OPEN-LOOP FREQUENCY RESPONSE ANALYSIS OF A JACKETED KETTLE WITH INTERNAL HEATING/COOLING COIL

BY

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ABSTRACT

This work was performed to investigate the response and stability of a jacketed kettle with internal heating/cooling coils and glasteel thermowell. This system was first analyzed mathematically by deriving transfer functions for each component and then combining all component equations together to give the overall system transfer function. The mathematical model was then checked by frequency response analysis of the system. Bode plots of the system were prepared and the effect of the thermowell was analyzed.

Nyquist plots of the system were used as the criteria for determining stability, and obtaining the open-loop gain necessary to give optimum closed-loop response. The system was found to be completely stable over the frequency range investigated.

Condensate throttling in the coil was the method used to control the kettle temperature during the frequency response tests. The advantages and disadvantages of this method of control are discussed.

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INTRODUCTION

For many years the Chemical Engineer working in the processing field was only concerned with the kinetic and equilibrium design of a process. The control of the process was not considered, and it was assumed that the operator would determine how fast a change must be made in the process to compensate for unexpected disturbances in the system. As the design of plants began moving in the direction of increased automation for closer control and reduced manpower operating costs, the systems-engineering approach to design became as important to the process as the kinetic and equilibrium considerations.

The objective of the systems-engineer is to perform a dynamic analysis of the system components to determine the best way to measure the controlled variable with minimum time lags, and give the maximum gain to the system without causing instability.

There are several methods such as mathematical derivation, frequency response testing, step response testing, pulse testing, etc., that can be used to make a dynamic analysis of a system. The objective of any dynamic analysis is to determine the transfer function for the system or a component. The transfer function of a system or component is defined as the ratio of the change of the output to the input which caused the change. The transfer functions are then plotted on a Bode diagram which gives a complete picture of the system or component. With the Bode diagram complete the engineer can determine the optimum modes of control, gain, and stability.

The mathematical derivation of transfer functions requires the writing of a differential or partial differential equation which completely expresses all of the system characteristics. Because, in practice, most systems are too complex to be handled by standard mathematical techniques, simplifying assumptions are made to reduce equations to lower order linear systems. Generally, the method used is to break the system down into components which can be analyzed separately. If one component does not depend on conditions in the other components, the transfer functions for the components can be multiplied to give the overall response of the system.

In frequency response testing a sine wave of constant amplitude and frequency is used as the input signal to the component under test. If the component is linear the output signal should also be a sine wave, but the amplitude and phase will be different in accordance with the transfer function. The test is conducted at various frequencies and the results obtained can be plotted directly on a Bode diagram. Generally, for complex systems, it is extremely difficult to back out the transfer function from a Bode diagram. Knowing the transfer function, however, is not necessary if the engineer is only interested in selecting control modes and determining stability.

The step response testing method involves applying an instantaneous change to the input which is then held constant. The resulting output shape and its time relationship is noted. The output form can then be approximated to a transfer function by assuming first and/or second order time constants. Although this technique is not as comprehensive as frequency response testing it has the advantage of only requiring as much time as the longest time constant of the system.

The work of this thesis involved the investigation of the response and stability of a jacketed kettle with internal heating/cooling coil and glass lined thermowell by mathematically deriving the transfer function for each component in the system and then performing a frequency response analysis of the system to determine if the mathematical model initially derived accurately described the system.

Although much work has been done with jacketed kettles there is a lack of any data on systems using coils in the vessel. The use of coils in vessels is a common technique used to obtain better control of polymerization processes. These processes require heat from a jacket to bring the process up to temperature, but as polymerization begins, the reaction becomes highly exothermic requiring rapid cooling. Kettle jackets have a large amount of inertia, thus making it almost impossible to obtain rapid cooling. The coil inside a vessel thus provides a second heat transfer surface which can provide instantaneous cooling at the time the polymerization takes place. It is for this reason that the jacketed kettle with an internal coil was evaluated.

APPROACH TO PROBLEM

The control of polymerization processes is very difficult because, as the reaction proceeds, heat must be supplied by the system, but, after a certain period of time, the reaction begins to polymerize and, at this point, the reaction becomes exothermic and the system must be capable of removing heat almost instantaneously. One common technique used to accomplish this is to allow the vessel jacket to supply the heat and provide internal coils to remove heat. To stimulate this process a jacketed vessel with an internal coil was used. The jacket was held at constant temperature while the coil provided a varying heating/cooling load to the system. Condensate throttling was the method used to control the coil heating/cooling input.

Another important consideration which must be given to the control of polymerization processes is the response of the temperature sensing device used. Because most polymerization processes are studied under batch conditions and tend to be corrosive, glass lined vessels are generally used. This type vessel uses a glass lined thermowell as the temperature sensing device. Thermowells of this design have slow response and contribute to the problem of providing rapid cooling when polymerization occurs. For this reason, a glass lined thermowell with a bare wire thermocouple was placed in the kettle as the sensihg element and its effect on the total response and stability of the system was studied.

Method of Attacking the Problem

The system was analyzed by deriving mathematical transfer functions for each component. The component transfer functions were then multiplied to give the resultant system transfer funtion and this result was plotted on Bode diagrams.

The Bode diagrams were used to determine the effect of the thermowell on the system and the maximum open-loop gain which could be used before the system would become unstable.

Nyquist diagrams were used to determine the stability of the open-loop system and also the necessary open-loop gain to give an optimum closed-loop response.

A Nichols chart was used to obtain the direct closed-loop function from plotting the direct open-loop transfer function.

A frequency response analysis of the system was performed and the results plotted on a Bode diagram. The results of the frequency response test were used as a check of the mathematical model to be certain that all elements of the system were considered.

THEORETICAL ANALYSIS

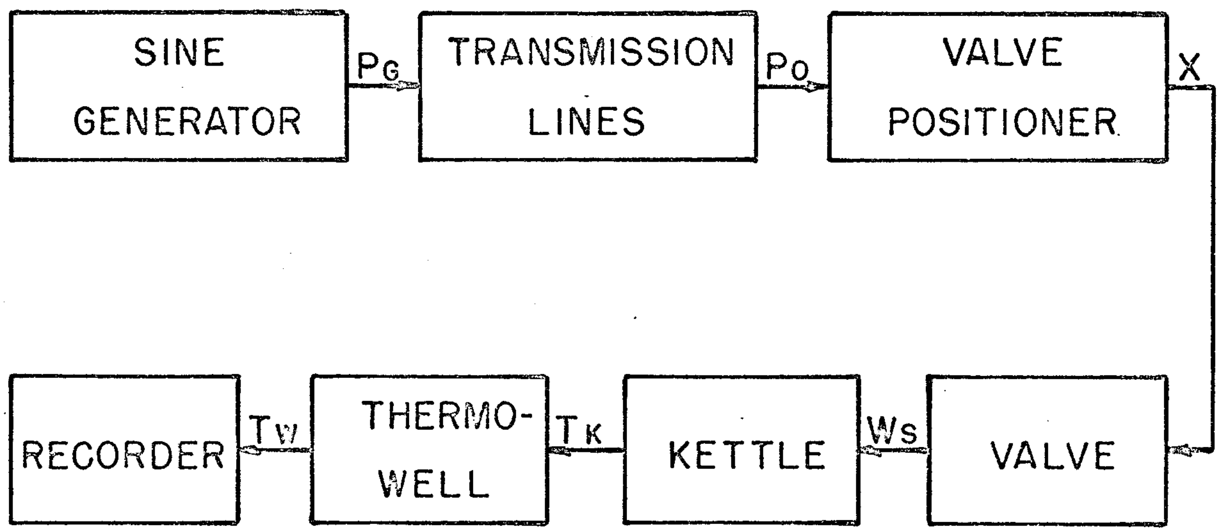
General Theory

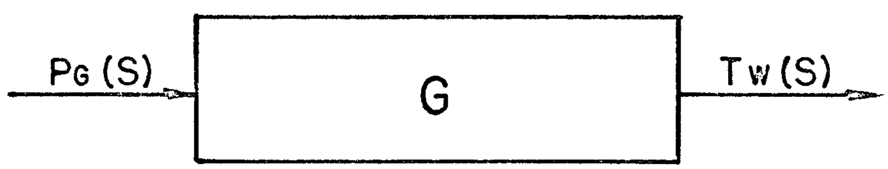
In order to perform a dynamic analysis of a system it is first necessary to determine which elements in the system will have a major influence. Once the major elements are known they should be arranged in a block diagram which shows their interaction. Each block contains an input and output, and a mathematical relationship which is the transfer function. In addition to organization of elements the block diagram is also useful for determining the overall consistency of mathematical units. Figure 1 gives the block diagram for the system studied.

In order to use the analysis of elements to determine the overall system transfer function, each element must not depend upon conditions in the other elements. The elements are then considered non-interacting and the individual transfer functions can be multiplied to give the overall system transfer function. In this system the heated tank element has no interaction with the thermowell element since the heat transfer to the thermowell can be neglected when the heat balance equation for the tank is written.[1]

1. Process Control by P. Harriott, New York: McGraw-Hill, 1964, p. 47.

FIGURE I
BLOCK DIAGRAM





The following gives the mathematical derivation of the transfer function for each element listed in Figure 1.

Kettle transfer function. In order to be certain the jacketed kettle with internal coils is a linear system the following conditions were used.

- The kettle contains enough water so the steam coil heat capacity is negligible.
- The kettle has a mixer which keeps the water at a uniform temperature.
- The steam throttling valve has linear characteristics.[2]

The heat flow into the tank is by the steam condensing in the coil and the heat is removed by the water flowing through the jacket of the kettle. The energy balance for the system is written:

Heat Input = Heat Output + Accumulation

$$w\lambda = UA(T_K - T_1) + MC_p \frac{dT_K}{d\theta} \quad (1)$$

where

w = steam flow rate

λ = latent heat of vaporization of steam

U = kettle overall heat transfer coefficient

2. Process Dynamics and Feedback Control by Ernest Doeblin, New York: McGraw-Hill, 1952, p. 127.

A = kettle effective cooling area

T_K = temperature of water in kettle

T_1 = inlet jacket water temperature

M = mass of water in the kettle

C_p = heat capacity of water

θ = time

Dividing through by UA, and rearranging gives

$$\frac{w\lambda}{UA} + T_1 = T_K + \frac{MC_p}{UA}\frac{dT_K}{d\theta} \tag{2}$$

Differentiating Equation 2 with respect to time

$$\frac{\lambda}{UA}\frac{dw}{d\theta} + \frac{dT_1}{d\theta} = \frac{dT_K}{d\theta} + \frac{MC_p}{UA}\frac{d^2T_K}{d\theta^2} \tag{3}$$

Taking the Laplace transform of Equation 3 holding the independent variable w constant and the other independent variable T_1 zero at time zero,

$$\frac{\lambda}{UA}\left(s\frac{w}{s} - w_0\right) + s\mathcal{L}T_1 - 0 = s\mathcal{L}T_K\left(1 + \frac{MC_p}{UA}s\right) \tag{4}$$

For T_1 constant and $w_0 = 0$,

$$\frac{\mathcal{L}[T_K]}{\mathcal{L}[w]} = \frac{\lambda/UA}{\frac{MC_p}{UA}s + 1} \tag{5}$$

<u>Thermowell transfer function.</u> The thermowell was a 1" steel well with a 0.06" glass coating. The differential equations governing the flow of heat through the thermo-

well are:

$$\frac{dQ}{d\theta} = K_\omega A_\omega (T_K - T) \qquad (6)$$

$$dQ = C_\omega dT \qquad (7)$$

where Q = quantity of heat

K_ω = thermowell overall heat transfer coefficient

A_ω = thermowell effective heating area

T_K = temperature of water in kettle

T = a variable temperature at any time

C_ω = thermowell heat capacity

θ = time

Substituting for dQ, dividing through by C_ω, and taking the Laplace transform gives

$$T\left(s + \frac{K_\omega A_\omega}{C_\omega}\right) = \frac{K_\omega A_\omega}{C_\omega} T_K \qquad (8)$$

Dividing by $\frac{K_\omega A_\omega}{C_\omega}$ and rearranging we get

$$\frac{\mathcal{L}[T]}{\mathcal{L}[T_K]} = \frac{1}{\frac{C_\omega}{K_\omega A_\omega} s + 1} \qquad (9)$$

Valve and operator transfer function. The valve used in this process was linear in its operation. Since this valve is fast acting compared to the system, the stem friction and inertia can be neglected, and the equation reduces to zero order.[3]

