williams is a function of branch diameter and mass flow rate. For certain branches, the combination of diameter and mass flow rate results in a closer agreement between the response curves. The choice between Moody's or Hazen-Williams' frictional head loss correlation is a matter of personal preference. In any event, both correlations yield comparable results for practical purposes.

2) The steady-state values of branch flows and frictional head losses obtained from the present analysis, using Hazen-Williams' correlation are in a good agreement with those of Reference (20) which is also based on the same correlation. This close agreement between this analysis and Reference (20) verifies the correctness of the new technique presented in this study.

The apparent discrepancy between two frictional head losses (for branch numbers 5 and 6) is most likely due to data round-off in Reference (20).

3) Referring to the numerical stability analysis presented earlier, and noting that the system minimum time constant, based on steady-state flow, is 2 sec. (for branch 26), it can be concluded that the application of a constant time increment equal to 2.0/20. or h = .1 sec. throughout the analysis ensures a convergent solution for the problem in hand. Setting h = .1 sec., the steady-state conditions are reached after 482 iterations with a relative flow convergence error of less than .006 lb/sec^2. Decreasing the time increment to h = .05 sec. does not appreciably change the dynamic results; while increasing the time increment to h = .2 sec. affects the system response curves. This test verifies the correctness of the numerical stability analysis presented earlier in this study.

4) To provide a comparison between the computational time required by this new method against the computer program presented in Reference (20), both programs were run on an RCA Spectra 70/45 digital computer. For the steady-state analysis, since the dynamic behavior is of no interest, the time increment can be increased to the same order of the system minimum time constant

(3 sec.). This increase in the time increment reduces the actual branch time constants and speeds up the computer running time without adversely affecting the steady-state results. The computer running time required to obtain steady-state solutions for the above-mentioned hydraulic network problem is 108 sec. for the present program and 134 sec. for the program presented in Reference (20).

5) The dynamic curves shown on Figures 2 and 3 have the following physical significance. Let us assume that at time t = 0 the branch mass flow rates are those typically indicated in the Figures, and the input pressure and flow at branch 1 as well as the output flows at branches 30, 31, 32, 34 and 35 remain constant. The mass flow rate time histories in the system branches will then undergo a transient behavior typically shown in the above Figures and will approach asymptotically to final steady-state values. In other words, in the present study, both the steady-state and transient solutions are obtained by a time-dependent dynamic analysis. Values of dependent variables at intermediate time points represent the actual system behavior during a transient and are, therefore, physically meaningful. In contrast, the Hardy Cross and the direct solution methods obtain the final steady-state values after a number of iterations (9). For these

analyses, values of dependent variables at these intermediate points have no physical significance.

VIII. CONCLUSIONS

A new technique for the steady-state and transient analyses of isothermal, incompressible flow in networks is presented. In this method, the pressure drop versus pressure, nodal continuity and branch momentum equations are expressed in matrix form for networks with any arbitrary configuration. These equations are then solved to obtain explicit relations for unknown nodal pressures and branch mass flow rates.

In contrast with the conventional steady-state network analysis methods, the present technique can be extended to the transient analysis of compressible flow in networks with heat transfer and phase change. This extension is currently under development by other investigators at the Newark College of Engineering.

To ascertain the accuracy of the solution, a numerical stability and convergence analysis is performed. This analysis provides an estimate for the upper bound of the time increment needed for a stable and convergent solution.

The new technique is applied to the analysis of a water distribution network. The results obtained are in agreement with the available solution.

IX. RECOMMENDATIONS

The new technique, presented in this study, can be utilized in the development of more efficient and more economical engineering tools needed for the design of more complex fluid systems. Two such applications are described hereunder:

1) This study can be extended to analyze thermal systems involving compressible flow with heat transfer and phase change. The new program will then be able to analyze power plant and processing plant networks under steady-state and transient conditions more efficiently and more economically than is presently possible.

2) The present program can determine branch mass flow rates and nodal pressures for a system with given configuration and pipe diameters. The actual design of hydraulic networks requires the selection of pipe diameters to achieve a set of hydraulic and economic objectives. The present program can be extended to treat this design problem with improved efficiency and economy.

The present version of HYTRAN program may be improved by incorporating the following modifications:

1) A subroutine may be added to the program to compute the upper bound of the time increment automatically during

the dynamic analysis. This feature is in contrast
with the presently hand-computed time increment used
as an input data to the program. The procedure for
the calculation of this upper bound is given in this
study.

2)   A laminar pressure drop versus flow relationship may
     be added to the program to provide a more accurate
     calculation of frictional losses during laminar flow
     regimes.

3)   Expansion and contraction losses can be made dependent
     on the flow direction. The present version of HYTRAN
     assumes identical losses in both flow directions.

X. NOMENCLATURE

A   Connection matrix for branches and nodes as defined in the text

a   Branch flow cross-sectional area, ft^2

C   Hazen-Williams' coefficient

c   A constant defined in the text

D   Branch hydraulic diameter, ft

F   Pressure loss due to friction, lb/ft^2

f   Friction factor

g   Acceleration of gravity, 32.2 ft/sec^2

H   Transducer pressure differential (pump pressure rise), ft

h   Time increment, sec

K   A constant defined by equation (11)

L   Branch actual length, ft

L' Branch equivalent length (Branch actual length plus additional lengths allowed for fittings), ft

M   A matrix defined by equation (9)

$\dot{m}$ Branch mass flow rate, lb/sec

$\dot{m}_{io}$ Nodal source and sink (input and output) mass flow rate (positive for source and negative for sink), lb/sec

p   Nodal pressure, psi

R_n Branch Reynolds Number ($= D \dot{m}^o/a\mu$)

t   Time, sec

Z   Nodal elevations, ft

Greek Symbols

γ A quantity defined in the text

Δp Pressure drop along the positive flow direction, psi

$\delta\dot{m}$ A small variation of the branch mass flow rate, lb/sec

ΔZ Branch elevational difference along the positive flow direction (elevation of node at the end of the arrow minus elevation of the node at the head of the arrow), ft

ϵ Pipe roughness, ft

μ Fluid viscosity, lb/sec - ft

ρ Fluid density, lb/ft^3

τ Branch time constant, sec

Superscripts

o   Present value

T   Transpose of a matrix

Subscripts

Min   Minimum

Miscellaneous Symbols

$[\ ]$ Contains matrices

$\{\ \}$ Contains arrays

XI. REFERENCES

1) Adams, R.W., "Distribution Analysis by Electronic Computer"; Journal of the Institute of Water Engineers, Vol. 15, 1961, pp. 415-428.

2) Barker, C.L., "Use of the McIlroy Analyzer on Water Distribution Systems"; Journal of the American Water Works Association, Vol. 50, January 1958, pp. 15-20.

3) Barlow, J.F., and E. Markland, "Computer Analysis of Pipe Networks"; Proceedings of the Institution of Civil Engineers, Vol. 43, May-August 1969, pp. 249-259.

4) Benedict, R.P., and N.A. Carlucci, "Flow with Losses"; Journal of Engineering for Power, Paper No. 63-WA-105, 1963, pp. 1-13.

5) Branin, Jr., F.H., "The Algebraic-Topological Basis for Network Analogies and the Vector Calculus"; IBM Technical Report TR 00.1495, July 25, 1966, pp. 1-66.

6) Dillingham, J.H., "Computer Analysis of Water Distribution Systems, Part 2"; Water and Sewage Works, Vol. 114, No. 2, February 1967, pp. 43-45.

7) Fuls, G.M., "Flow Transients Resulting from a Loss of Pumping Power in a Pressurized Water Nuclear Reactor"; ASME Publication No. 70-WA/NE-4, 1970, pp. 1-5.

8) Graves, Q.B., and D. Branscome, "Digital Computers for Pipeline Network Analysis"; Journal of the Sanitary Engineering Division, ASCE, Vol. 84, No. SA2, April 1958, pp. 1608-1 -- 1608-18.

9) Hoag, L.N., and G. Weinberg, "Pipeline Network Analysis by Electronic Digital Computer"; Journal of American Water Works Association, Vol. 49, 1957, pp. 517-524.

10) Kiyose, J., "Development of Flow Computers for Network Analysis in Japan"; Journal of the American Water Works Association, Vol. 57, October 1965, pp. 1346-1353.

11) Marlow, T.A., et al., "Improved Design of Fluid Networks with Computers"; Journal of the Hydraulic Division, ASCE, HY4, 1966, pp. 43-61.

12) Martin, D.W., and G. Peters, "The Application of Newton's Method to Network Analysis by Digital Computers"; Journal of the Institute of Water Engineers, Vol. 17, pp. 115-129.

13) McIlroy, M.S., "Direct Reading Electric Analyzer for Pipeline Network Analysis"; Journal of the American Water Works Association, Vol. 42, April 1950, p. 347.

14) McPherson, M.B., and J.V. Radziul, "Water Distribution Design and the McIlroy Network Analyzer"; Journal of the Hydraulics Division, ASCE, Vol. 84, No. HY2, 1958, pp. 1588-1 -- 1588-15.

15) Moody, L.F., "Friction Factors for Pipe Flow"; Transactions of ASME, Vol. 66, 1944, p. 671.

16) Nahavandi, A.N., "The Loss-of-Coolant Accident Analysis in Pressurized Water Reactors"; Nuclear Science and Engineering, Vol. 36, 1969, pp. 159-188.

17) Otter, J.R.H., "Dynamic Relaxation of Shell Theory Equations for Arch Dams"; Theory of Arch Dams edited by J.R. Rydzewski, Proceedings of the International Symposium held at Southampton University, England, 1964, pp. 313-318.

18) Pitchai, R., "A Model for Designing Water Distribution Pipe Networks"; Thesis presented to Harvard University, at Cambridge, Mass. in 1966 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

19) Reisman, A., "On a Systematic Approach to the Analysis and Synthesis of Complex Systems Involving the Unsteady Flow of Fluids"; ASME Publication No. 64-WA/FE-36, 1964, pp. 1-12.

20) Rosenhan, A.K., "Computer Analysis of Flow in Pipe Networks"; ASME Publication No. 69-WA/PVP-9, 1969, pp. 1-23.

21) Shamir, U., and C.D.D. Howard, "Water Distribution Systems Analysis"; Journal of the Hydraulics Division, ASCE, Vol. 94, No. HY1, January 1968, pp. 219-234.

22) "System/360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's Manual" International Business Machines Corporation.

XII. APPENDIX 1. ILLUSTRATIVE EXAMPLE
FOR MATHEMATICAL FORMULATION

To demonstrate the mathematical formulation presented
in this study, an illustrative example for a simplified
network shown below is worked out in detail.
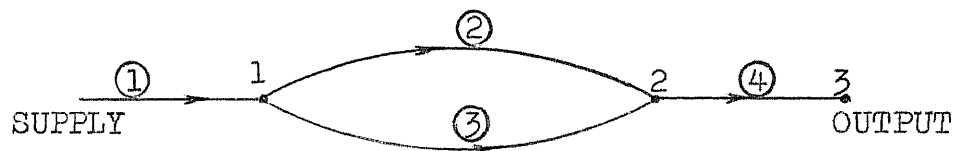
For this configuration, equation (1) takes the following
form:

$$
\begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \end{Bmatrix}
=
\begin{bmatrix} -1 & 0 & 0 \\ +1 & -1 & 0 \\ +1 & -1 & 0 \\ 0 & +1 & -1 \end{bmatrix}
\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix}
\tag{17}
$$

One may verify the correctness of equation (17) by a sim-
ple expansion as follows:

$$
\begin{aligned}
\Delta P_1 &= 0. &- P_1 \\
\Delta P_2 &= P_1 &- P_2 \\
\Delta P_3 &= P_1 &- P_2 \\
\Delta P_4 &= P_2 &- P_3
\end{aligned}
\tag{18}
$$

Equations (18) are obviously the branch pressure drop
versus nodal pressure relations. It should be noted that
the supply pressure is considered as the reference pres-
sure for all nodal pressures. Furthermore, equation (2)
takes the following form.

$$
\begin{bmatrix} -1 & +1 & +1 & 0 \\ 0 & -1 & -1 & +1 \\ 0 & 0 & 0 & -1 \end{bmatrix}
\begin{Bmatrix} \dot{m}_1 \\ \dot{m}_2 \\ \dot{m}_3 \\ \dot{m}_4 \end{Bmatrix}
+ \begin{Bmatrix} 0 \\ 0 \\ \dot{m}_{3o} \end{Bmatrix}
= \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}
\tag{19}
$$

One may verify the correctness of equation (19) by a simple expansion as follows:

$$
\begin{aligned}
-\dot{m}_1 + \dot{m}_2 + \dot{m}_3 &= 0 \\
-\dot{m}_2 - \dot{m}_3 + \dot{m}_4 &= 0 \\
- \dot{m}_4 + \dot{m}_{3o} &= 0
\end{aligned}
\tag{20}
$$

Equations (20) are obviously the nodal continuity equations for the system under consideration. It should be noted that $\dot{m}_1$, $\dot{m}_2$, $\dot{m}_3$ and $\dot{m}_4$ are the unknown branch flows; while $\dot{m}_{3o}$ is the known flow output at node 3. Similarly, equation (3) takes the following form:

$$
\begin{Bmatrix} \dfrac{\dot{m}_1 - \dot{m}_1^o}{h} \\[2ex] \dfrac{\dot{m}_2 - \dot{m}_2^o}{h} \\[2ex] \dfrac{\dot{m}_3 - \dot{m}_3^o}{h} \\[2ex] \dfrac{\dot{m}_4 - \dot{m}_4^o}{h} \end{Bmatrix}
= \begin{Bmatrix} \dfrac{a_1 g}{L_1} (144\ \Delta P_1 + \rho \Delta Z_1 + \rho H_1 - F_1) \\[2ex] \dfrac{a_2 g}{L_2} (144\ \Delta P_2 + \rho \Delta Z_2 + \rho H_2 - F_2) \\[2ex] \dfrac{a_3 g}{L_3} (144\ \Delta P_3 + \rho \Delta Z_3 + \rho H_3 - F_3) \\[2ex] \dfrac{a_4 g}{L_4} (144\ \Delta P_4 + \rho \Delta Z_4 + \rho H_4 - F_4) \end{Bmatrix}
\tag{21}
$$

It can be easily seen that equations (21) constitute the branch momentum equations. As shown earlier, solving equations (17), (19) and (21) provide explicit relations for unknown nodal pressures and branch mass flow rates. Applying equations (8) and (9) to the present case, one obtains

$$
[M] = (144gh)\begin{bmatrix} -1 & +1 & +1 & 0 \\ 0 & -1 & -1 & +1 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} a_1/L_1 & 0 & 0 & 0 \\ 0 & a_2/L_2 & 0 & 0 \\ 0 & 0 & a_3/L_3 & 0 \\ 0 & 0 & 0 & a_4/L_4 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ +1 & -1 & 0 \\ +1 & -1 & 0 \\ 0 & +1 & -1 \end{bmatrix} \quad (22)
$$

Performing the multiplications indicated above, yields

$$
[M] = (144gh)\begin{bmatrix} (a_1/L_1)+(a_2/L_2)+(a_3/L_3) & -(a_2/L_2)-(a_3/L_3) & 0 \\ -(a_2/L_2)-(a_3/L_3) & (a_2/L_2)+(a_3/L_3)+(a_4/L_4) & -(a_4/L_4) \\ 0 & -(a_4/L_4) & (a_4/L_4) \end{bmatrix}
$$

$$(23)$$

Inverting the above matrix, substituting into equation (8) and performing the indicated matrix operations, the nodal pressures are easily found. Subsequently, employing equations (1) and (6), the branch pressure drop and mass flow rates can be determined.