Analysis by a force balance:

$$P_c A_d = K_s \Psi \qquad (10)$$

$$\frac{A_d}{K_s} = \frac{\Psi}{P_c} \qquad (11)$$

where P_c = pressure on the operator dome

A_d = area of the operator piston

K_s = spring constant

Ψ = valve-stem displacement

In order to find a transfer function involving P_c, we must build on the conservation equation for the system.

$$w_1 d\theta - w_2 d\theta = C dp \qquad (12)$$

where w_1, w_2 = mass flow rate in and out of valve

θ = time

C = capacitance

P = absolute pressure

Since there is no accumulation the flow in equals flow out. The equation for flow of a liquid through a

3. <u>Process Dynamics and Feedback Control</u> by Ernest Doeblin, New York: McGraw-Hill, 1962, pp. 90-91.

constriction is:

$$w = Y C_v a \sqrt{\frac{2 g_c \rho \Delta P}{1-\beta^4}} \tag{13}$$

where Y = 1.0 for liquids

C_v = discharge coefficient for valve

a = cross sectional area for flow

ρ = density of liquid

ΔP = pressure drop across valve

β = ratio of constriction diameterto pipe diameter

If the valve is a linear one:

$$a = a_0 + B\psi \tag{14}$$

Substituting into Equation 13 for a and ψ

$$w = C_v\left[a_0 + B\left(\frac{P_c A_d}{K_s}\right)\right]\sqrt{\frac{2 g_c \rho \Delta P}{1-\beta^4}} \tag{15}$$

The transfer function between w and P_c, $\frac{\mathcal{L}[w]}{\mathcal{L}[P_c]}$ is $\frac{\partial w}{\partial P_c} = \frac{dw}{dP_c}$ (assuming no time lags) for w_0, a_0, ψ_0, P_{c_0} all = 0.[4]

$$\frac{\mathcal{L}[w]}{\mathcal{L}[P_c]} = \frac{C_v A_d B}{K_s}\sqrt{\frac{2 g_c \rho \Delta P}{1-\beta^4}} = K \tag{16}$$

Transmission line transfer function. Upon investigation of the transmission line transfer function, it was found

4. Process Dynamics and Feedback Control by Ernest Doeblin, New York:McGraw-Hill, 1962, p. 129.

to be negligible compared to the other elements of the system. The development of the equation and numerical values are presented in the appendix.

Numerical Analysis

Kettle transfer function. The kettle transfer function is:

$$\frac{T_K(s)}{w(s)} = \frac{\lambda/UA}{\left(\frac{MC_p}{UA}\right)s+1} \tag{17}$$

The only quantity which must be calculated is U, the overall heat transfer coefficient, as the other values can be obtained from the literature or by calibration of the equipment.

The equation used to calculate the overall heat transfer coefficient for a jacketed kettle with agitation is:

$$\frac{h_j D_j}{k} = .36\left(\frac{L^2N\rho}{\mu}\right)^{2/3}\left(\frac{c\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{.14}\ 5 \tag{18}$$

For water in the kettle at 100°F. average temperature,

h_j = heat transfer coefficient, jacket side

D_j = diameter of vessel (ft.)=1 ft.

k = thermal conductivity = .3625 for water at 100°F.

c = specific heat = 1.0 Btu/#

μ = viscosity = 1.815 #/ft.hr.

5. Process Heat Transfer by Donald Kern, New York: McGraw-Hill, 1950, p. 718.

ρ = density = 62.4 #/ft.^{3}

N = revolutions per hour = 4500 RPH

L = length of agitator paddle (ft.) = .416 ft.

$$h_j = (.36)(.3625)\left[\frac{(.416)^2(4500)(62.4)}{1.815}\right]^{2/3}\left[\frac{(1)(1.815)}{.3625}\right]^{1/3}$$

$$h_j = 202 \frac{BTU}{HR\ FT^2\ {}^\circ F}$$

Next h_o, the outside heat transfer coefficient, must be calculated. For this situation the jacket will be treated as an anulus of a double pipe heat exchanger.

The flow area of the jacket is

$$a = \pi (D_2^2 - D_1^2)/4 \qquad (19)$$

where $D_2 = \frac{O.D.}{D_j} = \frac{13''}{12''/FT} = 1.08\ FT$

$D_1 = \frac{I.D.}{D_j} = \frac{12''}{12''/FT} = 1\ FT$

$$a = (3.14)\left[(1.08)^2 - (1)^2\right]/4 = 1.0\ FT^2$$

The equivalent diameter is

$$D_e = (D_2^2 - D_1^2)/D_1 = \frac{\left[(1.08)^2 - (1.0)^2\right]}{1} = 1.27\ FT$$

The mass velocity through the jacket is

$$G = \frac{w}{a} = \frac{347\ \#/HR}{1.0\ FT^2} = 347 \frac{\#}{HR\ FT^2}$$

The Reynolds number for flow through the jacket is

$$N_{Re} = \frac{D_e G}{\mu} = \frac{(1.27)(347)}{1.815} = 243$$

Therefore, since the Reynolds number is $<$ 2100 the equation for streamline flow will be used to calculate h_o.

$$\frac{h_o D_e}{k} = 1.86\left[\frac{4}{\pi}\frac{wc}{kL}\right]^{1/3}\left[\frac{\mu}{\mu_w}\right]^{.14} \qquad (20)$$

where L = length of straight side of kettle = $\frac{18''}{12''}$/ft.

$$h_o = \frac{1.86\left[\frac{4}{\pi}\frac{(347)(1)}{(.356)(1.5)}\right]^{1/3}(1)(356)}{1.27}$$

$$h_o = 13.6 \ \frac{BTU}{HR\ FT^2\ {}^\circ F}$$

The overall heat transfer coefficient for the jacketed kettle is

$$U_c = \frac{h_j h_o}{h_j + h_o} = \frac{(202)(13.6)}{(202 + 13.6)} = 128 \ \frac{BTU}{HR\ FT^2\ {}^\circ F}$$

As standard design practice, a fouling factor, R_d of 0.005, will be used. Using R_d an averall heat transfer coefficient U_D is determined.

$$R_d = 0.005$$

$$h_d = \frac{1}{R_d} = 200$$

$$U_D = \frac{U_c U_D}{U_c + U_D} = \frac{(128)(200)}{128 + 200} = 78$$

Substituting values into the transfer function equation gives

$$\frac{T_K}{\omega_s} = \frac{\lambda / uA}{\frac{MC_p}{uA} s + 1}$$

where λ = 1150.7 Btu/# for 5 PSIG steam

A = 4.7 ft.^{2}

λ/uA = 3.14

$\frac{MC_p}{uA}$ = time constant = 12 minutes

$$\frac{T_K}{\omega_s} = \frac{3.14}{12s+1}$$

The 12 minute time constant compares favorably with a 14.5 minute time constant obtained by actually performing a step response test of the kettle.

Thermowell transfer function. The thermowell transfer function equals

$$\frac{T}{T_K} = \frac{1}{\frac{C_\omega}{K_\omega A_2} s + 1} \qquad (21)$$

where

K_ω = thermal conductivity of thermowell

Since

$$\frac{1}{K_w} = \frac{1}{k_{glass}} + \frac{1}{k_{steel}} + h_{water}$$

and

$$k_{glass} = \frac{6 \text{ Btu(in.)}}{\text{hr.ft}^2 \text{ }^\circ\text{F.}} = \frac{6}{\text{thickness of glass in inches}}$$

$$= \frac{6}{.06} = 100$$

$$k_{steel} = \frac{360 \text{ Btu(in.)}}{\text{hr.ft.}^2 \text{ }^\circ\text{F.}} = \frac{360}{\text{thickness of steel wall}}$$

$$= \frac{360}{.153} = 2360$$

$$h_{water} = \frac{1}{h_j} = \frac{1}{202} = 0.005$$

Substituting the above values

$$\frac{1}{K_w} = .005 + \frac{1}{100} + \frac{1}{2360} = .0154$$

K_w = 65 Btu/hr.ft.^{2} $^\circ$F.

C_w = sum of capacities of glass and steel volumes of the thermowell

C_w was found to be equal to .20 Btu/$^\circ$F.

A_2 = thermowell effective heating area
= .142 ft.^{2}

Substituting the above values into Equation 21 gives

$$\frac{T}{T_K} = \frac{1}{1.3\,S+1}$$

The thermowell time constant compares favorably to an actual step response test which gave a thermowell time constant of 1 minute.

Valve and operator transfer function. The transfer function for the valve and operator equals

$$\frac{W}{P_c} = \frac{C_v A_d \beta}{K_s}\sqrt{\frac{2g_c\,\rho\,\Delta P}{1-\beta^4}} \qquad (22)$$

where $\frac{A_d}{K_s} = \frac{\mathcal{y}}{P_c}$

The value of $\frac{A_d}{K_s}$ may be evaluated on the basis that the valve has a full travel of 3/8" when 15 psi air pressure is applied to the operator and the valve is linear over this range. Therefore, $\frac{A_d}{K_s} = \frac{3/8 - 0}{12 - 0} = .0313$

B is defined as the slope of the line which relates the valve stem travel to the cross sectional area of flow. (When the valve is full open ($x = 3/8"$) the cross sectional area for flow is .306 in.^{2}) Since the valve is linear, and with the stem closed the cross sectional area equal to 0,) $B = \frac{.306 - 0}{3/8 - 0} = .81$.

C_v is the valve coefficient determined by the manufacturer's data and is equal to .01.

ΔP is the pressure drop across the valve and equals 720#/ft.^{2}.

B is the ratio of the construction diameter to the pipe diameter. $B = \frac{5/32}{.180} = .83$

Substituting the above values into Equation 22

$$\frac{W}{P_c} = (.01)(.0313)(.81)\sqrt{\frac{(2)(32.2)(62.4)(720)}{1 - (.83)^4}} = .48$$

Overall system transfer function. Multiplying the preceeding equations for the elements of the system gives an overall transfer function:

$$G = \frac{T_W}{P_G} = \frac{(3.14)(.48)}{(12s+1)(1.3s+1)} \qquad (23)$$

The above equation must now be evaluated to determine amplitude ratio and phase shift. These values are necessary to construct the Bode diagram.

For a first order system of the form

$$G = \frac{R}{\tau s+1}$$

The amplitude ratio $|(G)| = \frac{R}{\sqrt{1+w^2\tau^2}}$ and (24)

the phase angle $\angle G$ = -arc tan $\omega\tau$. (25)

Using Equations 24 and 25 the amplitude ratio and phase angle for each element of the system was calculated. Since the Bode diagram is a semi-log plot the overall system response can be obtained by plotting each element and adding each point to obtain the resultant.

The amplitude ratio used in plotting a Bode diagram of a sinusoidal transfer function is generally given in decibels.

$$\text{Decibels} = \text{db} = 20 \log_{10}(G) \ . \qquad (26)$$

Table 1 gives the summary of the calculated values of amplitude ratio and phase angle.

Table 1

Amplitude Ratio and Phase Angle for Mathematical Analysis of Elements

W(cycles/min.)	Kettle		Thermowell		Valve & Operator		Resultant	
	|G|db	ϕ	|G|db	ϕ	|G|db	ϕ	|G|db	ϕ
.001	9.94	- 1.0	0	0	-6.38	0	3.56	- 1.0
.01465	9.8	-12.0	0	0	-6.38	0	3.42	- 12.0
.0294	9.2	-23.0	0	- 1.0	-6.38	0	2.82	- 24.0
.0586	7.6	-40.4	0	- 3.0	-6.38	0	1.22	- 43.4
.1175	4.03	-59.7	- .09	- 6.8	-6.38	0	-2.44	- 66.5
.234	- .92	-73.6	- .3	-13.0	-6.38	0	- .76	- 86.6
.469	- 7.96	-81.6	- .82	-25.0	-6.38	0	-15.16	-106.6
.938	-13.35	-86.0	-2.73	-43.2	-6.38	0	-22.46	-129.2

Figures 2, 3 and 4 give the Bode diagrams obtained from the data in Table 1.

Figure 5 is a Nyquist plot for the overall system. The Nyquist plot is the method used on this system to determine stability. The test for stability using the Nyquist method requires that the amplitude ratio be less than 1 when the system goes through -180° of phase shift. Upon examination of Figure 5 the system has an amplitude ratio of 1 at -52°, thus giving the system 126° of phase margin. This amount of phase margin makes the system completely stable over the frequency range investigated.

Open-loop gain setting. The Nyquist plot is also a good method of determining the open-loop gain which should be applied to the system to give a closed-loop system which would have fast response and good stability. Figure 6 shows the Nyquist plot with a line constructed at 130° (M=1.3). The value of 130° is picked to give the closed-loop response a 50° plase margin which is generally used as a good compromise between stability and response. A circle is then constructed to be tangent to the open-loop response and the M=1.3 line. The open-loop gain is then determined by measuring the distance from the origin to the point of tangency. The open-loop gain multiplied by the measured distance must equal 1.0.

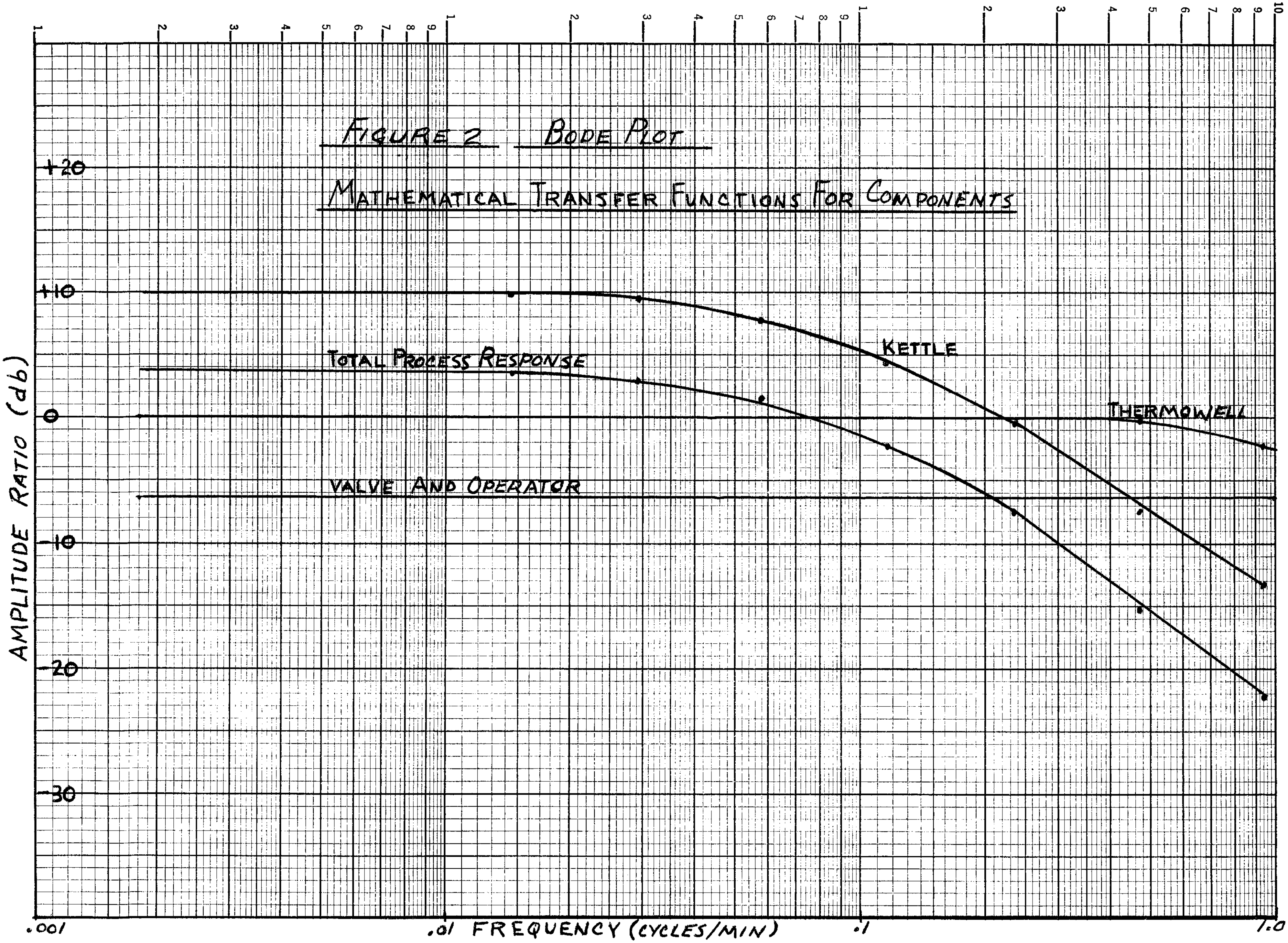


FIGURE 2 BODE PLOT

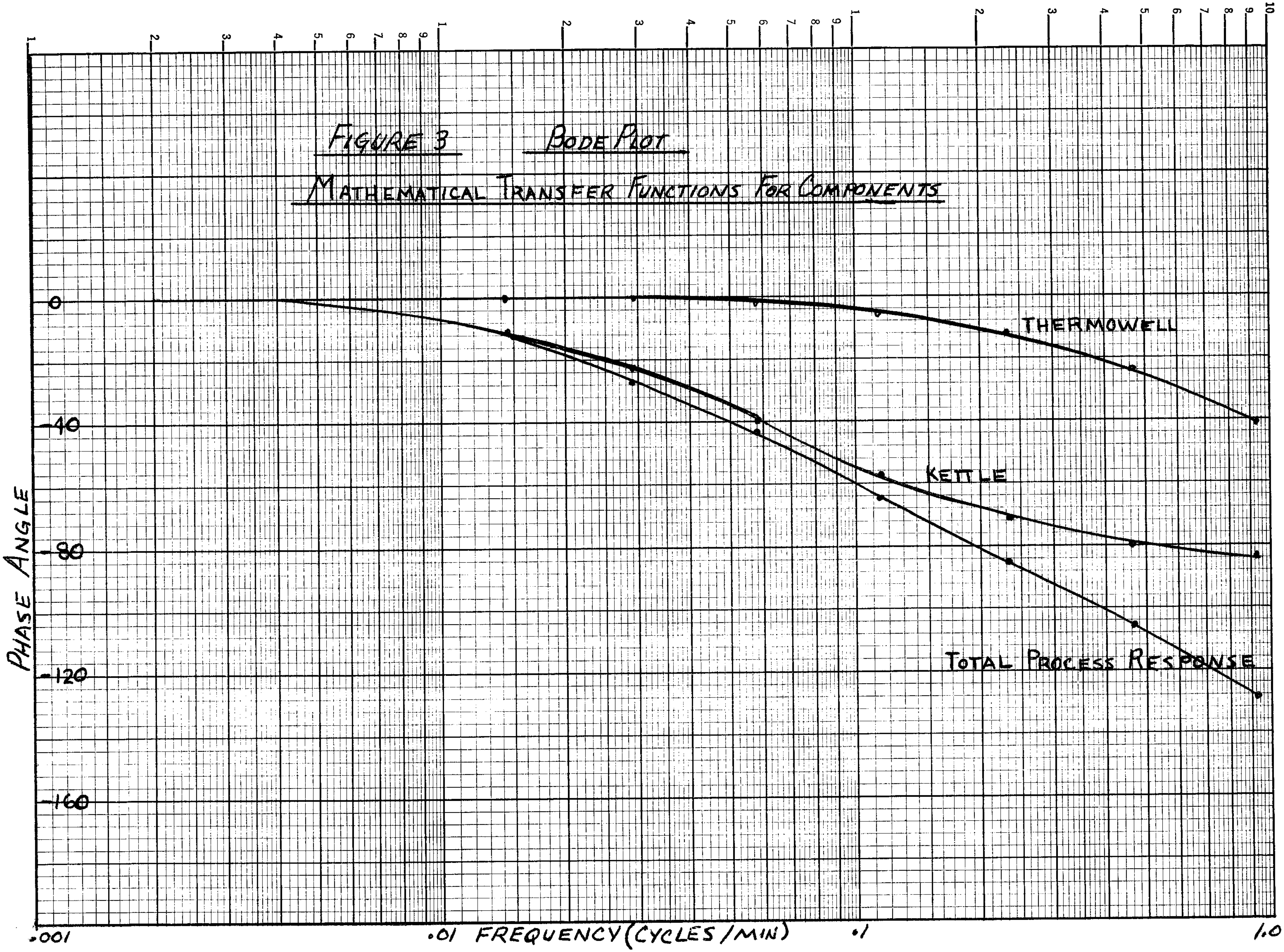


FIGURE 3
BODE PLOT
MATHEMATICAL TRANSFER FUNCTIONS FOR COMPONENTS
0
-40
-80
-120
-160
PHASE ANGLE
THERMOWELL
KETTLE
TOTAL PROCESS RESPONSE
.001
.01
FREQUENCY (CYCLES/MIN)
.1
1.0

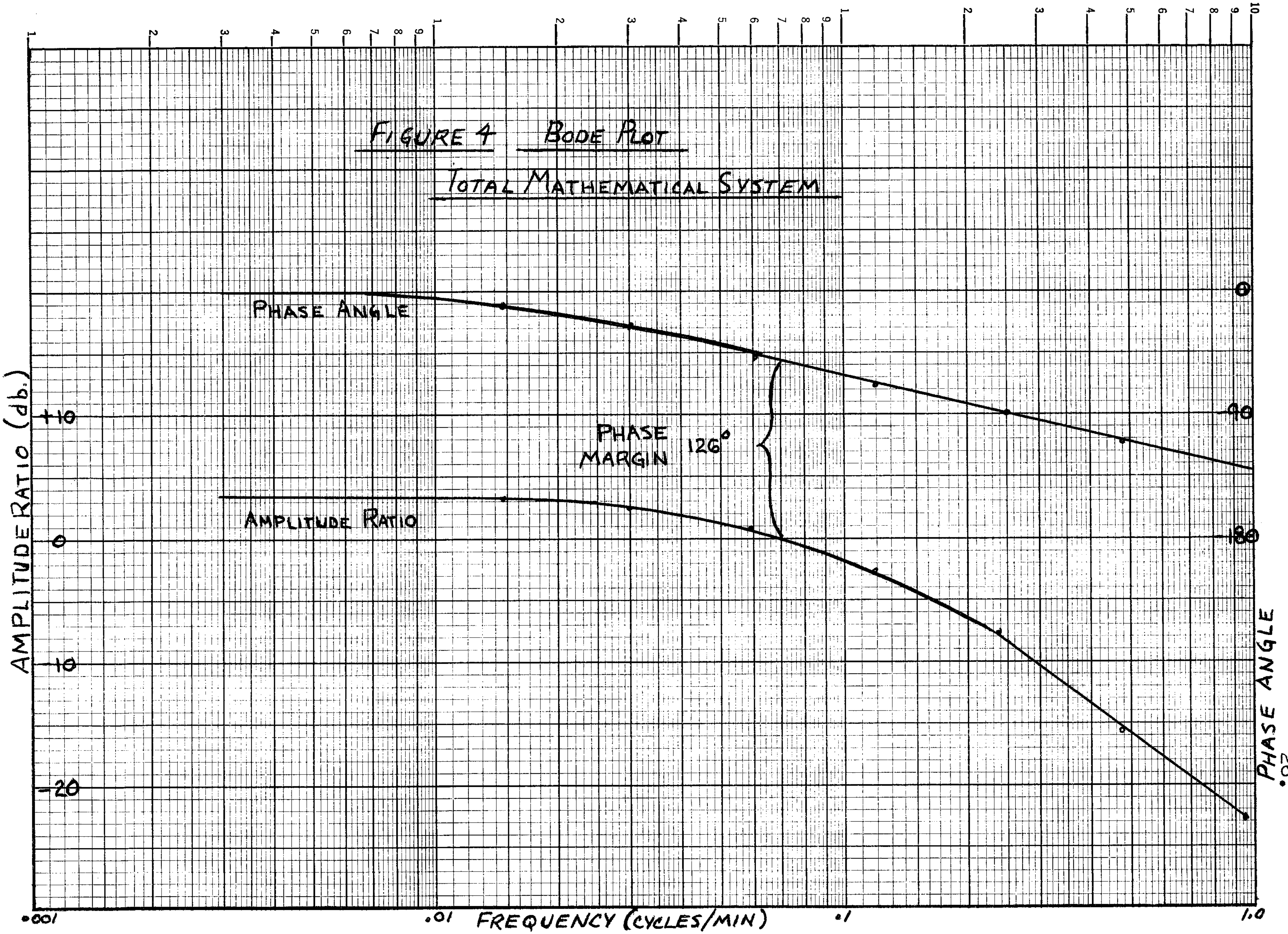


FIGURE 4 BODE PLOT
TOTAL MATHEMATICAL SYSTEM
PHASE ANGLE
AMPLITUDE RATIO
PHASE MARGIN 126°
AMPLITUDE RATIO (db.)
+10
0
-10
-20
PHASE ANGLE
0
90
180
FREQUENCY (CYCLES/MIN)
.001
.01
.1
1.0