PART TWO

USER'S MANUAL

The program HYTRAN (Hydraulic Transient Analysis)
consists of a main program and several subroutine linked as
shown in Figures 4 through 16. A tabular outine of the main
program and its subroutines together with their functions
is presented below and followed by a more detailed descrip-
tion.

MAIN PROGRAM	FUNCTION
MAIN	Hydraulic Transient Analysis

SUBROUTINES	FUNCTION
MINV	Matrix Inversion
GMPRD	Matrix Multiplication
GMSUB	Matrix Subtraction
GMTRA	Matrix Transposition
SI FUNCTION	Linear Interpolation
DPRINT	Matrix Printout

Main Program

A detailed description of the MAIN program is presen-
ted in the flow charts on Figures 4 through 15. This program
is employed to perform the following two main functions:

1) Mathematical operations performed once at the begin-
    ning of the program which are described below:

    a) Conversion of input data from engineering units
        to ft-lb-sec. units.

    b) Assembly of the transpose of the connection matrix
        A^T from CONP(J) and CONN(J) arrays defined later
        in the description of input data.

    c) Transposition of the transpose of the connection
        matrix A^T, to obtain the connection matrix A.

```
                              ┌──────────────────────────────────┐        ┌─────────────────────┐
                              │ READ EIRUN                       │        │ KOUNT=1             │
                              │ PRINT EIRUN                      │        │ TIME=0              │
                              │ READ XIMAX,XJMAX,XNMAX           │        │ TPR=0               │
     ╭───────╮                │ PRINT XIMAX,XJMAX,XNMAX          │        │ IMAX=XIMAX          │       ╭───╮
     │ START │ ──────────────▶│ READ COUNT,ERROR,DELTAT,DTPR     │───────▶│ JMAX=XJMAX          │──────▶│ A │
     ╰───────╯                │ PRINT COUNT,ERROR,DELTAT,DTPR    │        │ KMAX=IMAX*JMAX      │       ╰───╯
                              │ READ RHO,VISC,ROUGH,GRAV         │        │ NMAX=XNMAX          │
                              │ PRINT RHO,VISC,ROUGH,GRAV        │        │ NCOUNT=COUNT        │
                              │ READ FLAG1                       │        └─────────────────────┘
                              │ PRINT FLAG1                      │
                              └──────────────────────────────────┘
```

```
                 ┌────────────────────────────────┐
                 │ READ XL(J) J=1,JMAX            │
                 │ PRINT J,XL(J) J=1,JMAX         │
                 │ READ XLP(J) J=1,JMAX           │
                 │ PRINT J,XLP(J) J=1,JMAX        │              DO 1100 N=1,NMAX
                 │ READ D(J) J=1,JMAX             │         ┌──────────────────────────┐
                 │ PRINT J,D(J) J=1,JMAX          │         │                          │
                 │ READ EMO(J) J=1,JMAX           │         │                      ┌─────────────────────┐
                 │ PRINT J,EMO(J) J=1,JMAX        │        ╱ ╲                     │ READ HEAD(M,N)      │
   ╭───╮         │ READ CONP(J) J=1,JMAX          │       ╱   ╲                    │ M=1,10              │      ╭───╮
   │ A │────────▶│ PRINT J, CONP(J) J=1,JMAX      │──────▶⟨(NMAX)⟩────────────────▶│ PRINT M,HEAD(M,N)   │─────▶│ B │
   ╰───╯         │ READ CONN(J) J=1,JMAX          │       ╲   ╱                    │ M=1,10              │      ╰───╯
                 │ PRINT J,CONN(J) J=1,JMAX       │        ╲ ╱                     └─────────────────────┘
                 │ READ Z(I) I=1,IMAX             │         │
                 │ PRINT I,Z(I) I=1,IMAX          │         │
                 │ READ XM(I) I=1,IMAX            │         ▼
                 │ PRINT I,XM(I) I=1,IMAX         │       ╭──────╮
                 │ READ PB(N) N=1,NMAX            │       │ 1102 │
                 │ PRINT N,PB(N) N=1,NMAX         │       ╰──────╯
                 └────────────────────────────────┘
```
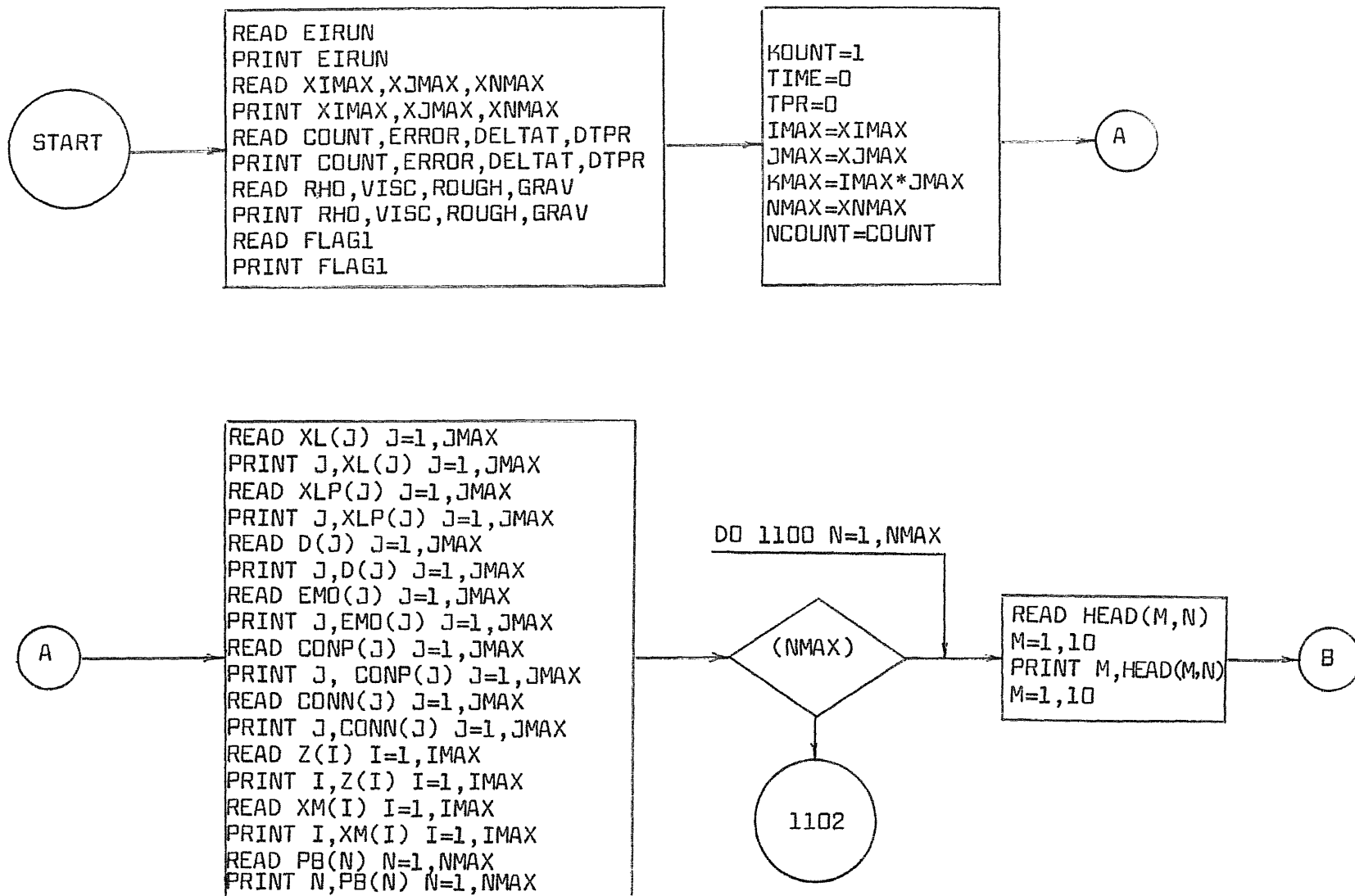
Figure 4
Flow Chart of HYTRAN MAIN Program

Figure 5
Flow Chart of HYTRAN MAIN Program

Figure 6
Flow Chart of HYTRAN MAIN Program

Figure 7

Flow Chart of HYTRAN MAIN Program

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Figure 8
Flow Chart of HYTRAN MAIN Program

DO 9929 J1=1,KMAX

$$(N) \rightarrow (25) \rightarrow \boxed{J=0} \rightarrow \boxed{J=J+1} \rightarrow \langle (J-JMAX) \rangle \xrightarrow{+} (9927) \rightarrow (O)$$

-,0

$$(O) \rightarrow \boxed{J=1} \rightarrow (9928) \rightarrow \boxed{\begin{array}{c} PROD(J1)=A(J1)* \\ GAMMA(J) \end{array}} \rightarrow (9929) \rightarrow (P)$$

$$(P) \rightarrow \langle (FLAG1-1.) \rangle \xrightarrow{0} (2221) \rightarrow \boxed{\begin{array}{c} CALL\ DPRINT \\ (PROD,JMAX, \\ IMAX) \end{array}} \rightarrow (2222) \rightarrow (Q)$$

+,—

Figure 9
Flow Chart of HYTRAN MAIN Program
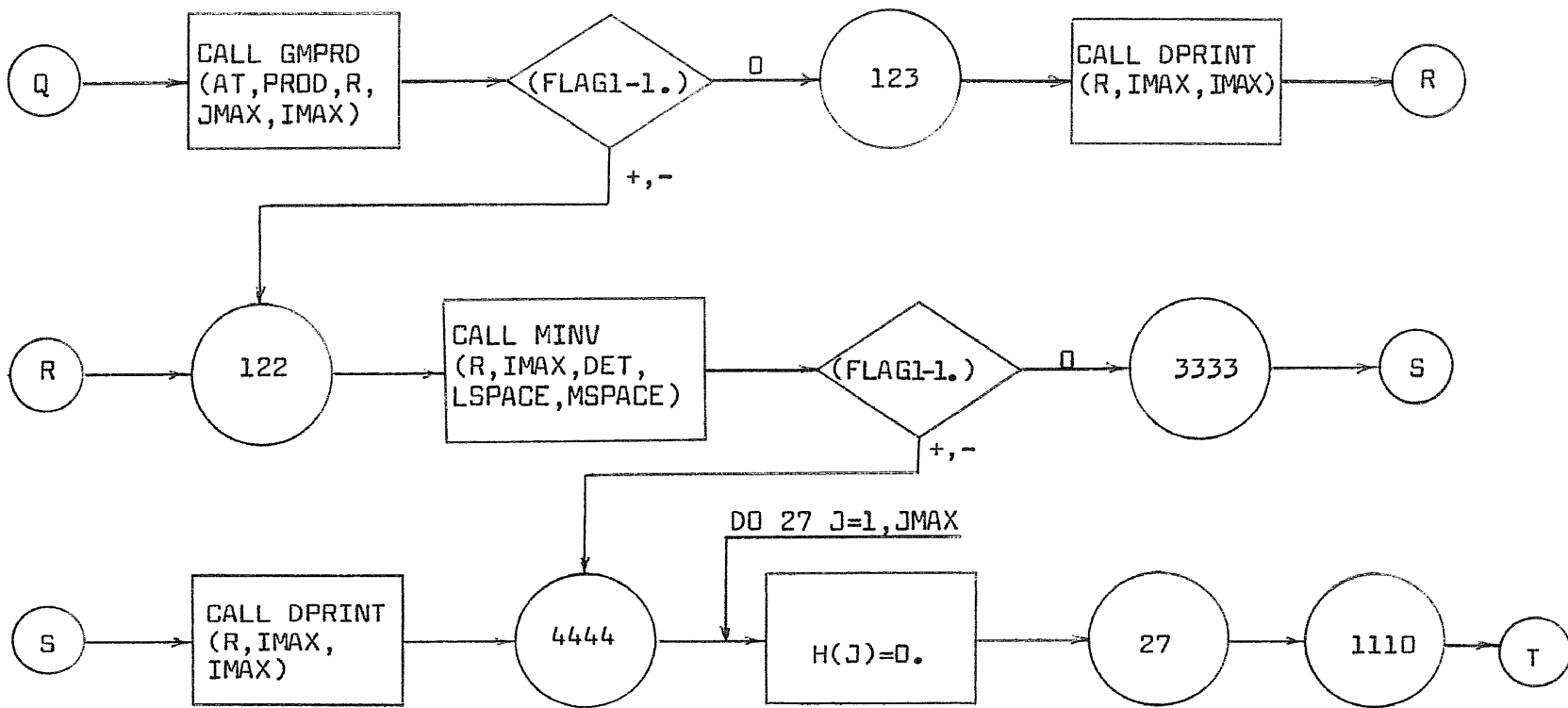
Figure 10