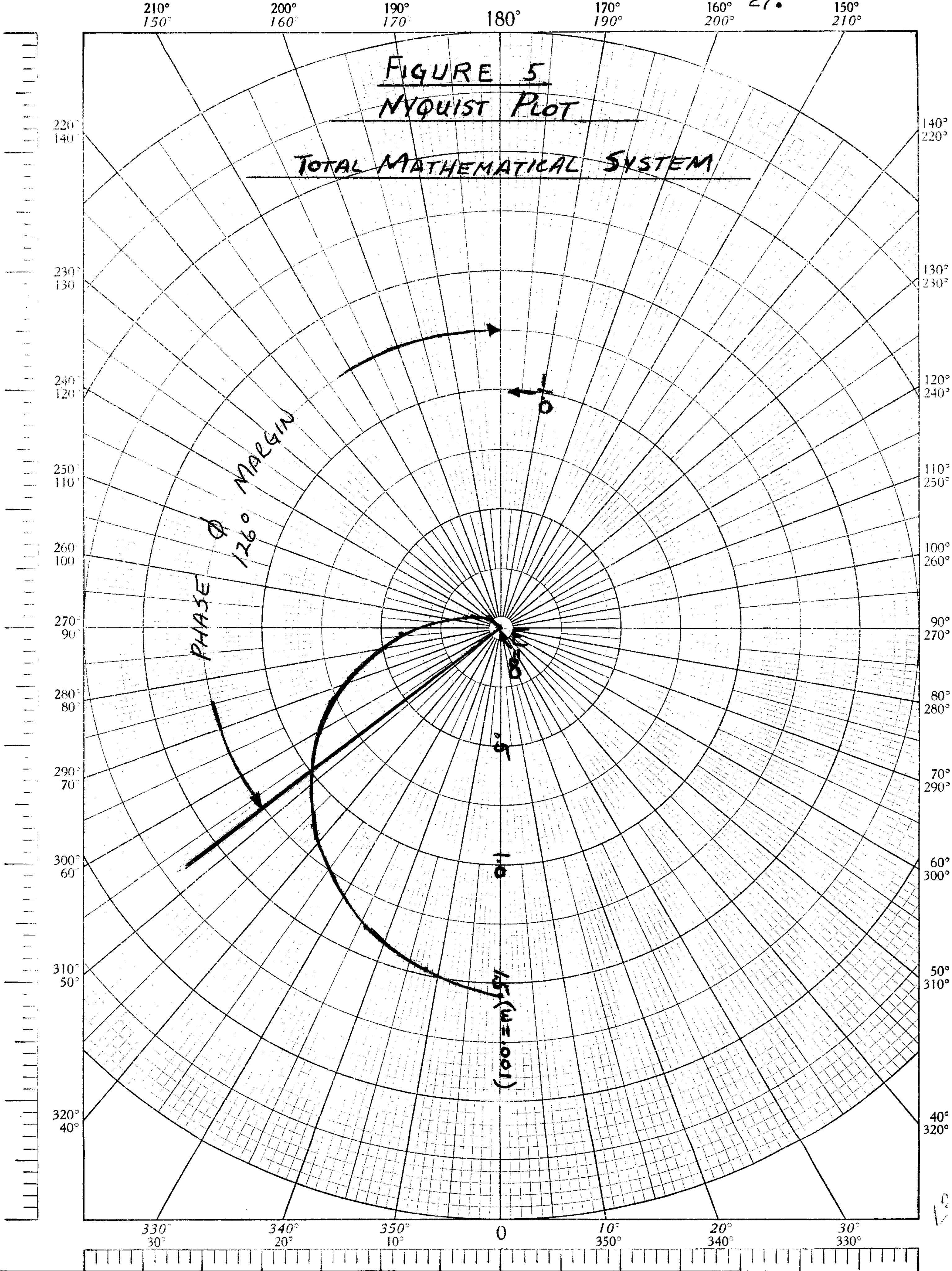
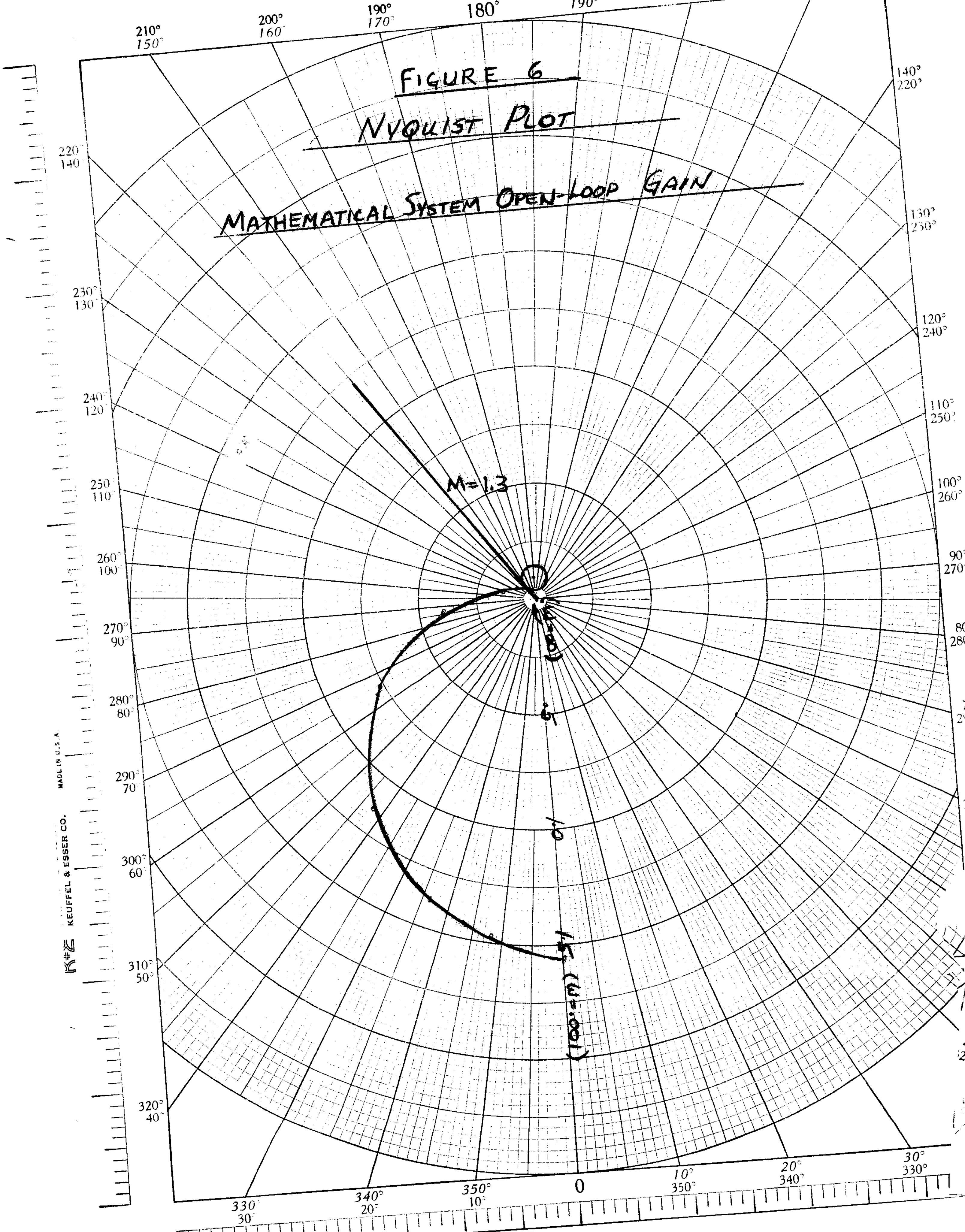


FIGURE 5
NYQUIST PLOT
TOTAL MATHEMATICAL SYSTEM

FIGURE 6

NYQUIST PLOT

MATHEMATICAL SYSTEM OPEN-LOOP GAIN



The value of open-loop gain from Figure 1 which would give a responsive-stable closed-loop system was found to be 20.

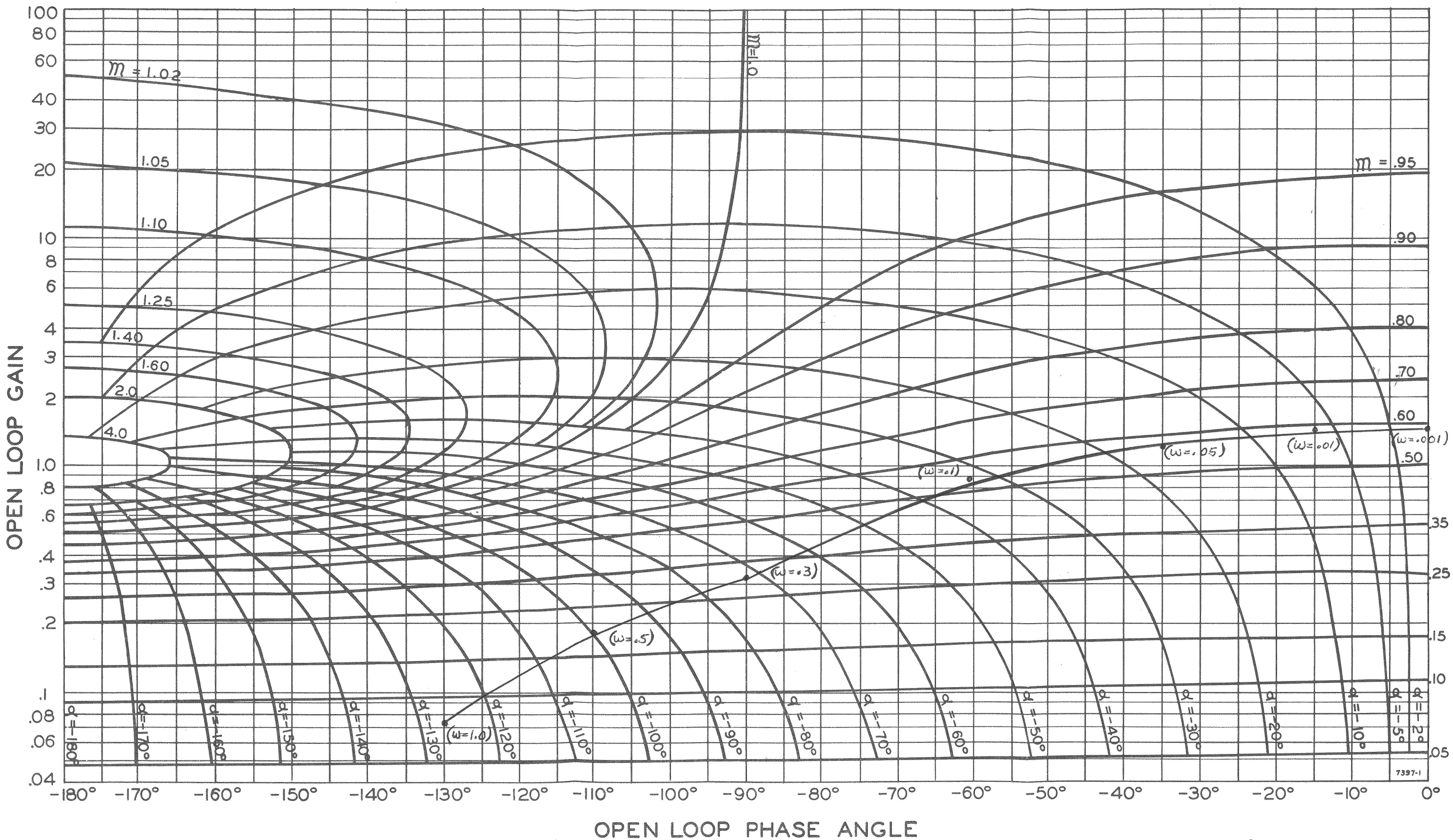
<u>Gain setting by logarithmic methods.</u> In the previous section the Nyquist plot was used to obtain the open-loop gain. The Nichols chart method is a similar method using logarithmic coordinates to set open-loop gain.

The Nichols charts require plotting the direct open-loop transfer function and the chart will give the direct closed-loop transfer function. The peak amplitude ratio and the resonant frequency can be obtained from the Nichols chart.

The recommended closed-loop gain of 1.3 is the value used for design and the open-loop transfer function gain is adjusted to provide this value. Figure 7 shows the Nichols plot for the system studied. The open-loop transfer function can be moved up or down on the chart to position the peak amplitude ratio on the closed-loop gain of 1.3. If the open-loop transfer function plotted on Figure 7 is moved up until the $W=1.0$ value rests on $m=1.3$ the amount of gain change necessary can be read directly as the vertical displacement in decibels required to reach $m=1.3$. In this system it is seen that open-loop gain of 20 db

FIGURE 7

NICHOLS CHART



m – CLOSED LOOP GAIN
α – CLOSED LOOP PHASE ANGLE

Taylor Instrument Companies
ROCHESTER 1, NEW YORK

would shift the closed-loop transfer function to the $M = 1.3$ value. The resonant frequency in the frequency at which $m = 1.3$ occurs. In this system the resonant frequency is 1.0 cycles/minute.

The Nichols method for open-loop gain setting cannot be applied directly to systems with frequency-dependent elements in the feedback path. The mathematical derivation for the Nichols method uses the concept that if the open-loop transfer function is described as $/G/$, then the corresponding closed-loop transfer function obtained by the Nichols method is of the form $\left(\frac{G}{1+G}\right)$.

Thermowell analysis. Attention to the thermowell is important because this type of well contributed the second largest time constant in the system. The thermowell in this system had a time constant of 1 minute. This time constant was not critical in this process, but, if it were used in a faster acting system it could become the predominant factor, especially with regard to selecting control instruments which depend entirely upon the speed of response of the temperature input to the controller.

By manipulation of the thermowell time constant an attempt was made to make the system become unstable.

Upon investigation of the system it was found that the thermowell could not force the system to go unstable even if the thermowell time constant was made the predominant factor.

Figure 8 shows a plot of the system with the kettle and thermowell time constants equal. It is seen that, at even this level, the system still possesses 80° of phase margin.

In order for the thermowell to possess a 14 minute time constant the well would have to contain a 1" glass coating which is not used in commercial equipment.

Maximum open-loop gain. When designing a control system it is important to apply the maximum gain the system can tolerate without causing instability. A basic analysis of a system will show that the influence of any disturbances is inversely proportional to the magnitude of the gain. This is to say that disturbances will have very little effect on a control system which possesses a large amount of gain.

Figure 9 is a Bode plot of the system assuming that the thermowell time constant is infinitely greater than the kettle time constant and the kettle has the maximum gain applied to cause the system to become marginally stable. This analysis shows that the system could be

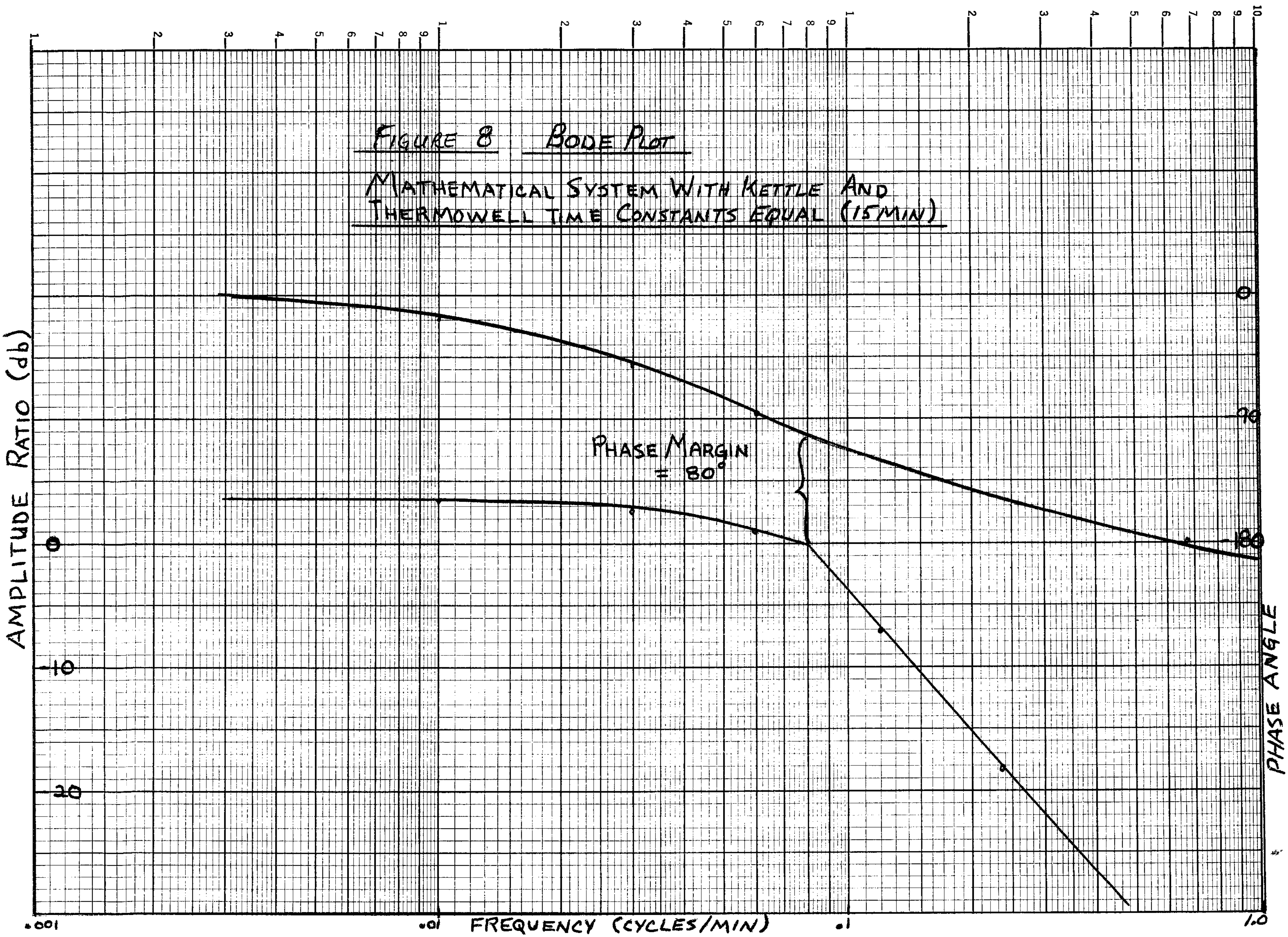


FIGURE 8 BODE PLOT
MATHEMATICAL SYSTEM WITH KETTLE AND THERMOWELL TIME CONSTANTS EQUAL (15 MIN)
PHASE MARGIN = 80°
AMPLITUDE RATIO (db)
PHASE ANGLE
FREQUENCY (CYCLES/MIN)
0
-10
-20
0
-90
-180
.001
.01
.1
1.0

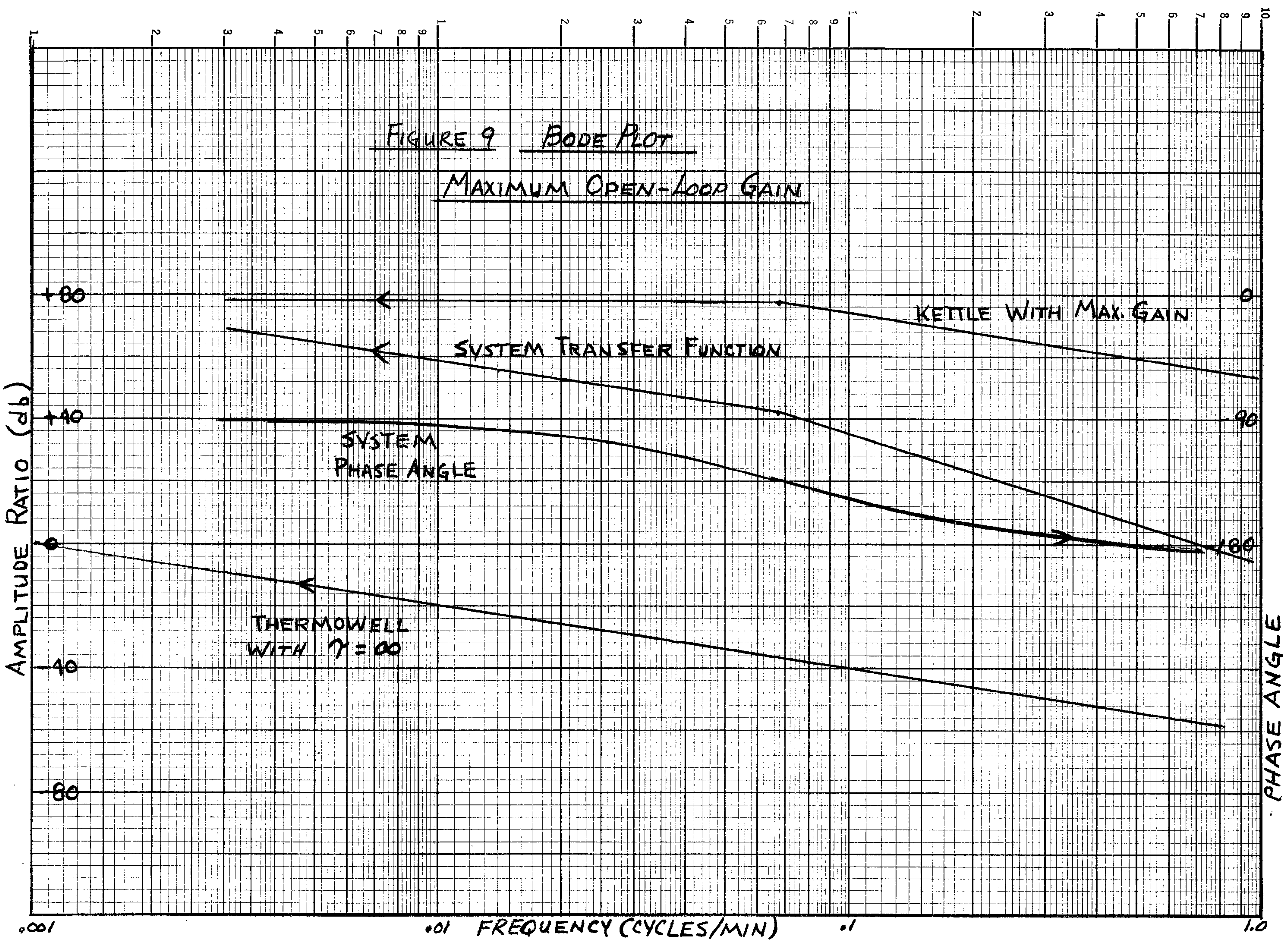


FIGURE 9 BODE PLOT
MAXIMUM OPEN-LOOP GAIN
KETTLE WITH MAX. GAIN
SYSTEM TRANSFER FUNCTION
SYSTEM PHASE ANGLE
THERMOWELL WITH γ = ∞
+80
+40
0
-40
-80
AMPLITUDE RATIO (db)
PHASE ANGLE
0
-90
-180
.001
.01
.1
1.0
FREQUENCY (CYCLES/MIN)

given 78 db of gain before becoming unstable and that any disturbances with this much gain would have little effect on the process.

Frequency response analysis. In order to determine how accurately the mathematical model described the system, a frequency response analysis was performed. The equipment and experimental procedure used will be described later.

Table 2 gives the amplitude ratio and phase angle data obtained experimentally.

Figures 10 and 11 give the Bode diagram for the frequency response data.

Figure 12 presents a Bode diagram comparing the mathematical system with the frequency response analysis.