Flow Chart of HYTRAN MAIN Program

DO 28 N=1,NMAX

T → (NMAX) →+,− 142 → J=PB(N) →

```
GPMX(I)=GPM(I,N)
HEADX(I)=HEAD(I,N)
H(J)=SI(GPMX,HEADX,
         EMO(J),10)
```
→ U

DO 29 J=1,JMAX

U → 28 → 143 →

```
REYN(J)=D(J)*ABS(EMO(J))/AREA(J)*VISC)
FRIC(J)=.0055*(1.+(2.*10E4*ROUGH/D(J)
         10E6/REYN(J)**.333)
FF(J)=((FRIC(J)*XLP(J)*ABS(EMO(J))*EMO(J))/
       (D(J)*2.*32.2*RHO*RHO*(AREA(J)**2)))
F(J)=FF(J)−ZB(J)
SUM(J)=GAMMA(J)*(F(J)+H(J))−EMO(J)
```
→ V

V → 29 →

```
CALL GMPRD (RR,SUM,RRR,IMAX,JMAX,1)
CALL GMSUB (RRR,RM,P,IMAX,1)
CALL GMPRD (A,P,DELP,JMAX,IMAX,1)
```
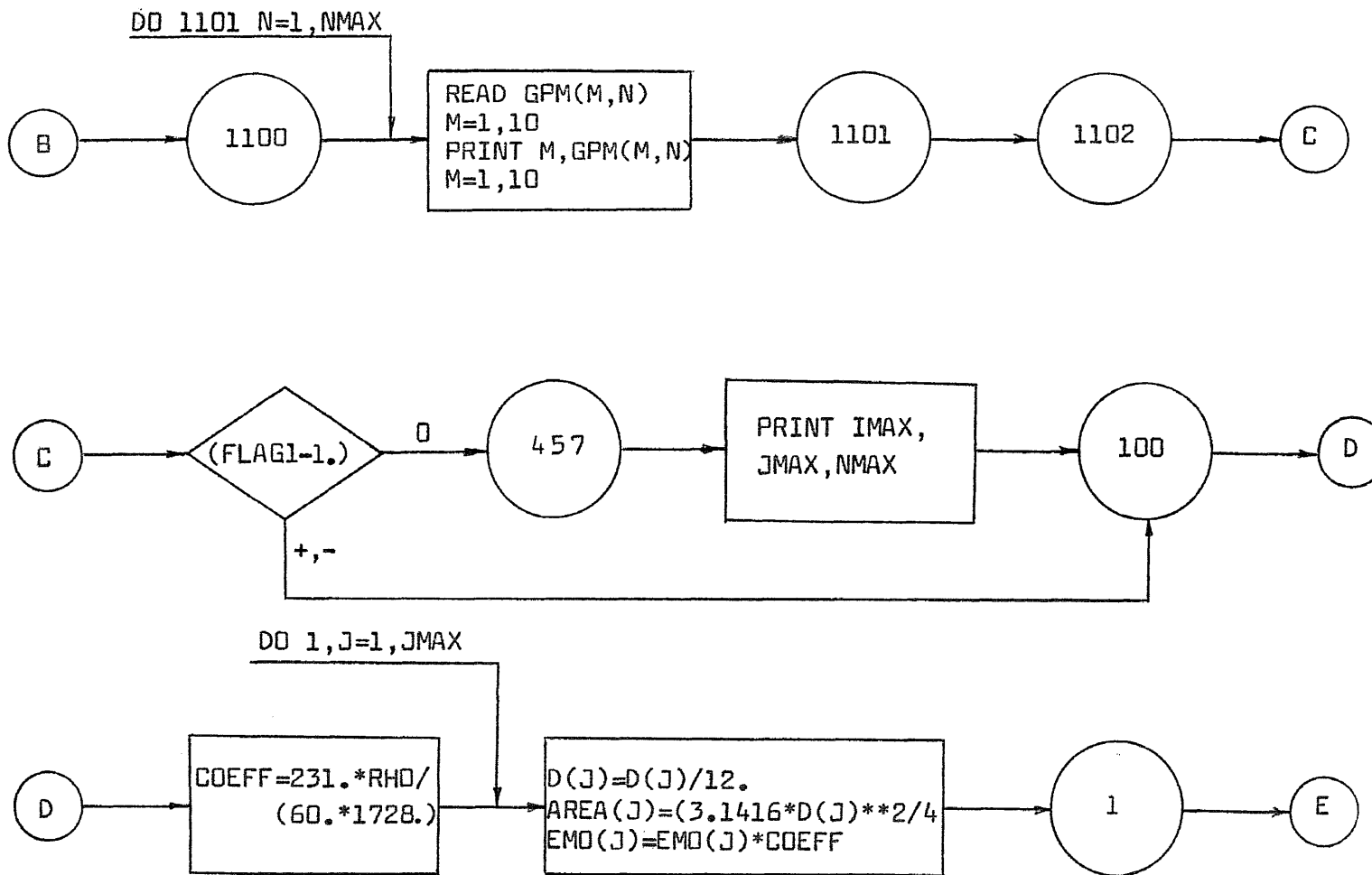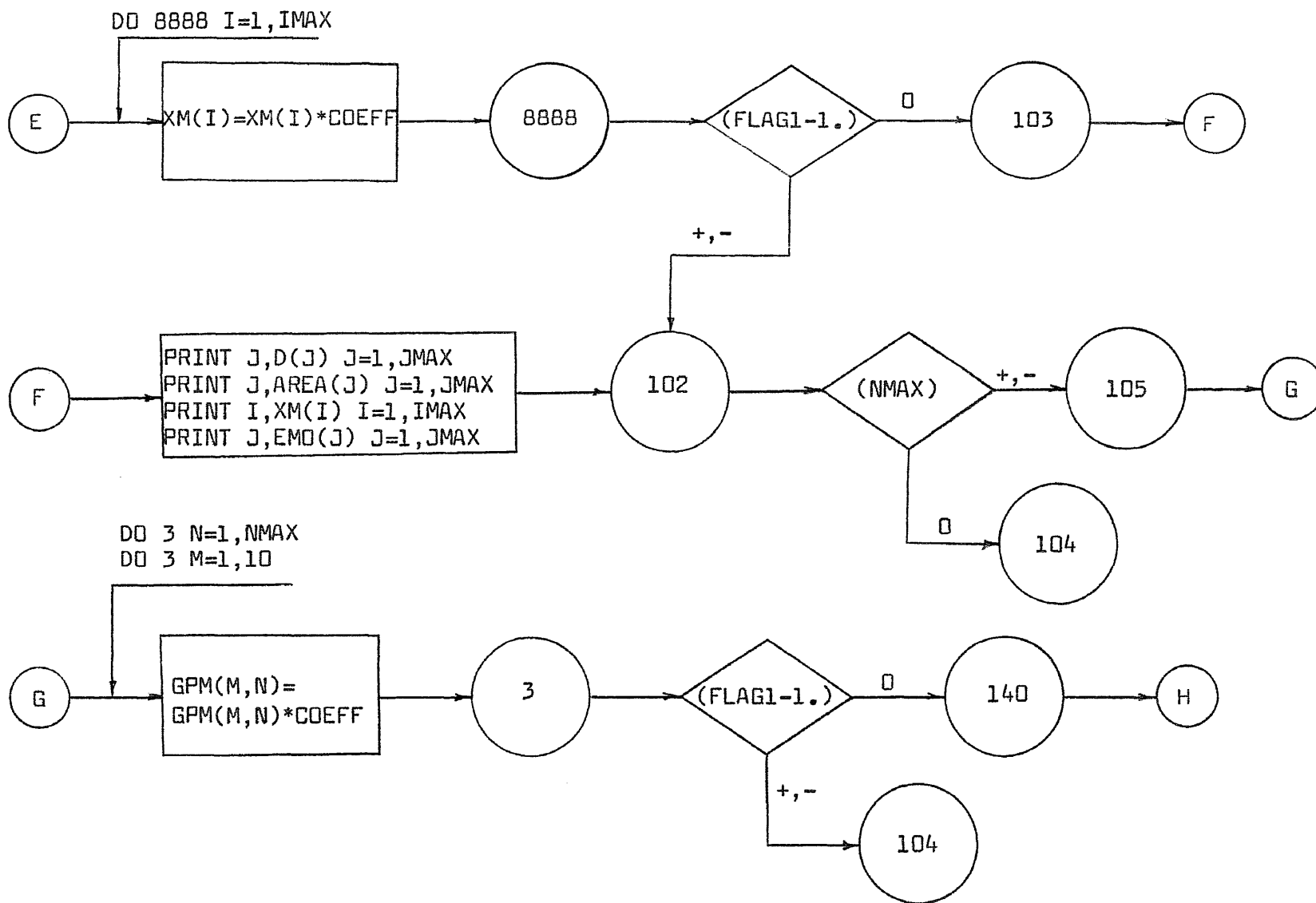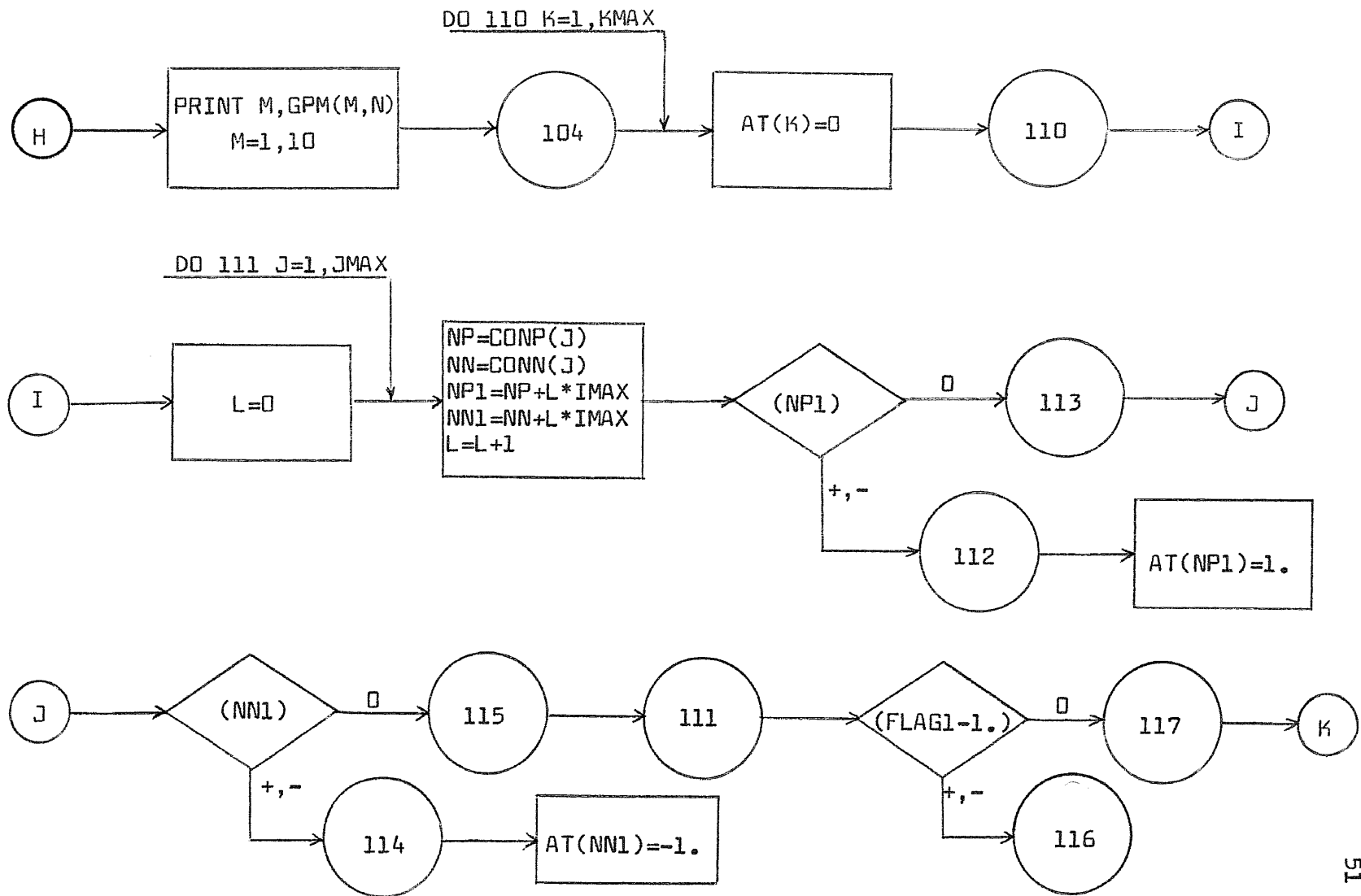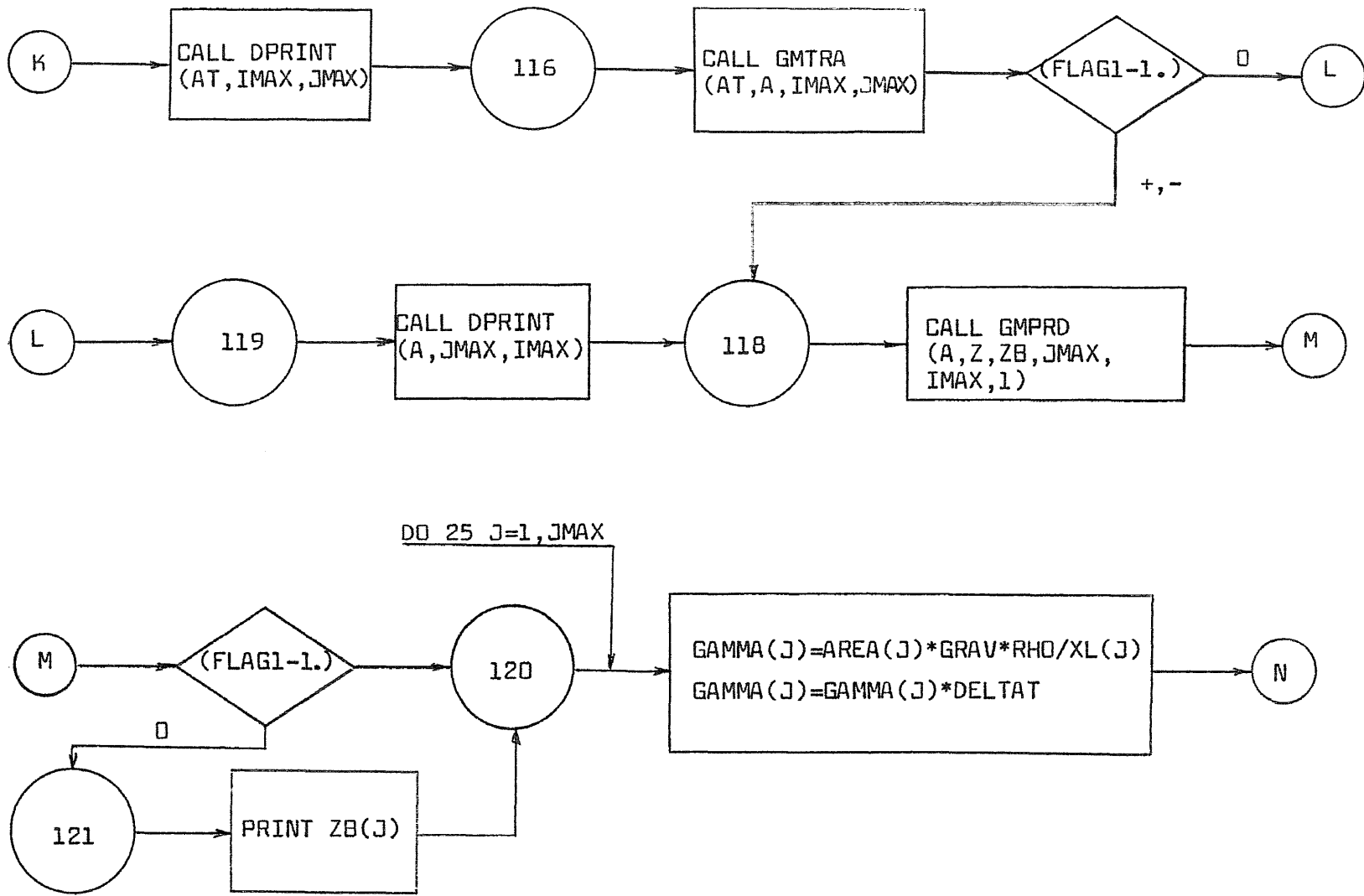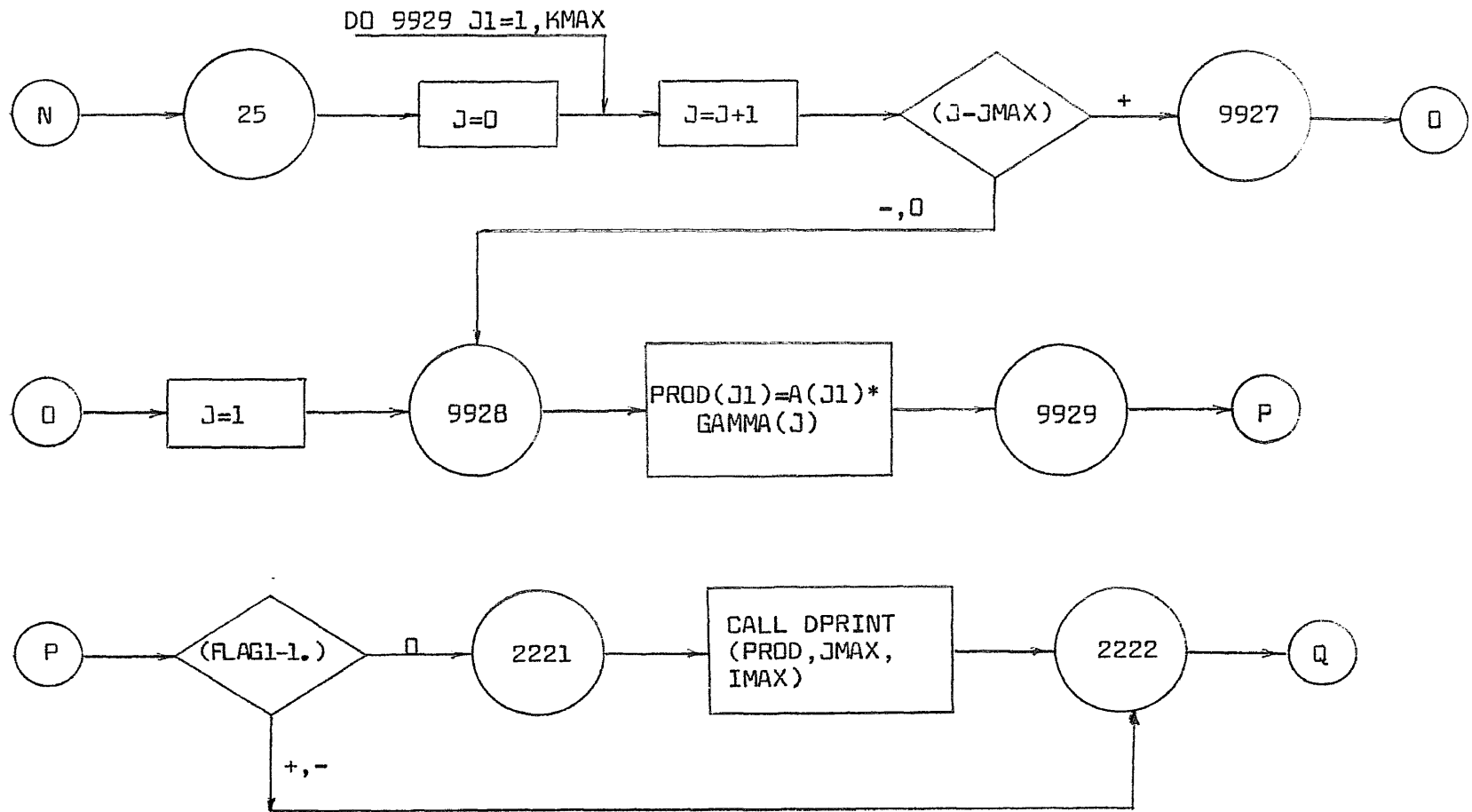→ W
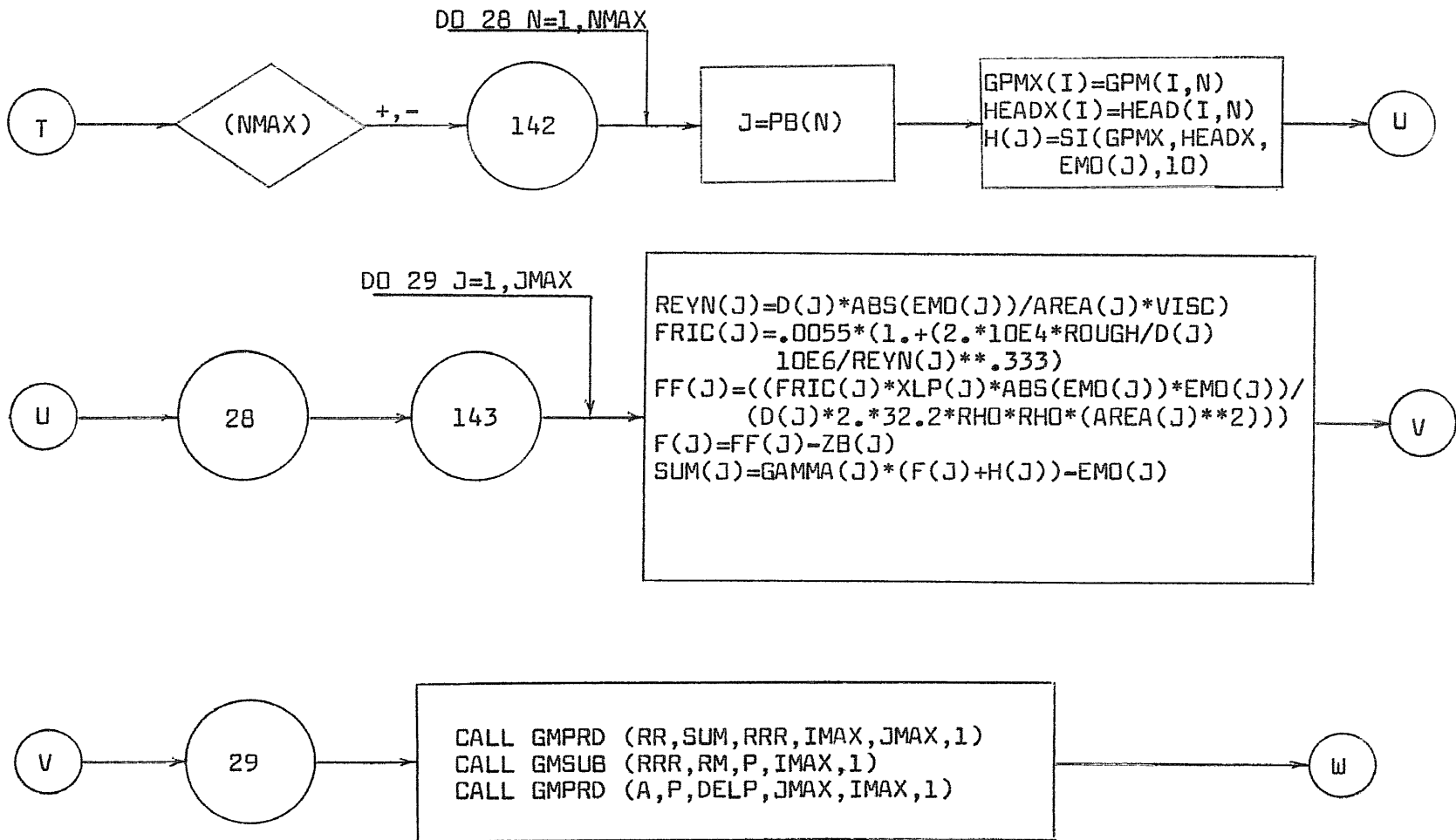
Figure 11

Flow Chart of HYTRAN MAIN Program

Figure 12

Flow Chart of HYTRAN MAIN Program

Figure 13

Flow Chart of HYTRAN MAIN Program

DO 32 J=2,JMAX

CC ----→ REL(J)=
ABS((EM(J)-
EMO(J))/DELTAT → 61 → REL(J)
-ERROR → -,0 → 32 → DD

+ → EE

DD → 475 →

```
PRINT TIME
PRINT J,EMG(J)  J=1,JMAX
PRINT J,EM(J)   J=1,JMAX
PRINT I,PP(I)   I=1,IMAX
PRINT J,FF(J)   J=1,JMAX
PRINT J,REL(J)  J=1,JMAX
PRINT KOUNT
```
→ STOP

EE → 201 → KOUNT-
NCOUNT → - → 202 → KOUNT=KOUNT+1
TIME=TIME+
DELTAT → FF

0,+ → GG
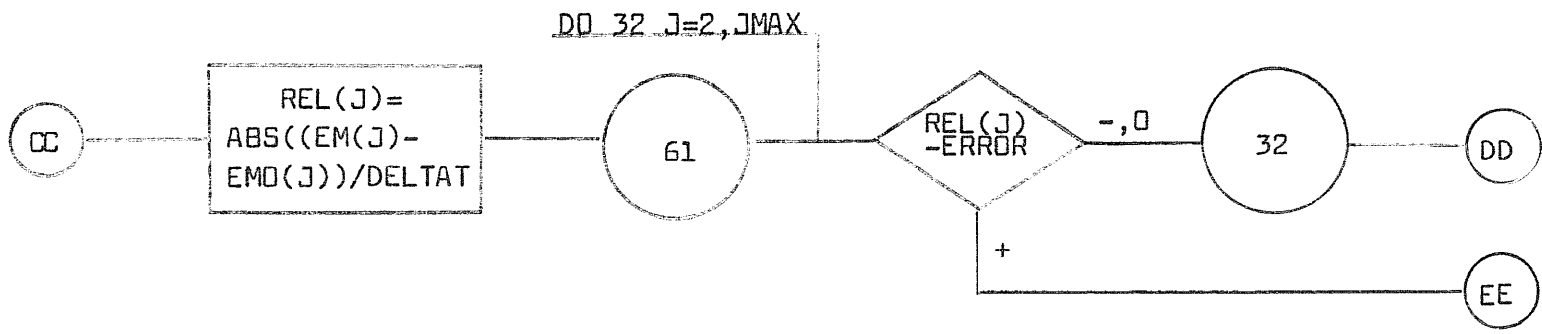
Figure 14

Flow Chart of HYTRAN MAIN Program

DO 10 J=2,JMAX

FF → EMO(J)=EM(J) → 10 → GO TO 1110 → END

GG → 203 → PRINT ERROR MESSAGE → STOP
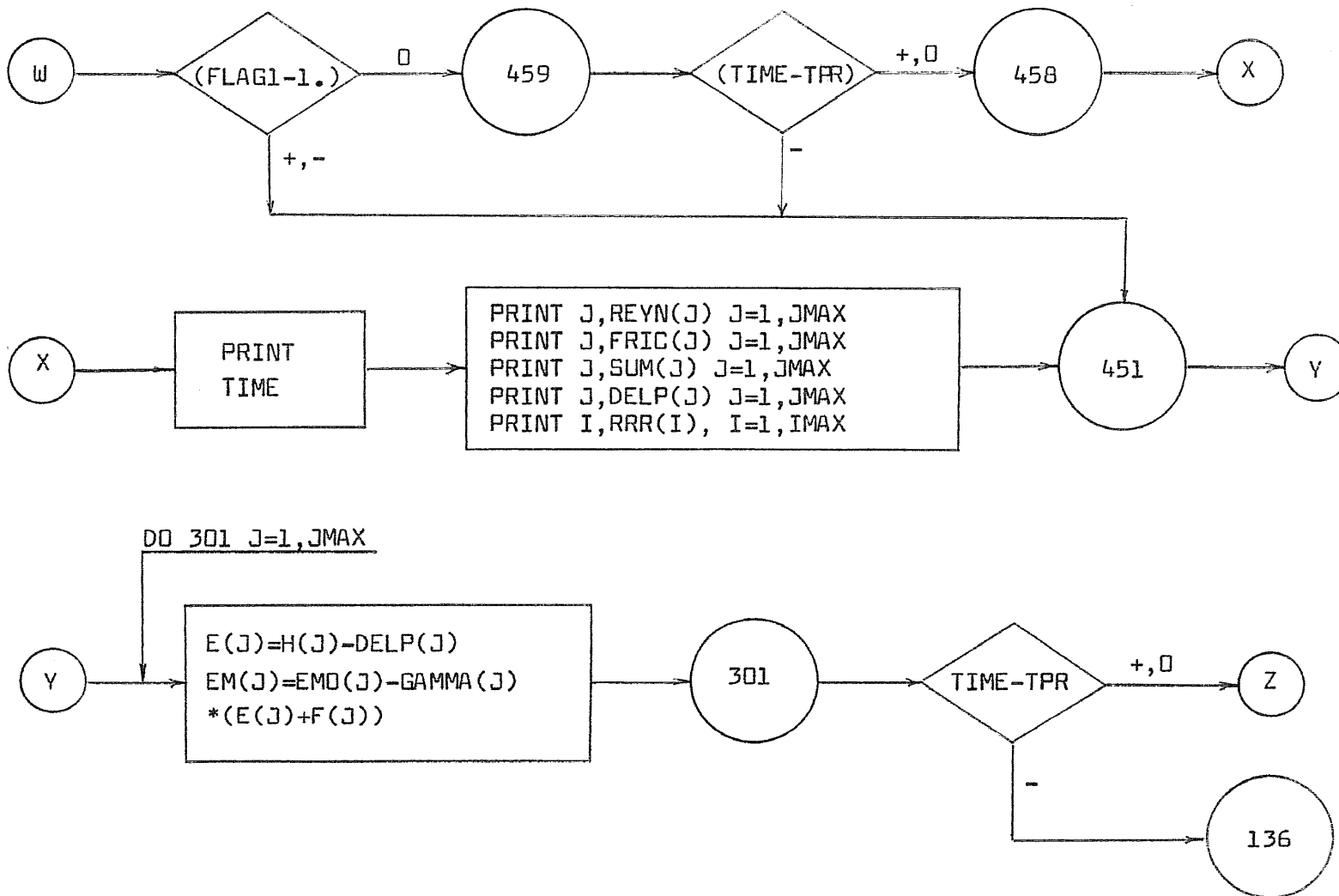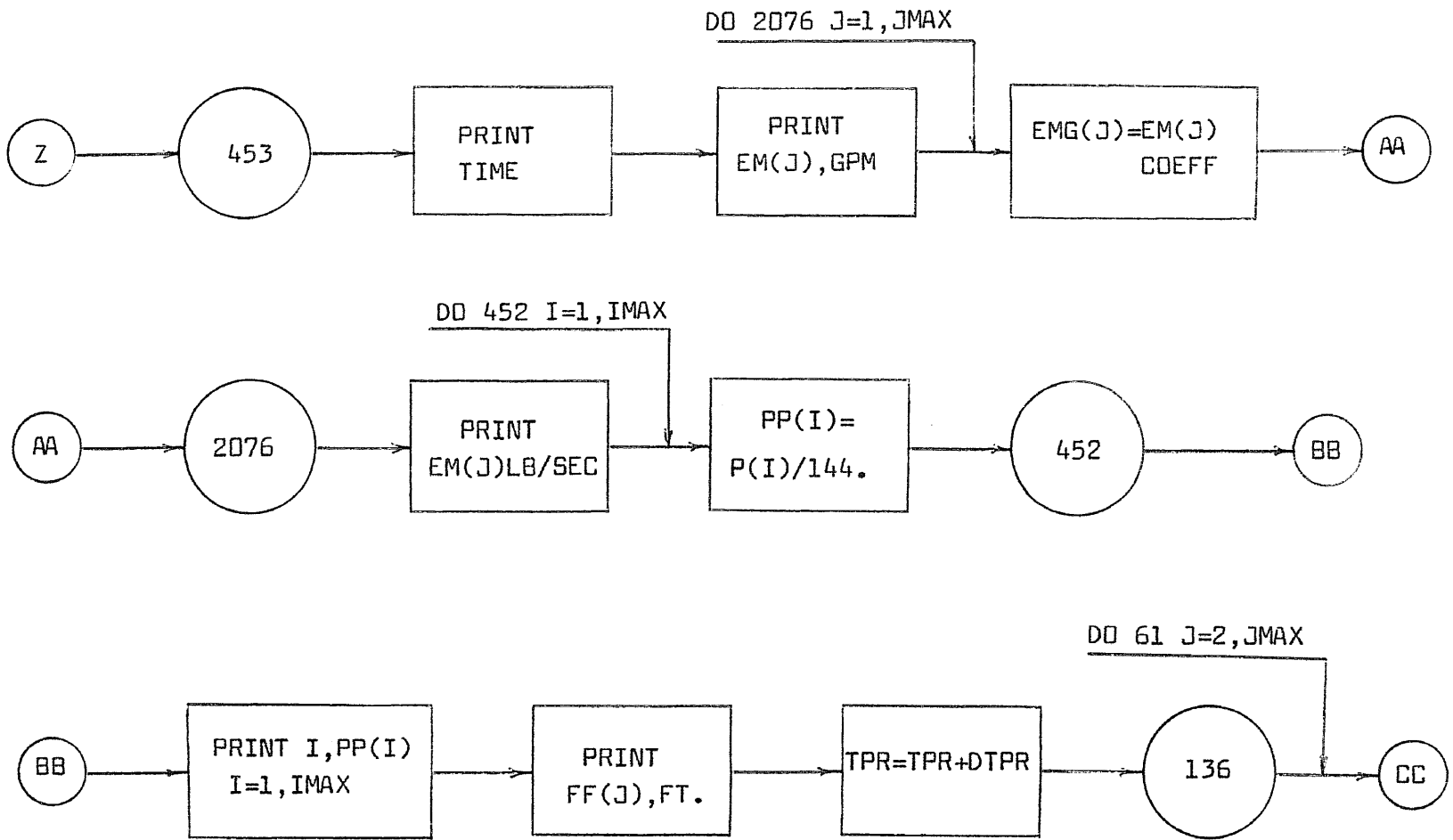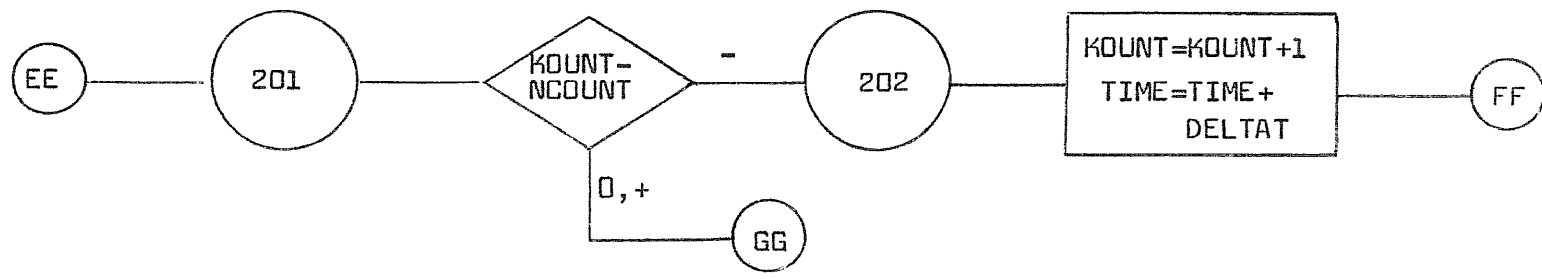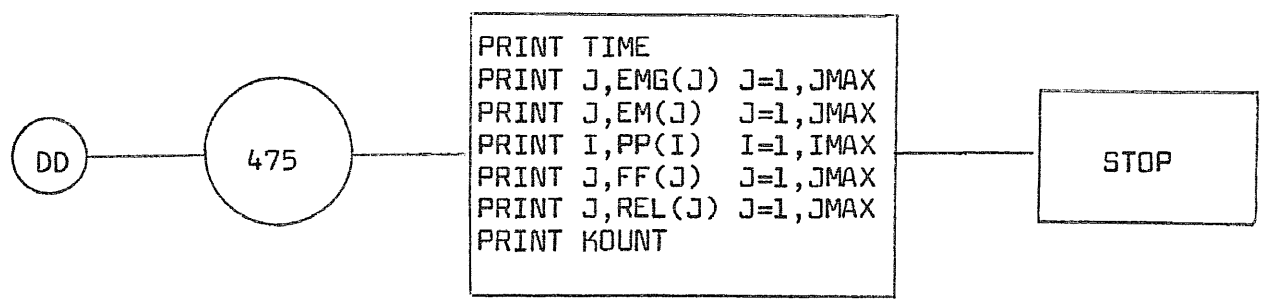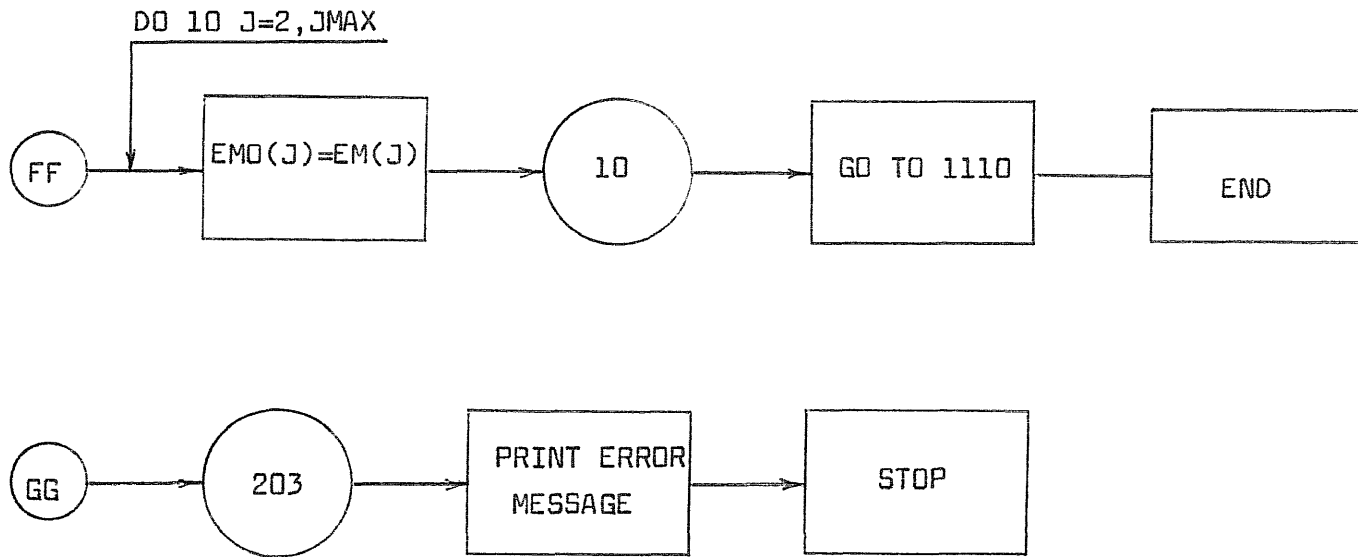
Figure 15

Flow Chart of HYTRAN MAIN Program
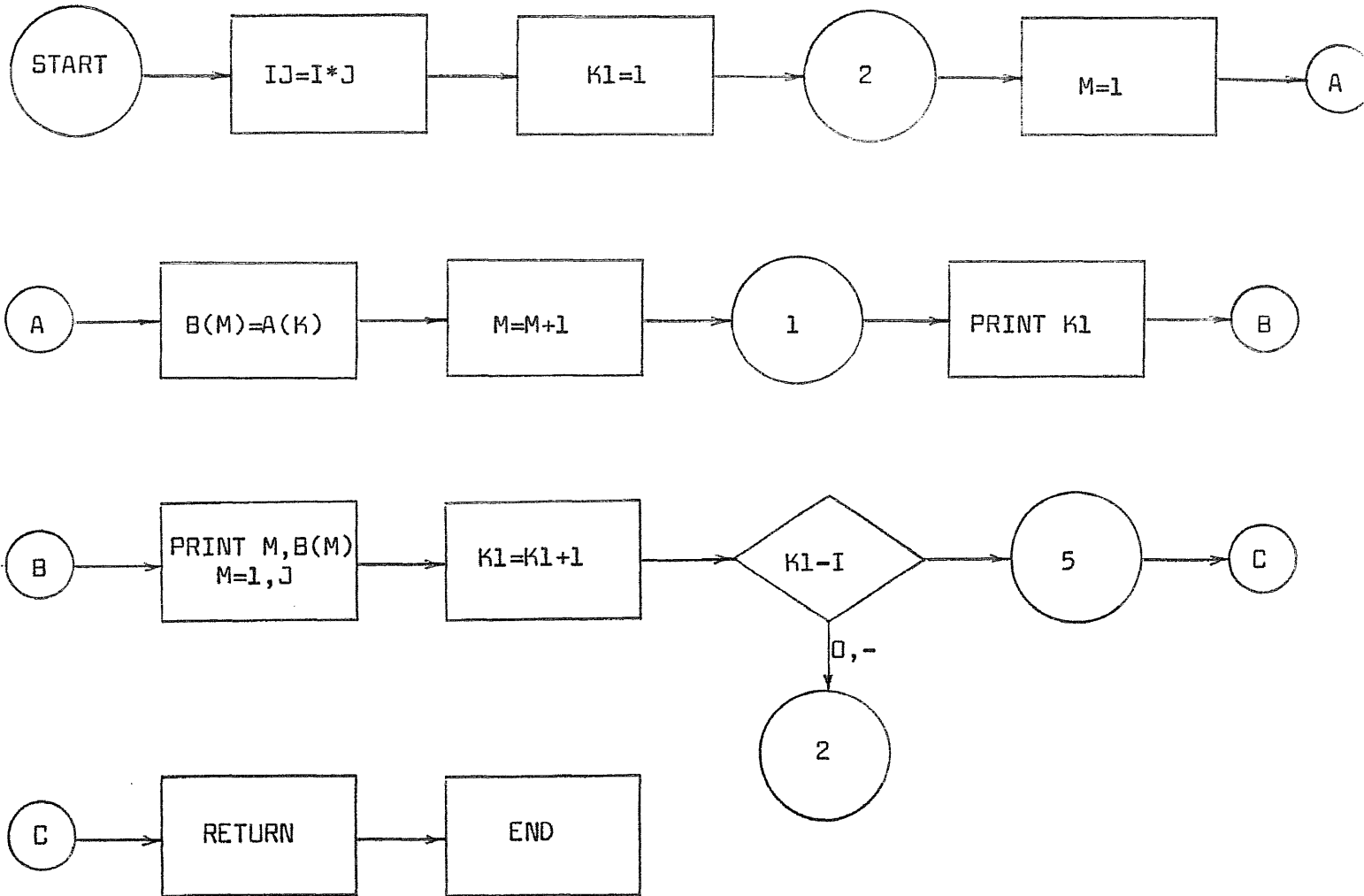
Figure 16
Flow Chart of Subroutine DPRINT

d) Multiplication of the connection matrix A by nodal elevation array Z to obtain branch height difference ΔZ.

e) Formation of matrix M defined by equation (9).

f) Formation of products $M^{-1} . A^T$ and $M^{-1} . \dot{m}_{io}$ required by equation (8).