Table 2

AMPLITUDE RATIO AND PHASE ANGLE FOR FREQUENCY RESPONSE TESTING OF SYSTEM

ω (cycles/min.)	Amplitude Ratio (db)	Phase Angle
.01465	- .45	-19.0
.0294	- 3.0	-32.5
.0586	-11.55	-52.3
.1175	-14.9	-57.8
.234	-24.62	-71.0
.469	-37.0	-76.5

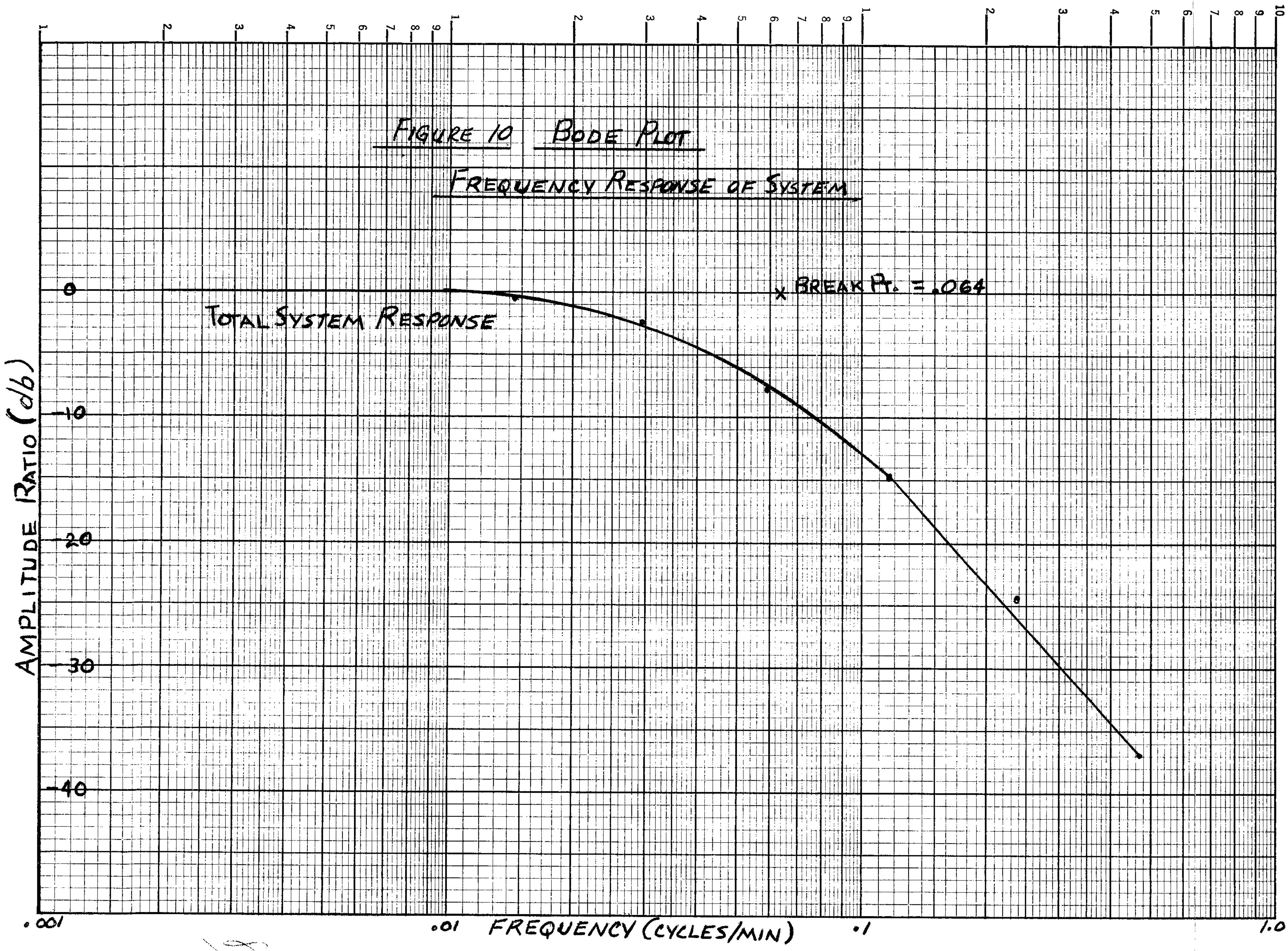


FIGURE 10 BODE PLOT
FREQUENCY RESPONSE OF SYSTEM
x BREAK PT. = .064
TOTAL SYSTEM RESPONSE
AMPLITUDE RATIO (db)
0
-10
-20
-30
-40
.001
.01
.1
1.0
FREQUENCY (CYCLES/MIN)

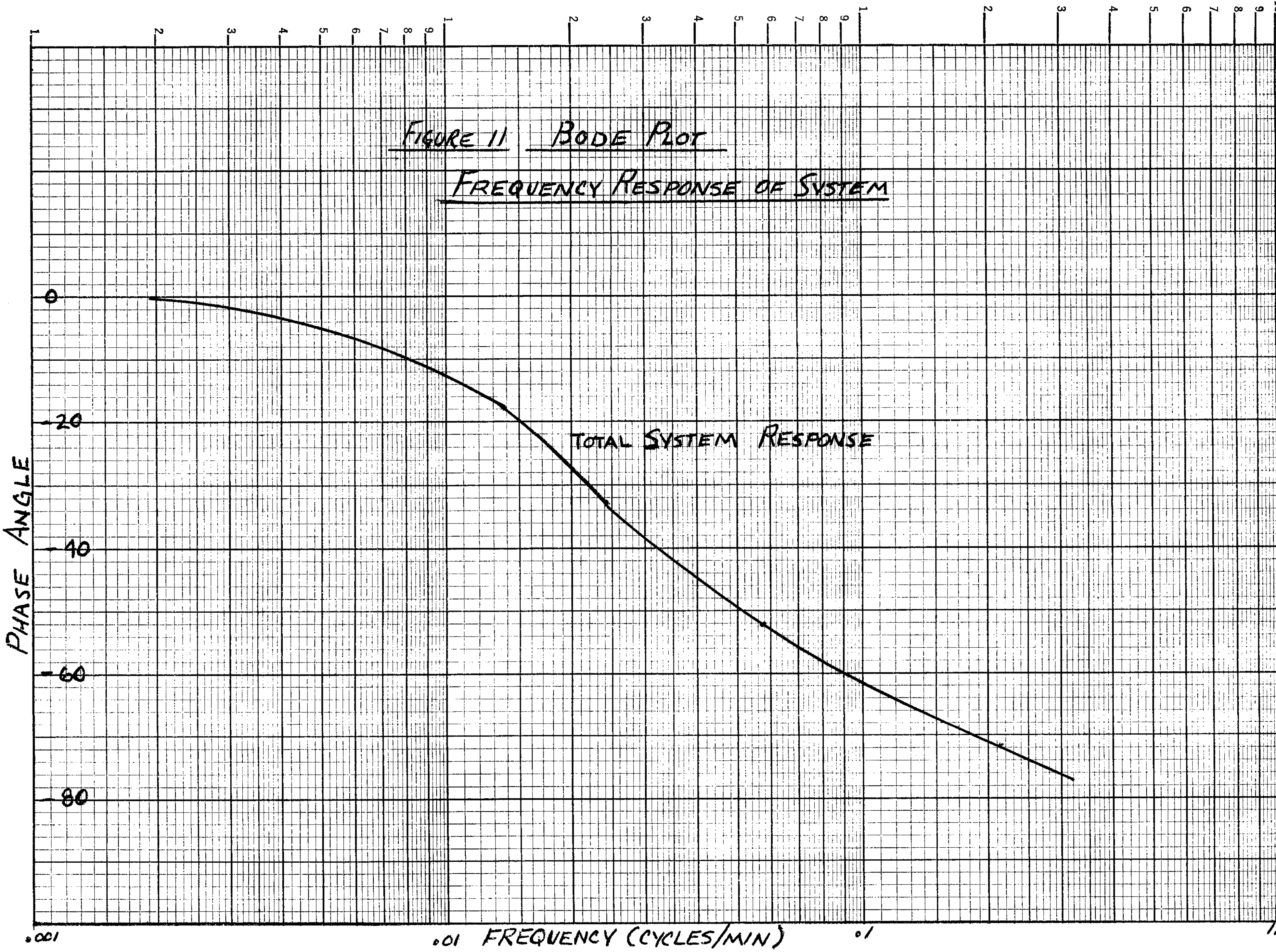
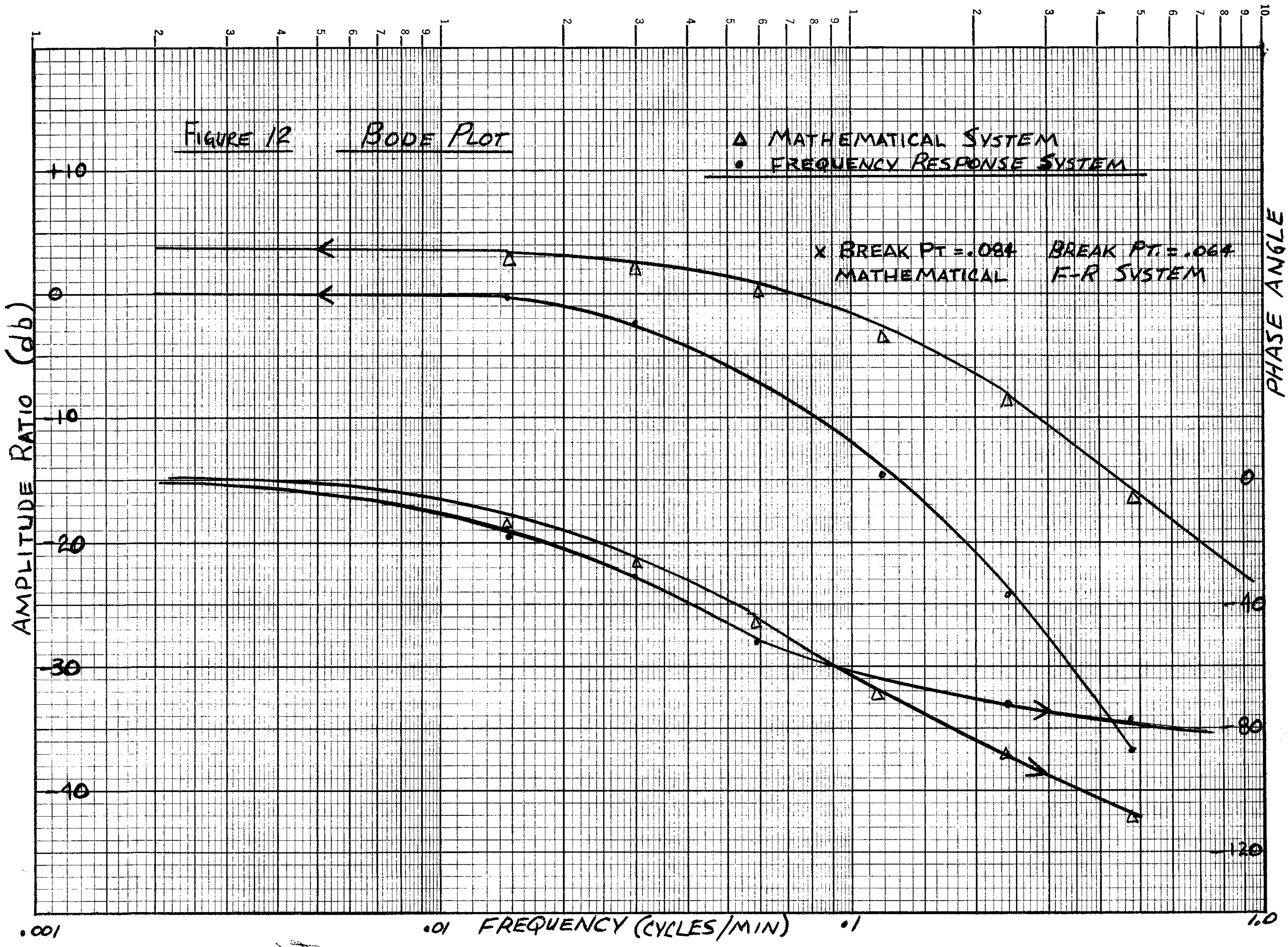


FIGURE 11 BODE PLOT

FREQUENCY RESPONSE OF SYSTEM

KEUFFEL & ESSER CO.



38.

Figure 13 is a Nyquist plot of the data obtained by the frequency response test.