2) Iterative operations performed after each time increment as described below:

a) Calculation of branch static, transducer and frictional heads and their multiplication by γ.

b) Summation of the above terms and the present values of the branch mass flow rates required by equation (8).

c) Multiplication of $M^{-1} . A^T$ by the result obtained under (b) above and subtraction of $M^{-1} . \dot{m}_{io}$ from this product to form the pressure array p as indicated by equation (8).

d) Calculation of branch pressure drop array Δp from equation (1).

e) Calculation of branch mass flow rate array from equation (6).

MINV Matrix Inversion Subroutine

This subroutine is used to invert matrices. It is used only once at the beginning of the program to invert matrix M designated by equation (9). MINV is a standard

subroutine from reference (22) and its calling sequence is:

CALL MINV (A,N,D,L,M)

A = Input matrix, destroyed in computation
and replaced by resultant inverse.

N = Order of matrix A

D = Resultant determinant (not used in this
analysis)

L = Work vector of length N

M = Work vector of length N

GMPRD   Matrix Product Subroutine

This subroutine is used for the multiplication of
matrices, as discussed above. It is a standard subroutine
from reference (22) and its calling sequence is:

CALL GMPRD (A,B,R,N,M,L)

A = Name of first input matrix

B = Name of second input matrix

R = Name of output matrix

N = Number of rows in A

M = Number of columns in A and rows in B

L = Number of columns in B

GMSUB   Matrix Subtraction Subroutine

This subroutine is used for the subtraction of two
matrices. It occurs once in the program to perform the
matrix subtraction indicated by equation (8). It is a
standard subroutine from reference (22) and its calling
sequence is:

CALL GMSUB (A,B,R,N,M)

A = Name of first input matrix

B = Name of second input matrix

R = Name of output matrix

N = Number of rows in A, B and R

M = Number of columns in A, B and R

GMTRA   Matrix Transposition Subroutine

This subroutine is used to transpose matrices. It is
used once in the program to transpose the connection
matrix A to A^T. It is a standard subroutine from refer-
ence (22) and its calling sequence is:

CALL GMTRA (A,R,N,M)

A= Name of matrix to be transposed

R = Name of resultant matrix

N = Number of rows of A and columns of R

M = Number of columns of A and rows of R

SI FUNCTION   Linear Interpolation Subroutine

This function is used to interpolate values in the
head versus volumetric flow tables for transducers (pumps).
It is a standard subroutine and its calling sequence is:

FUNCTION SI (XTBL, YTBL, X, N)

XTBL = Independent variable table

YTBL = Dependent variable table

X = Value of independent variable

N = Number of points in table

DPRINT   Matrix Printout Subroutine

This subroutine, charted on Figure 16, is used for printing the following matrices: A^T, A, M and its inverse. These matrices can be printed at the users option, for debugging purposes, by setting appropriate values for FLAG 1 as shown in the description of input data. The calling sequence of this subroutine is:

    CALL DPRINT (A,I,J)

    A = Name of matrix to be printed

    I = Number of rows in matrix

    J = Number of columns in matrix

The input data can be divided into two groups:

1) Non-subscripted variables

2) Subscripted variables

A brief description of each input variable appears in the program listing to aid in checking the program. A more detailed description of input data is presented in this section. The procedure used for labeling the input data is described below:

All non-subscripted variables are identified by the prefix "FIX", followed by two numbers. The first number indicates the card number and the second indicates the order of the variable on that card. For example, FIX 2-3 XNMAX denotes that the variable XNMAX is a non-subscripted variable appearing as the third value on card number 2. All subscripted variables are identified by a prefix "VAR" followed by one number. For example, VAR 7 Z denotes that Z is a subscripted variable appearing as the seventh set of arrays in the input data. Each card contains 6 values of Z(I). All input data are entered using the FORMAT E10.3.

Non-subscripted Variables

FIX 1-1 EIRUN Run Number. This quantity is used for identification of successive computer runs.

FIX 2-1 XIMAX Maximum Number of Nodes. This number represents the maximum number of nodes in the system being

analyzed. These nodes are the points on Figure 1 denoted by the non-encircled numbers.

FIX 2-2 XJMAX Maximum Number of Branches. This value represents the maximum number of branches in the system being analyzed. These are the lines on Figure 1 denoted by the encircled numbers. As can be noted from Figure 1, the branch numbers do not have to correspond with node numbers.

FIX 2-3 XNMAX Maximum Number of Transducers (Pumps). This quantity describes the maximum number of transducers (pumps) in the system being analyzed. It should be noted that any flow device or restrictions whose flow versus head characteristics are known can be included in this category.

FIX 3-1 COUNT Maximum Allowable Number of Iterations. This number represents the maximum number of iterations allowed in the analysis and is a safety device to stop the computer in the event of program error. The value of COUNT is calculated by

$$\text{COUNT} = \frac{\text{Maximum problem physical time}}{\text{DELTAT}}$$

Since the maximum problem physical time is not known before the problem solution, a suggested value of COUNT is 1000. If more transient time is required, an error message will appear on the output sheet and COUNT should then be increased. Steady-state solutions generally use large

time increments and require fewer iterations. Thus, the
value of COUNT = 1000 is generally adequate for steady-
state analyses.

FIX 3-2 ERROR Steady-State Flow Convergence Error.
This value represents the steady-state flow convergence
error. Steady-state conditions are considered to be
reached when

$$\frac{\dot{m} - \dot{m}^0}{h} \leq \quad \text{ERROR}$$

FIX 3-3 DELTAT Time Increment, Sec. This variable
specifies the time increment for each integration step.
It is equivalent to h, presented earlier in the text.
The criterion for its selection is discussed in the sec-
tion on "Numerical Stability Analysis". The time incre-
ment is selected by first calculating all branch time
constants using equation (14) and then setting the time
increment equal to 1/20 of the minimum time constant as
specified by equation (15).

FIX 3-4 DTPR Printout Interval, Sec. This number
specifies the desired time interval for the printout of
the transient solution. It is used to avoid the printing
of results after every iteration and minimize computer
running time. For the transient case, a suggested value
of 10 times DELTAT proved satisfactory for this quantity.
For the steady-state solution when only the final conver-
ged value is desired, DTPR should be of the order 1000

times DELTAT.

FIX 4-1 RHO Fluid Density lbs/cu.ft. This number spe-
cifies the density of the fluid flowing in the hydraulic
network.

FIX 4-2 VISC Fluid Viscosity lbs/ft-sec. This number
represents the viscosity of the fluid in the system.

FIX 4-3 ROUGH Pipe Absolute Roughness, ft. This
quantity represents the absolute roughness of pipes used
in the network. The program divides this quantity by
the hydraulic diameter to obtain relative roughness.

FIX 4-4 GRAV Gravitational Acceleration ft/sec^2. This
number represents the force of gravity acting on the
system under investigation (32.2 ft/sec^2).

FIX 5-1 FLAG 1 Printout Indicator. This parameter
is used primarily for debugging purposes. It controls
the printing of the following variables defined later in
the description of the output data.

    IMAX, JMAX, NMAX, D(J), AREA(J), XM(I), EMO(J),
    GPM(M,N), A^T, A, ZB(J), PROD(J1), R, REYN(J),
    FRIC(J), F(J), SUM(J), RR, E(J), RRR, P(I), DELP(J)

Assigning a value of 1 to FLAG 1 prints the above varia-
bles and a value of 0 eliminates printing.

Subscripted Variables

VAR 1 XL(J) Branch Actual Length, Ft. These values represent the actual developed length of each pipe branch.

VAR 2 XLP(J) Branch Equivalent Length, Ft. These values define the branch equivalent length composed of actual length XL(J) plus the resistance due to valves, fittings, expansions and contractions and other flow restrictions expressed in terms of pipe length.

VAR 3 D(J) Branch Hydraulic Diameter, Inches. These values represent the hydraulic diameter of each branch. The hydraulic diameter for round pipes is equal to their actual inside diameter. The hydraulic diameter for pipes other than circular cross section is equal to four times their volume divided by their wetted surface.

VAR 4 EMO(J) Branch Initial Mass Flow Rate, GPM. These values define the initial mass flow rates in each branch. For steady-state analysis, these values are assumed without any restriction except that continuity equations must be satisfied at each node. For transient analysis, the prevailing steady-state values should be entered. These values may be obtained either from a previous run on this program or from other sources.

VAR 5 CONP(J) Positive Branch Node, VAR 6 CONN(J) Negative Branch Node. A node is defined as any point in

the system at which three or more flows meet or network geometric dimensions change. A branch is defined as a line connecting two nodes. The nodes and the branches of the hydraulic network are numbered separately starting from 0 for nodes and 1 for branches without skipping any number. The node numbered 0 should correspond to the point in the system where the pressure is known. The branch number 1 corresponds to the branch connected to node 0. Every branch in the system is given an arbitrary positive orientation or direction designated by an arrow. The flow is considered positive when it is along this positive direction and negative when it is against this positive direction. The node located at the tail of the arrow is designated as the "sending node" for that branch and the node located at the head of the arrow is designated as the "receiving node". The value of CONP(J) and CONN(J) represent the node number corresponding to branch J for the sending node and the receiving node, respectively. For example, the values of CONP(J) and CONN(J) for J = 36 and J = 37 in Figure 1 will become:

	J = 36	J = 37
CONP(J)	9	8
CONN(J)	10	9

The CONP(J) and CONN(J) arrays are used by the program to assemble the connection matrix. All CONP(J) nodes contribute a value of +1 to the connection matrix, and all CONN(J) nodes contribute a -1 to the connection matrix.

VAR 7 Z(I) Node Elevation, Ft. These values denote
the algebraic value of height of each node in the system
with respect to node 0 taken as a datum.

VAR 8 XM(I) Nodal Mass Flow Rate, GPM. These values
represent flow rates entering or leaving the system nodes.
Flow entering a system node is designated as negative.
Flow leaving the system node is designated as positive.
In an open system, these flows represent the network
load and are known. In a closed system, these values
are zero and must be entered as such.

VAR 9 PB(N) Branch Locations of Transducers (Pumps).
These variables define the branch location of the trans-
ducers in the system. A typical input data card would
appear as follows:

 7   10   12   22   37   4

This would indicate that transducer number 1 is located
in branch 7, transducer number 2 is located in branch
10,...etc. If there are no pumps in the system, this
array may be omitted by setting XNMAX equal to zero. This
will omit the entry of these values.

VAR 10 HEAD(M,N) Transducer Head, Ft. This array
specifies the head variation for each transducer in the
system. The value of M represents the running index for
the points in the table for transducer number N.

<u>VAR II GPM(M,N) Transducer Volumetric Flow Rate, GPM</u>.

This array specifies the volumetric flow rate variation for each transducer in the system. These values together with those in VAR 10 define the flow versus head characteristics of the transducer. The value of transducer head used in the analysis is determined by the interpolation subroutine (SI FUNCTION) described earlier.

III. DESCRIPTION OF OUTPUT DATA

The output data can be conveniently divided into the following groups:

1) Input Data Printout

All input data are printed out and labeled using variable names defined in the description of input data.

2) Debugging Printout

At the users option, under the control of FLAG 1, described in the input data, the following calculated variables may be printed out for debugging or checking purposes. These data are classified into two groups:

a) Debugging printout for mathematical operations performed once at the beginning of the program labeled as indicated below:

IMAX, JMAX, NMAX

These values are equivalent to XIMAX, XJMAX and XNMAX, described in the input data, after being converted to fixed mode.

D(J) Branch Hydraulic Diameter, Ft.

These values represent the branch hydraulic diameters after being converted to feet.

AREA(J) Branch Cross-Sectional Area, Ft.^{2}

These values specify the calculated branch flow cross-sectional area.

XM(I), EMO(J), GPM(M,N)

These quantities represent mass flow rates, as defined previously in the description of input data, after conversion from GPM to lb/sec.

A^T Transpose of System Connection Matrix

This matrix is generated using the input variables CONN and CONP and is printed under subroutine DPRINT.