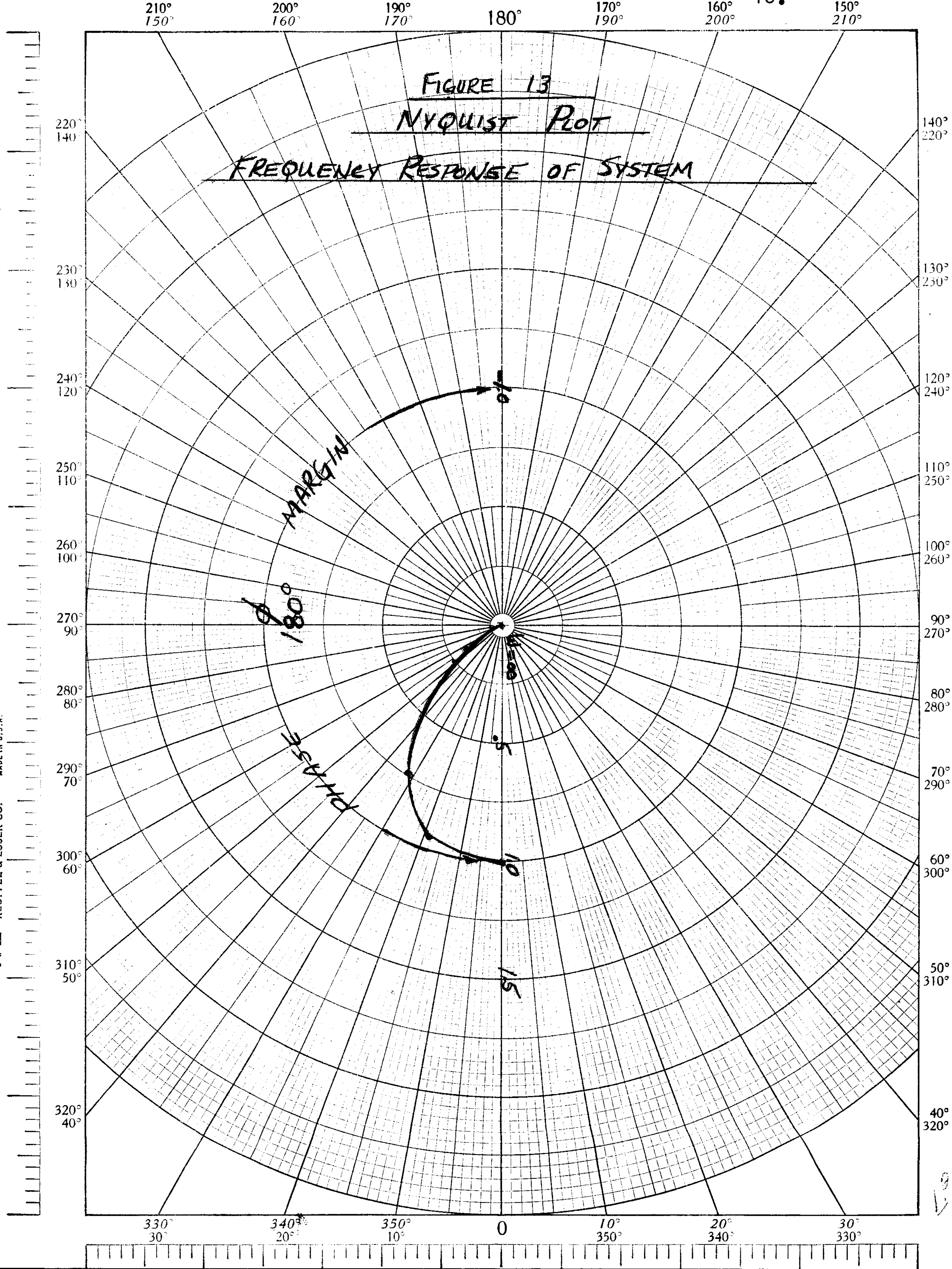


FIGURE 13
NYQUIST PLOT
FREQUENCY RESPONSE OF SYSTEM

DISCUSSION OF RESULTS

Comparison of Mathematical and Experimental Systems

When analyzing any system the mathematical model of the system is desirable because the equations can be manipulated on paper for various conditions much faster than experimental data can be obtained. It is possible to determine the mathematical equations from the frequency response test data, but this is sometimes difficult and inaccurate. The best approach is to derive the equations first and then use the frequency response test to confirm these equations.

In order to develop these equations it is essential to have a complete understanding of the elements which will have a significant effect on the system. In this work the elements were easily identified and were simple, so that the order of the equations was known before the derivation was started.

The mathematical equations as derived for this system were second order and had a slope of -40 db/decade. The frequency response test also is important because it may show disturbances in the system at higher frequencies which the mathematical equations could not predict. In this system the frequency response test did not show any disturbances in the process at higher frequencies and, therefore, the equations developed are correct as derived.

There was a discrepancy in terms of gain between the mathematical and experimental Bode diagrams. This error is probably caused by inaccuracy in the valve-operator transfer function. The only gain values in the system were contributed by the kettle and the valve-operator and these should cancel each other if there is to be no gain as indicated by the frequency response tests. The valve-operator is suspect because it is based upon the rather complicated equation for flow constriction and this equation is only as accurate as such things as discharge coefficients, area ratios, etc. can be calculated.

The system as studied was completely stable over the range investigated. As shown previously, the thermowell has no effect upon the stability of the system although thermowells of this nature with extremely large time constants will make the control of a process rather sloppy, especially if the reaction or process exhibits a critical point which must be determined quickly and accurately.

The only possible way to make the system unstable would be to design too much gain into the controller which may be selected for the system. This system can take a large gain which is good, since disturbances will have little effect on the system.

Source of Disturbances

The experimental equipment as set up had only one disturbance which could not be eliminated from the test. The water flow rate to the jacket was metered through a rotameter but the building water pressure and temperature would vary slightly, depending upon consumption. This had little direct effect of the data since amplitude ratios and phase angles would remain constant, but the flow rate and temperature fluctuations cause the total curves to shift up or down and it was then more difficult to determine when steady state conditions were reached.

Condensate Throttling

Condensate throttling was the method used to control this system. This method of control has the advantage of allowing a relatively simple and small valve to be used. The main problem with this method of control is that it does produce non-linearities in the system which could be significant in some processes. The mathematical equations used in this report used a linear approach to describe the flow of heat to the kettle, but if the equations are developed in detail accounting for the changing heat transfer area and partially filled sections of coil, the result is a second order differential equation with non constant coefficients.

This method of control should be considered very carefully before being applied to other systems. If the process time constant is less than the equipment used, dead times may be encountered because of the relatively slow action of condensate throttling. Dead times can cause even the most stable of systems to become unstable.

EXPERIMENTAL APPARATUS

The experimental equipment consisted of a low frequency sinusoidal signal generator which applied a 3-15 psig air signal to a Mason Neilon control valve. The control valve was placed on the outlet of a steam coil located in the jacketed kettle. The control valve opening and closing throttled the steam condensate out of the coil thus causing the kettle temperature to rise and fall.

The jacketed kettle with internal coil was equipped with agitator and variable speed drive. A glass-steel thermowell was positioned in the kettle and a bare wire thermocouple was placed inside the well. The thermocouple was weighted at the bottom to insure a positive contact with the bottom of the inside steel wall, thus eliminating air insulation effects. A second bare wire thermocouple was placed in the kettle in a position near the thermowell. This thermocouple was used to measure the kettle response without the effect of the thermowell.

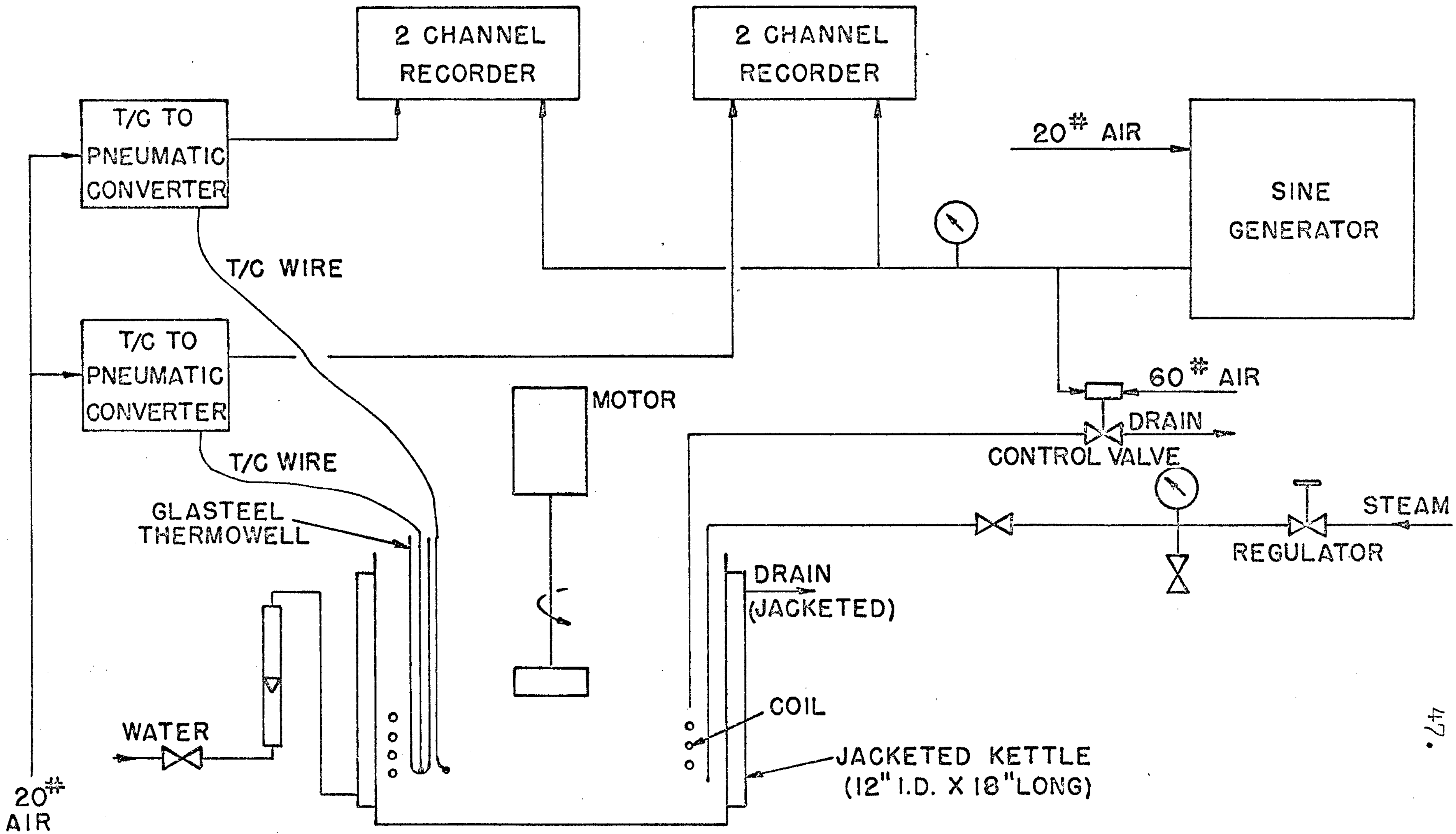
Both thermocouples were connected to millivolt-to-pneumatic transmitters which converted the thermocouple millivolt signal to a 3-15 psig pneumatic signal.

Two pneumatic recorders were used to record the results. Each recorder had two pens; one used to record the generator output signal and the other to measure the kettle output coming from the transmitter.

By varying the signal frequency from the sinusoidal generator it is possible to simultaneously conduct a frequency response analysis of the kettle-thermowell system as well as the kettle alone.

Figure 14 shows a schematic of the experimental system.

FIGURE 14
EXPERIMENTAL EQUIPMENT LAYOUT



EXPERIMENTAL PROCEDURE AND RESULTS

Experimental Procedure

The experimental procedure was the same for all tests except the frequency was changed between runs.

- Purge air lines of all water.
- Adjust air pressure, 20 psig to all instruments, and 60 psig to the control valve.
- Fill jacketed kettle with water.
- Adjust water flow rate to jacket of the kettle.
- Turn on electric power to sinusoidal generator and adjust frequency to be used for the test.
- Turn on electric power to the two recorders, and also to the millivolt-to-pneumatic transmitters.
- Check thermocouples by subjecting them to body heat to be certain that the transmitter-recorder system is operable.
- Check thermowell-thermocouple to be certain the tip of the thermocouple is touching the bottom of the thermowell.
- Adjust the steam pressure to the coil to 5 psig.

Experimental Results

Figure 15 shows a typical frequency response test record. From this recorded frequency response data the Bode diagrams (Figures 10 and 11) were constructed.