A   System Connection Matrix

This matrix is obtained by transposing matrix A^T, using GMTRA subroutine.

ZB(J) Branch Elevational Difference Array, Ft.

These values represent the elevational difference measured along the positive flow direction. For example, referring to Figure 1, the branch elevational difference for branch 38 is $Z(18)-Z(12)$.

PROD Matrix

This matrix is the product of the following two matrices--the first matrix having branch γ's along its diagonal and zero elsewhere, and the other, the system connection matrix A, as indicated by equation (9).

R Matrix

This matrix is the product of the transpose of the system connection matrix A^T times the PROD matrix, represented by matrix M in equation (9).

R^{-1} Matrix

This matrix is the inverse of the R matrix defined above.

RR Matrix

This matrix is defined as the product of the inverse of the R matrix times the transpose of the connection matrix A^T as required by equation (8).

RM Array

These values constitute an array obtained by multiplying the inverse of the R matrix times the XM array defined earlier.

b) Debugging printout for mathematical operations performed after each time integration labeled as indicated below:

REYN(J) Reynolds Number

Reynolds Numbers are calculated as the product of hydraulic diameter times mass flow rate divided by the product of the flow cross-sectional area times fluid viscosity. They are used to determine friction factors in the system branches.

FRIC(J) Friction Factor

These quantities are calculated from equation (5) based on Moody's Method to determine frictional head loss in pipes.

These values denote an intermediate calculation and
represent the quantity contained in the first set of braces
on the right-hand side of equation (8).

DELP(J) Total Branch Pressure Difference lb/ft^2

These values represent the total branch pressure
difference computed from equation (1).

RRR Array

These values constitute an array obtained by multi-
plying RR matrix by SUM array, both defined earlier.

3) Variables Defining the Problem Solution

These variables are printed after every DTPR seconds
and define the problem transient solution. They are
labeled as indicated below:

EM(J)	Branch Mass Flow Rate, GPM
EM(J)	Branch Mass Flow Rate, lb/sec
P(I)	Nodal Pressures, lb/in^2
FF(J)	Branch Frictional Pressure Drop, Ft

4) Final Steady-State Printout

The variables are printed only when steady-state con-
ditions are reached. They are labeled as indicated below.

REL(J) Steady-State Convergence Error

These values represent the slope of the mass flow
rate time response curve at steady-state conditions.

KOUNT   Actual Number of Iterations

This value represents the actual number of iterations performed in order to reach steady-state conditions.

In addition to REL(J) and KOUNT, all information stated under paragraph (3) above is also printed out.

To employ the HYTRAN computer program effectively, the user should familiarize himself with sections on the mathematical formulation, numerical stability analysis, and the presentation of results. He should then prepare a diagram of the system to be analyzed and assign a separate numbering system to nodes and branches as shown in Figure 1. The nodal point in the system where the pressure is known is numbered as node zero and the remaining nodes are numbered arbitrarily without skipping any number. It should be emphasized that node zero in the hydraulic network simulates a reservoir with known pressure and infinite water capacity. The algebraic sum of all XM(I) will enter the system at node zero and the nodal pressures in the system are determined with respect to the reservoir pressure considered as datum.

The user should then prepare the input data in accordance with the format and description given in part two, section II. These input data are then punched on cards and assembled with the HYTRAN program together with a number of control cards. These control cards vary from one computer organization to the next. A sample of the deck assembly for RCA Spectra-70 is given below:

```
                                run time sec, no of output lines
// JOB........., HYTRAN,...................................,
// PARAM LIST = YES, DEBUG = YES
// FORTRN
HYTRAN PROGRAM DECK
// EXEC
INPUT DATA DECK
```

After the program has been run, the user should
verify the following points:

1) Correctness of Input Data and KOUNT Error Message

The input data printed on output sheets should be
verified for possible errors in punching. Furthermore,
the output sheets should be examined for a possible error
message on KOUNT and proper action taken to rectify this
error condition as described in the description of input
data.

2) Convergence of the Transient Solution

The branch mass flow rates are then examined and plot-
ted. If the integration time step is properly selected
according to the numerical stability analysis described
earlier, the time variation of the branch flow rate will
be smooth and will not exhibit an oscillating pattern.
However, since the numerical stability criterion depends
on the approximate values of steady-state mass flow rates,
and these values are not exactly known before the problem
solution, the mass flow rate in the branch with the small-
est time constant may at times become oscillatory or non-
convergent. When such a situation occurs, one should,
using the run performed, adjust the estimate of steady-
state mass flow rates to decrease the branch time constants
and therefore reduce the integration time step. To verify
the convergence of the transient solution, the user may
employ one of the following techniques:

a) Determine the integration time increment based on steady-state mass flow rates. If this integration time increment is close to the one initially employed, the solution has converged.

b) Enter a value of time increment equal to one half of the previous time increment and compare the transient results on mass flow rates.

When the user is not interested in the transient solution and he wishes to obtain only the final steady-state solution, the time increment can be increased to the order of the system minimum time constant or even higher, leading to a large reduction in computer running time.

3) <u>Convergence of the Final Steady-State Solution</u>

The convergence of the final steady-state values can be determined by examining the branch mass flow rate curves plotted earlier. If these curves approach asymptotically to constant steady-state values, the final steady-state solution has been reached. If not, the value of the steady-state convergence error (ERROR) should be reduced.

To assist the user in the modification of the HYTRAN program, a list of some key variables with their corresponding analysis notation and their definition is presented in this section.

Program Notation	Analysis Notation	Description
A	A	Connection matrix for branches and nodes as defined in the text
AREA	a	Branch flow cross-sectional area, ft^2
D	D	Branch hydraulic diameter, ft
FF	F	Pressure loss due to friction, lb/ft^2
FRIC	f	Friction factor
GRAV	g	Acceleration of gravity, 32.2 ft/sec^2
H	H	Transducer pressure differential (pump pressure rise), ft
DELTAT	h	Time increment, sec
XL	L	Branch actual length, ft
XLP	L	Branch equivalent length (Branch actual length plus additional lengths allowed for fittings), ft
R	M	A matrix defined by equation (9)
EM	$\dot{m}$	Updated value of branch mass flow rate, lb/sec
EMO	$\dot{m}^0$	Present value of branch mass flow rate, lb
XM	$\dot{m}_{io}$	Nodal source and sink (input and output) mass flow rate (positive for source and negative for sink), lb/sec
P	p	Nodal pressure, psi
REYN	R_n	Branch Reynolds Number ($= D\,\dot{m}^0/a\mu$)

Program Notation	Analysis Notation	Description
TIME	t	Time, sec
Z	Z	Nodal elevations, ft
GAMMA	γ	A quantity defined in the text
DELP	Δp	Pressure drop along the positive flow direction, psi
ZB	ΔZ	Branch elevational difference along the positive flow direction (elevation of node at the tail of the arrow minus elevation of the node at the head of the arrow), ft
ROUGH	ϵ	Pipe roughness, ft
VISC	μ	Fluid viscosity, lb/sec - ft
RHO	ρ	Fluid density, lb/ft^3
ERROR	-	Steady-state convergence error

VI. HYTRAN PROGRAM LISTING

The HYTRAN program is written in FORTRAN-IV for RCA Spectra Model 70/45 digital computer. A listing of the HYTRAN program appears in the following pages.

```
 1          PROGRAM HYTRAN
 2 C            MAIN PROGRAM  WATER NETWORK FIXED GEOMETRY TRANSIENT ANALYSIS
 3 C                    G.V.CATANZARO GRAD HE
 4          DIMENSION A(2000),XL(50),XLP(50),Z(40),D(50),PB(5),HEAD(10,5),
 5       1           GPM(10,5),FMO(50),XM(40),GPMX(10),HEADX(10)
 6          DIMENSION AT(2000),ZB(50),GAMMA(50),PROD(2000),R(1600),LSPACE(40),
 7       1           LSPACE(40),H(50),REYN(50),FRIC(50),F(50),SUM(50),RR(2000),
 8       2           RRR(40),P(40),DELP(50),EM(50),COMP(50),CONN(50),PEL(50),
 9       3           AREA(50),E(50),FF(50),EMG(50),PP(40),RM(40)
10 C
11 C  FIX NO.   NOTATION     DESCRIPTION                    UNITS
12 C
13 C  1-1       EIPUN        RUN NO.DESIGNATION
14 C
15 C. 2-1       XIMAX        MAX.NO.OF NODES
16 C  2-2       XJMAX        MAX.NO.OF BRANCHES
17 C  2-3       XMMAX        MAX.NO.OF PUMPS
18 C
19 C  3-1       COUNT        MAX.ALLOW.NO OF ITERATIONS
20 C  3-2       ERROR        STEADY STATE CONVERGENCE ERROR
21 C  3-3       DELTAT       TIME INCREMENT                 SEC.
22 C  3-4       DTPR         PRINTOUT INTERVAL
23 C
24 C  4-1       RHO          FLUID DENSITY                  LBM/CU-FT
25 C  4-2       VISC         FLUID VISCOSITY                LBM/FT*SEC
26 C  4-3       ROUGH        PIPE ABSOLUTE ROUGHNESS        INCHES
27 C  4-4       GRAV         GRAVITATIONAL ACCELERATION     FT/SEC**2
28 C
29 C  5-1       FLAG1        PRINTOUT  INDICATOR-1 PRINTS IMAX,JMAX,NMAX,
30 C                         D(J),AREA(J),XM(I),FMO(J),GPM(M,N),AT,A,ZB,
31 C                         PROD(J1),R,REYN(J),FRIC(J),F(J),SUM(J),RR,E(J)
32 C                         RRR,P,  0 SKIPS PRINT
33 C
34 C  VAR.NO.   NOTATION     DESCRIPTION                    NO.OF CARDS
35 C  1         XL(J)        BRANCH ACTUAL LENGTH,FT.       XJMAX/6
36 C  2         XLP(J)       BRANCH EQUIVALENT LENGTH,FT.   XJMAX/6
37 C  3         D(J)         BRANCH DIAMETER,INCHES         XJMAX/6
38 C  4         FMO(J)       BRANCH INITIAL MASS FLOW RATE,GPM XJMAX/6
39 C  5         COMP(J)      POSITIVE BRANCH NODE           XJMAX/6
40 C  6         CONN(J)      NEGATIVE BRANCH NODE           XJMAX/6
41 C
42 C  7         Z(I)         NODE ELEVATION,FT              XIMAX/6
43 C  8         XM(I)        NODE FLOW,GPM                  XIMAX/6
44 C
45 C  9         PB(N)        BRANCH NO. OF PUMPS            XMMAX/6
46 C  10        HEAD(M,N)    PUMP HEAD,FT                   2*XMMAX
47 C  11        GPM(M,N)     PUMP VOLUMETRIC FLOW,GPM       2*XMMAX
48 C***********************************************************************
49          NIN=5
50          NOUT=6
```

```
   51   9992 FORMAT(10X,I4)
   52   1000 FORMAT(2X,6(6X,E14.7))
   53   1001 FORMAT(1E10.3)
   54   1002 FORMAT(2E10.3)
   55   1003 FORMAT(3E10.3)
   56   1004 FORMAT(4E10.3)
   57   1005 FORMAT(5E10.3)
   58   1006 FORMAT(6E10.3)
   59   1007 FORMAT(6(I5,1X,E14.7,2X))
   60 C      ***READ AND PRINT NONSUBSCRIPTED VARIABLES***
   61        WRITE(NOUT,2054)
   62   2054 FORMAT('1',14X,('INPUT DATA              HYTRAN PROGRAM')
   63      1,/)
   64        READ(NIN,1001)EIRUN
   65        WRITE(NOUT,2001)
   66   2001 FORMAT(1H012X,'EIRUN')
   67        WRITE(NOUT,1000) EIRUN
   68 C
   69        READ(NIN,1003)XIMAX,XJMAX,XNMAX
   70        WRITE(NOUT,2002)
   71   2002 FORMAT(1H0,14X,'XIMAX          XJMAX          XNMAX')
   72        WRITE(NOUT,1000)XIMAX,XJMAX,XNMAX
   73 C
   74        READ(NIN,1004) COUNT,ERROR,DELTAT,DTPR
   75        WRITE(NOUT,2003)
   76   2003 FORMAT(1H0,14X,'COUNT          ERROR          DELTAT
   77      1     DTPR')
   78        WRITE(NOUT,1000)COUNT,ERROR,DELTAT,DTPR
   79 C
   80        READ(NIN,1004)RHO,VISC,ROUGH,GRAV
   81        WRITE(NOUT,2004)
   82   2004 FORMAT(1H0,14X,'RHO            VISC           ROUGH
   83      1     GRAV')
   84        WRITE(NOUT,1000)RHO,VISC,ROUGH,GRAV
   85 C
   86        READ(NIN,1001)FLAG1
   87        WRITE(NOUT,2005)
   88   2005 FORMAT(1H0,14X,'FLAG1')
   89        WRITE(NOUT,1000)FLAG1
   90        KOUNT=1
   91        TIME=0.
   92        TPR=0.
   93 C
   94 C
   95 C
   96        IMAX=XIMAX
   97        JMAX=XJMAX
   98        KMAX=IMAX*JMAX
   99        NMAX=XNMAX
  100        NCOUNT=COUNT
```

```
101 C        ***READ AND PRINT SUBSCRIPTED VARIABLES***
102          READ(NIN,1006) (XL(J),J=1,JMAX)
103          WRITE(NOUT,2006)
104   2006 FORMAT(1H0,10X,'XL(J)',/)
105          WRITE(NOUT,1007) (J,XL(J),J=1,JMAX)
106 C
107          READ(NIN,1006) (XLP(J),J=1,JMAX)
108          WRITE(NOUT,2007)
109   2007 FORMAT(1H0,10X,'XLP(J)',/)
110          WRITE(NOUT,1007) (J,XLP(J),J=1,JMAX)
111 C
112          READ(NIN,1006) (D(J),J=1,JMAX)
113          WRITE(NOUT,2008)
114   2008 FORMAT(1H0,10X,'D(J)',/)
115          WRITE(NOUT,1007) (J,D(J),J=1,JMAX)
116 C
117          READ(NIN,1006) (EMO(J),J=1,JMAX)
118          WRITE(NOUT,2009)
119   2009 FORMAT(1H0,10X,'EMO(J)',/)
120          WRITE(NOUT,1007) (J,EMO(J),J=1,JMAX)
121 C
122          READ(NIN,1006) (CDMP(J),J=1,JMAX)
123          WRITE(NOUT,2010)
124   2010 FORMAT(1H0,10X,'CDMP(J)',/)
125          WRITE(NOUT,1007) (J,CDMP(J),J=1,JMAX)
126 C
127          READ(NIN,1006) (CDNN(J),J=1,JMAX)
128          WRITE(NOUT,2011)
129   2011 FORMAT(1H0,10X,'CDNN(J)',/)
130          WRITE(NOUT,1007) (J,CDNN(J),J=1,JMAX)
131 C
132          READ(NIN,1006) (Z(I),I=1,IMAX)
133          WRITE(NOUT,2012)
134   2012 FORMAT(1H0,10X,'Z(I)',/)
135          WRITE(NOUT,1007) (I,Z(I),I=1,IMAX)
136 C
137          READ(NIN,1006) (XM(I),I=1,IMAX)
138          WRITE(NOUT,2013)
139   2013 FORMAT(1H0,10X,'XM(I)',/)
140          WRITE(NOUT,1007) (I,XM(I),I=1,IMAX)
141          READ(NIN,1006) (PB(N),N=1,NMAX)
142          IF(NMAX)2028,1102,2028
143   2028 WRITE(NOUT,2014)
144   2014 FORMAT(1H0,10X,'PB(N)',/)
145          WRITE(NOUT,1007) (N,PB(N),N=1,NMAX)
146          IF(NMAX)1200,1102,1200
147   1200 DO 1100 N=1,NMAX
148          READ(NIN,1006) (HEAD(M,N),M=1,10)
149          WRITE(NOUT,2015)
150   2015 FORMAT(1H0,10X,'HEAD(M,N)-N=',I3,/)
```

```
151          WRITE(NOUT,1007) (N,HEAD(M,N),M=1,10)
152   1100 CONTINUE
153 C
154          DO 1101   M=1,NMAX
155          READ(NIN,1006) (GPM(M,N),M=1,10)
156          WRITE(NOUT,2016)
157   2016 FORMAT(1H0,10X,'GPM(M,N)-N=',I3,/)
158          WRITE(NOUT,1007) (N,GPM(M,M),M=1,10)
159   1101 CONTINUE
160   1102 CONTINUE
161          IF(FLAG1-1)100,457,100
162    457 WRITE(NOUT,2053)
163   2053 FORMAT('1',14X,(' DEBUGGING PRINTOUT'),/)
164    101 WRITE(NOUT,2017)
165   2017 FORMAT(1H0,14X, 'IMAX     JMAX        NMAX')
166          WRITE(NOUT,1010)IMAX,JMAX,NMAX
167   1010 FORMAT(8X,3(6X,I3))
168    100 CONTINUE
169 C
170 C      ***CHANGE DIMENSIONS OF INPUT DATA TO FT-LB-SEC***
171 C
172          COEFF=(231.*RHO)/(60.*1728.)
173          DO 1 J=1,JMAX
174          D(J)=D(J)/12.
175          AREA(J)=(3.1416*D(J)**2/4.)
176          EMO(J)=EMO(J)*COEFF
177      1 CONTINUE
178          DO 8888 I=1,IMAX
179          XM(I)=XM(I)*COEFF
180   8888 CONTINUE
181          IF(FLAG1-1.)102,103,102
182    103 WRITE(NOUT,2018)
183   2018 FORMAT(1H0,10X,'D(J)',/)
184          WRITE(NOUT,1007) (J,D(J),J=1,JMAX)
185          WRITE(NOUT,2019)
186   2019 FORMAT(1H0,10X,'AREA(J)',/)
187          WRITE(NOUT,1007) (J,AREA(J),J=1,JMAX)
188          WRITE(NOUT,2050)
189   2050 FORMAT(1H0,10X,'XM(I)',/)
190          WRITE(NOUT,1007) (I,XM(I),I=1,IMAX)
191          WRITE(NOUT,2062)
192   2062 FORMAT(1H0,10X,'EMO(J)',/)
193          WRITE(NOUT,1007) (J,EMO(J),J=1,JMAX)
194    102 CONTINUE
195          IF(NMAX)105,104,105
196    105 DO 3 N=1,NMAX
197          DO 3 M=1,10
198          GPM(M,N)=GPM(M,N)*COEFF
199      3 CONTINUE
200          IF(FLAG1-1.)104,140,104
```

```
201       140 WRITE(NOUT,2016)
202           WRITE(NOUT,1007) (M,GPN(M,N),M=1,10)
203       104 CONTINUE
204 C
205 C         ***FORMATION OF CONNECTION MATRIX AT(K)
206           DO 110 K=1,KMAX
207           AT(K)=0.
208       110 CONTINUE
209           L=0
210           DO 111 J=1,JMAX
211           NP=COMP(J)
212           NN=CONN(J)
213           NP1=NP+L*IMAX
214           NN1=NN+L*IMAX
215           L=L+1
216           IF(NP1)112,113,112
217       112 AT(NP1)=1.
218       113 CONTINUE
219           IF(NN1)114,115,114
220       114 AT(NN1)=-1.
221       115 CONTINUE
222       111 CONTINUE
223           IF(FLAG1-1.)116,117,116
224       117 WRITE(NOUT,2029)
225      2029 FORMAT(1H0,10X,'AT',/)
226           CALL DPRINT(AT,IMAX,JMAX)
227       116 CONTINUE
228           CALL GMTRA(AT,A,IMAX,JMAX)
229           IF(FLAG1-1.)118,119,118
230       119 WRITE(NOUT,2030)
231      2030 FORMAT(1H0,10X,'A',/)
232           CALL DPRINT(A,JMAX,IMAX)
233       118 CONTINUE
234           CALL GMPRD(A,Z,ZB,JMAX,IMAX,1)
235           IF(FLAG1-1.)120,121,120
236       121 WRITE(NOUT,2031)
237      2031 FORMAT(1H0,10X,'ZB',/)
238           WRITE(NOUT,1007) (J,ZB(J),J=1,JMAX)
239       120 CONTINUE
240           DO 25 J=1,JMAX
241           GAMMA(J)=AREA(J)*GRAV*RHO/XL(J)
242           GAMMA(J)=GAMMA(J)*DELTAT
243        25 CONTINUE
244           J=0
245           DO 9929 J1=1,KMAX
246           J=J+1
247           IF(J-JMAX)9928,9928,9927
248      9927 J=1
249      9928 PROD(J1)=GAMMA(J)*A(J1)
250      9929 CONTINUE
```

```
251           IF(FLAG1-1.)2222,2221,2222
252     2221  WRITE(NOUT,2052)
253     2052  FORMAT(1H0,10X,'PROD(J1)',/)
254           CALL DPRINT(PROD,JMAX,IMAX)
255     2222  CALL GMPRD(AT,PROD,R,IMAX,JMAX,IMAX)
256           IF(FLAG1-1.)122,123,122
257      123  WRITE(NOUT,2032)
258     2032  FORMAT(1H0,10X,'R',/)
259           CALL DPRINT(R,IMAX,IMAX)
260      122  CONTINUE
261           CALL MINV(R,IMAX,DET,LSPACE,MSPACE)
262           CALL GMPRD(R,AT,RR,IMAX,IMAX,JMAX)
263           CALL GMPRD(R,XM,RM,IMAX,IMAX,1)
264           IF(FLAG1-1.)4444,3333,4444
265     3333  WRITE(NOUT,2033)
266     2033  FORMAT(1H0,10X,'R',/)
267           CALL DPRINT(R,IMAX,IMAX)
268      129  WRITE(NOUT,2034)
269     2034  FORMAT(1H0,10X,'RR',/)
270           CALL DPRINT(RR,IMAX,JMAX)
271      151  WRITE(NOUT,2074)
272     2074  FORMAT(1H0,10X,'RM')
273           WRITE(NOUT,1007) (I,RM(I),I=1,IMAX)
274     4444  CONTINUE
275           DO 27 J=1,JMAX
276           H(J)=0.
277       27  CONTINUE
278     1110  CONTINUE
279           IF(NMAX)142,143,142
280      142  DO 28 N=1,NMAX
281           J=PR(N)
282           GPMX(I)=GPM(I,N)
283           HEADX(I)=HEAD(I,N)
284           H(J)=SI(GPMX,HEADX,EMD(J),10)
285       28  CONTINUE
286      143  CONTINUE
287           DO 29 J=1,JMAX
288           REYN(J)=D(J)*ABS(EMD(J))/(AREA(J)*VISC)
289           FRIC(J)=.0055*(1.+(2.*10.**4*ROUGH/D(J)+10.**6/REYN(J))**.333)
290           FF(J)=((FRIC(J)*XLP(J)*ABS(EMD(J))*EMD(J))/(D(J)*2.*GPAV*RHO*RHO
291          1*(AREA(J)**2)))
292           F(J)=FF(J)-ZB(J)
293            SUM(J)=GAMMA(J)*(F(J)+H(J))-EMD(J)
294       29  CONTINUE
295           CALL GMPRD(RR,SUM,PRR,IMAX,JMAX,1)
296           CALL GMSUB(RRR,RM,P,IMAX,1)
297           CALL GMPRD(A,P,DELP,JMAX,IMAX,1)
298           IF(FLAG1-1)451,459,451
299      459  IF(TIME-TPR)451,458,458
300      458  WRITE(NOUT,2075)TIME
```

```
301    2075 FORMAT('1',14X,'DEBUGGING PRINTOUT AT TIME=',E14.7,/)
302     127 WRITE(NOUT,2021)
303    2021 FORMAT(1H0,10X,'REYN(J)',/)
304         WRITE(NOUT,1007) (J,REYN(J),J=1,JMAX)
305         WRITE(NOUT,2022)
306    2022 FORMAT(1H0,10X,'FRIC(J)',/)
307         WRITE(NOUT,1007) (J,FRIC(J),J=1,JMAX)
308         WRITE(NOUT,2024)
309    2024 FORMAT(1H0,10X,'SUM(J)',/)
310         WRITE(NOUT,1007) (J,SUM(J),J=1,JMAX)
311     444 WRITE(NOUT,2037)
312    2037 FORMAT(1H0,10X,'DELP,FT',/)
313         WRITE(NOUT,1007) (J,DELP(J),J=1,JMAX)
314     889 WRITE(NOUT,2035)
315    2035 FORMAT(1H0,10X,'ERR',/)
316         WRITE(NOUT,1007) (I,RRR(I),I=1,IMAX)
317     451 DO 301 J=1,JMAX
318         E(J)=H(J)-DELP(J)
319     301 EM(J)=EMU(J)-GAMMA(J)*(E(J)+F(J))
320         IF(TIME-TPR)136,453,453
321     453 WRITE(NOUT,2071)TIME
322    2071 FORMAT('1',14X,'OUTPUT VARIABLES AT TIME=',E14.7,/)
323     445 WRITE(NOUT,2026)
324    2026 FORMAT(1H0,10X,'EM(J),GPM',/)
325         DO 2076 J=1,JMAX
326         EMG(J)=EM(J)/COEFF
327    2076 CONTINUE
328         WRITE(NOUT,1007) (J,EMG(J),J=1,JMAX)
329         WRITE(NOUT,2079)
330    2079 FORMAT(1H0,10X,'EM(J),LBS/SEC',/)
331         WRITE(NOUT,1007) (J,EM(J),J=1,JMAX)
332         DO 452 I=1,IMAX
333         PP(I)=P(I)/144.
334     452 CONTINUE
335         WRITE(NOUT,2072)
336    2072 FORMAT(1H0,10X,'P(I),PSI',/)
337         WRITE(NOUT,1007) (I,PP(I), I=1,IMAX)
338         WRITE(NOUT,2055)
339    2055 FORMAT(1H0,10X,'FF(J),FT')
340         WRITE(NOUT,1007) (J,FF(J), J=1,JMAX)
341         TPR=TPR+DTPR
342     136 CONTINUE
343         DO 61 J=2,JMAX
344         REL(J)=ABS((EM(J)-EMU(J))/DELTAT)
345      61 CONTINUE
346         DO 32 J=2,JMAX
347         IF(REL(J)-ERROR)32,32,201
348      32 CONTINUE
349     475 WRITE(NOUT,2071)TIME
350         WRITE(NOUT,2026)
```

```
351          WRITE(NOUT,1007) (J,EMG(J),J=1,JMAX)
352          WRITE(NOUT,2079)
353          WRITE(NOUT,1007) (J,EM(J),J=1,JMAX)
354          WRITE(NOUT,2072)
355          WRITE(NOUT,1007) (I,PP(I), I=1,IMAX)
356          WRITE(NOUT,2055)
357          WRITE(NOUT,1007) (J,FF(J), J=1,JMAX)
358          WRITE(NOUT,2077)
359   2077  FORMAT('1',14X,'FINAL STEADY STATE PARAMETERS')
360          WRITE(NOUT,2027)
361   2027  FORMAT(1H0,10X,'RELATIVE ERROR',/)
362          WRITE(NOUT,1007) (J,REL(J),J=2,JMAX)
363          WRITE(NOUT,2051)
364   2051  FORMAT(1H0,10X,'KOUNT',/)
365          WRITE(NOUT,9999)KOUNT
366          STOP
367    201  IF(KOUNT-NCOUNT)202,203,203
368    203  WRITE(NOUT,2057)
369   2057  FORMAT(1H0,5X,'THE MAXIMUM ALLOWABLE NO. OF ITERATIONS HAS BEEN
370         1REACHED',/)
371          STOP
372    202  KOUNT=KOUNT+1
373          TIME=TIME+DELTAT
374          DO 10 J=2,JMAX
375          EMO(J)=EM(J)
376     10  CONTINUE
377          GO TO 1110
378          END
```