Figure 16 shows the results of a step input to the

FIGURE 15

TYPICAL FREQUENCY RESPONSE TEST

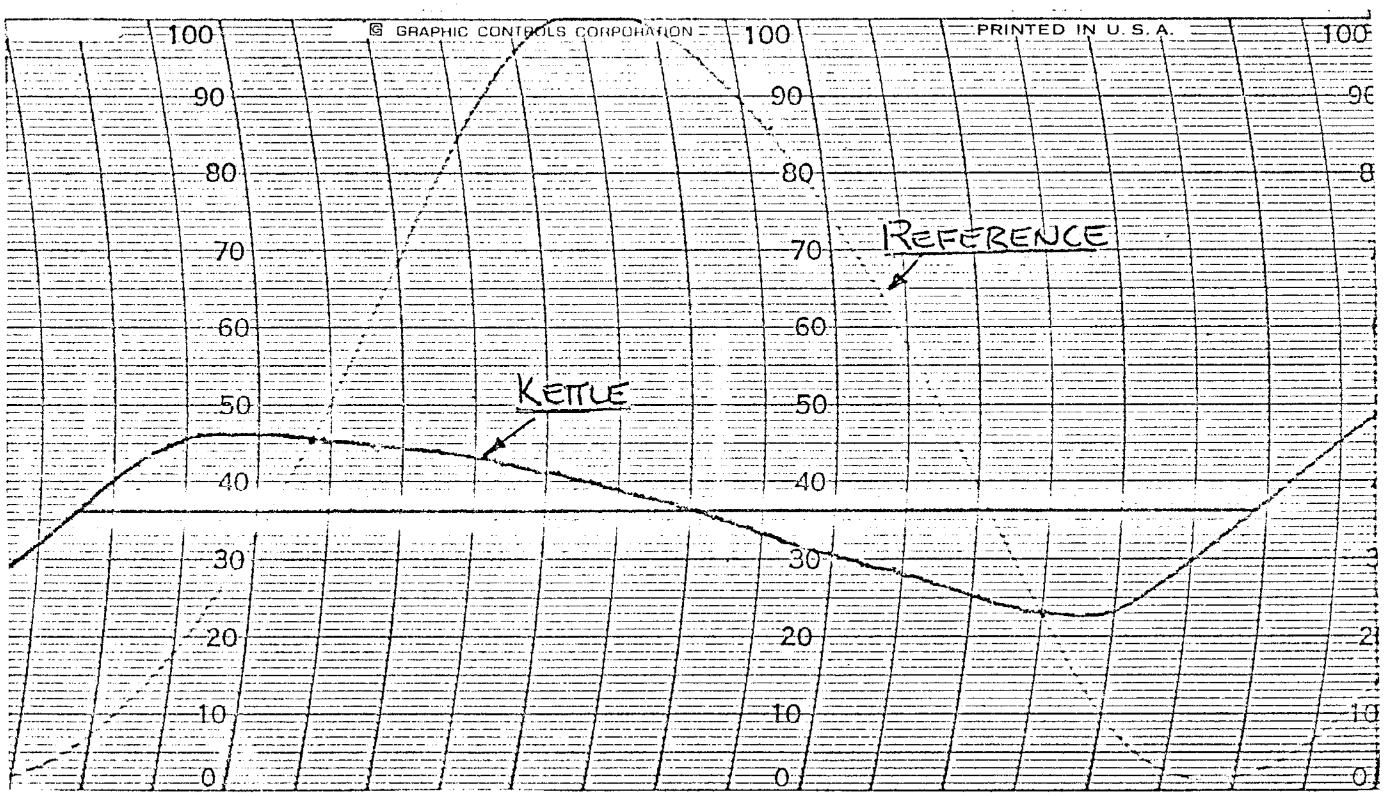
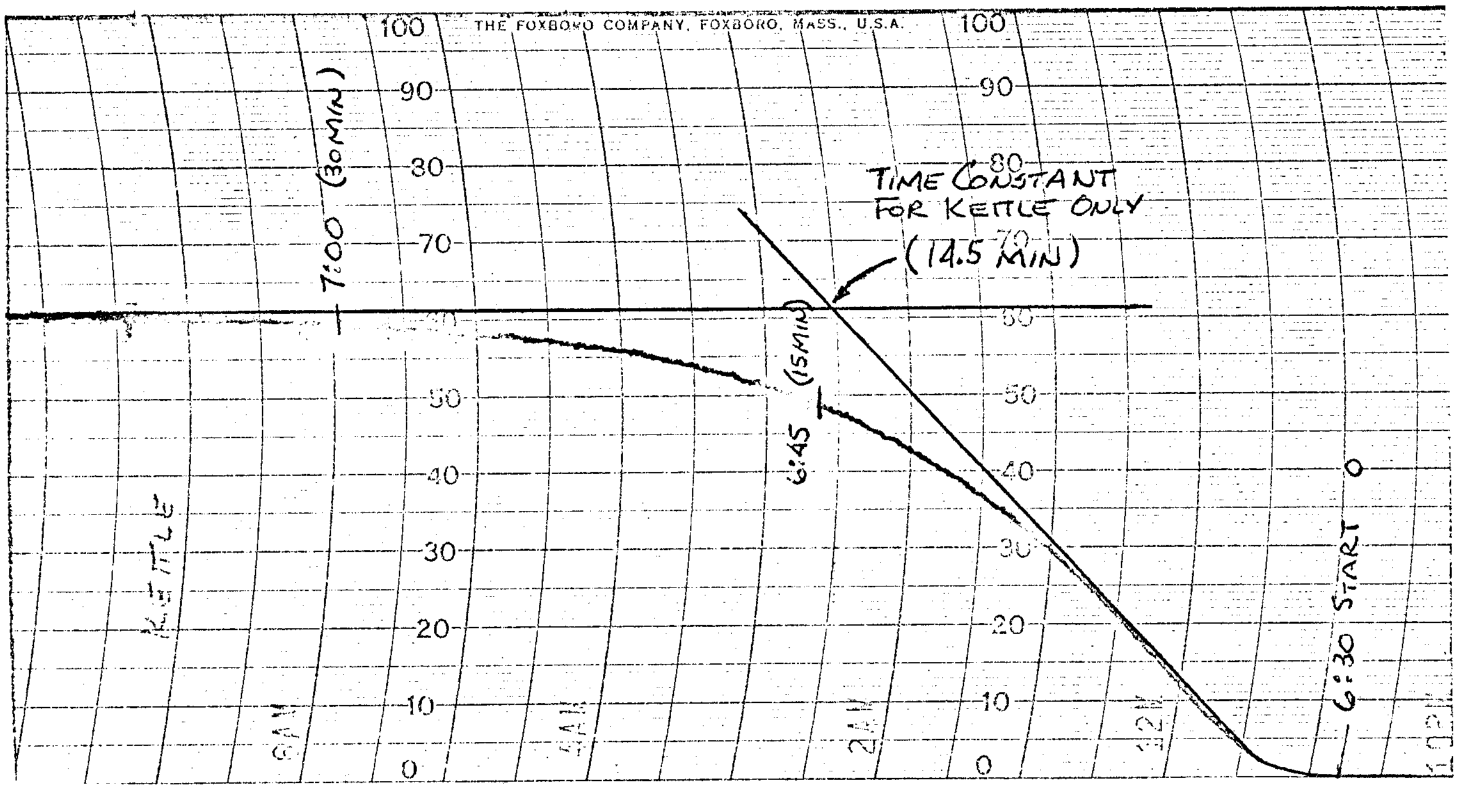


FIGURE 16

EXPERIMENTAL KETTLE TIME CONSTANT



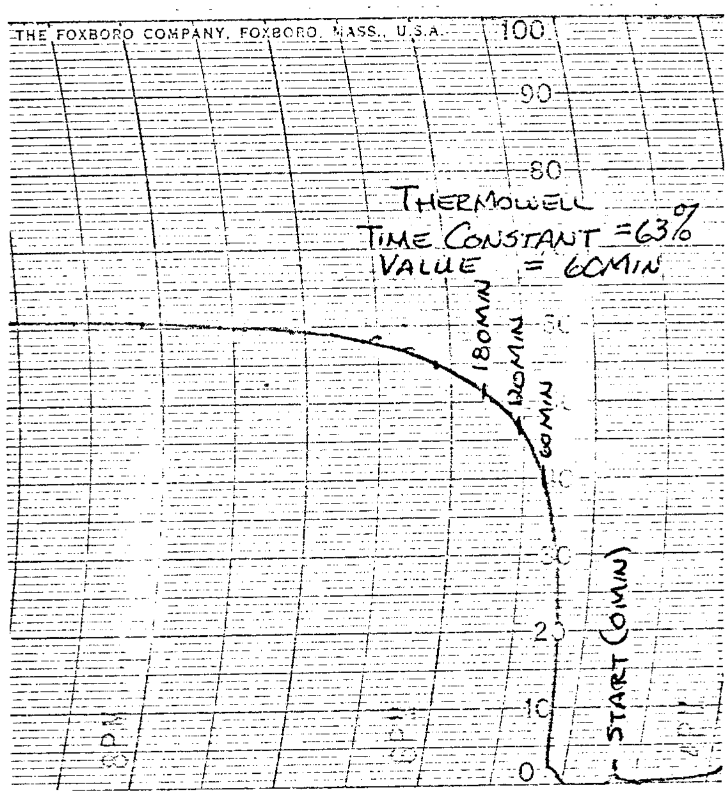
kettle without the thermowell being recorded. The experimental time constant of the kettle of 14.5 minutes was determined from this record by taking the time elapsed for the system to achieve 63.2% of the step input value.

Figure 17 shows the results of a step input to the thermowell without the kettle affecting the system.

This test was performed by heating the kettle to its maximum temperature without the thermowell being present. The kettle was then held at a constant temperature and the thermowell was placed into the kettle. Figure 17 shows that the thermowell time constant was 60 minutes. This was the time elapsed for the thermowell to achieve 63.2% of kettle temperature.

FIGURE 17

EXPERIMENTAL THERMOWELL TIME CONSTANT



CONCLUSIONS

Using the mathematical and frequency response methods of analysis on the jacketed kettle with internal heating/cooling coils the following can be concluded.

- Step response testing should be used to verify all significant mathematical equations and can also be used as the only analysis for systems with one large time constant. Only frequency response testing should be used as a check for overall systems order since it will show irregularities at higher frequencies not predicted by a mathematical system.

  The mathematical model is most useful for determining effects on the system by disturbances or isolating areas which should be investigated for possible improvement.
- The kettle had the largest time constant in the system, but the thermowell should receive the most attention when designing a control system because it has the direct effect on the controls and their response.
- Because the system studied is a lag response type system, application of a controller with rate action (a lead response system) would improve the overall system response.
- The system studied was found to be absolutely

stable. The thermowell time constant had no effect on the stability of the system due to the low gain of the overall process.

The only way to make the system unstable is to increase the open-loop gain, therefore reset action (a lag response system) must be applied with caution because it could put enough added gain in the open-loop response at low frequencies to cause instability. The thermowell should be given the most attention when the reset mode is applied.

- The open-loop gain necessary to give the optimum closed-loop response was found to be 20 db.

RECOMMENDATIONS

Based upon the work done on this process the following is recommended:

- Investigate the response of the thermowell for various methods of thermocouple positioning, such as oil filled, air insulation, etc.
- Investigate the response of the system using the jacket water flow as the controlled variable and compare this to condensate throttling.
- Develop all parameters for closed-loop operation and set up system and evaluate experimental vs. theory derived via open-loop techniques.

APPENDIX

Transmission Line Transfer Function

The pneumatic transmission line can be described using the basic electrical concepts of resistance, inductance, and capacitance.

The transmission line has resistance (R) in terms of pressure drop per unit length; capacitance (C) in terms of valve operators as well as the capacity of the line itself, and inductance (L) from the inertia of the gas.

The transfer function for the RLC system relates the pressure between the inlet and outlet of the line.[6]

$$\frac{d\varphi}{d\theta} = \frac{P_1 - P_2 - L\left(\frac{d^2\varphi}{d\theta^2}\right)}{R} \qquad (1)$$

where φ = volume of gas in capacity = CP_2.

Transforming and substituting for φ

$$R\varphi = P_1 - P_2 - Ls^2\varphi$$
$$P_2(RCs + LCs^2 + 1) = P_1 \qquad (2)$$
$$\frac{P_2}{P_1} = \frac{1}{sC(R + Ls + 1)}$$

Because the inductance is small for low frequencies

$$\frac{P_2}{P_1} = \frac{1}{(RCs + 1)} \qquad (3)$$

6. Process Control by P. Harriott, New York: McGraw-Hill 1964, pp. 204-207.

The above equation will be evaluated for ¼" tubing, 10 ft. long, at an average pressure of 9 psig.

Resistance. Using the Hagen-Poiseuille equation

$$\frac{\Delta P}{\mathcal{L}} = \frac{32 v \mu}{D^2 g_c} = \frac{128 F \mu}{\pi D^4 g_c} = \frac{128 \mu F^* P^*}{\pi D^4 g_c P} \qquad (4)$$

where F = volumetric flow

F* = flow at STP

$$R = \frac{128}{3.14} \frac{(1.21 \times 10^{-5} \frac{\#MASS}{FT\ SEC})(\frac{1}{12} \frac{IN}{FT})(14.7)}{(.188^4) IN^4 (32.2 \frac{\#MASS\ FT}{\#FORCE\ SEC^2})(23.7)}$$

$$R = 6.3 \times 10^{-4} \frac{PSI/FT}{SCI\ SEC} \times 10 = 6.3 \times 10^{-3} \frac{PSI}{SCI\ SEC}$$

Capacitance. Isothermal capacitance of a unit length of line is defined as the area divided by the standard pressure

$$C = \frac{A}{P^*} \frac{[\pi/4\,(.188)^2 \times 12]}{14.7} = .0224 \frac{SCI/FT}{PSI} \qquad (5)$$

$$C = .0224 \times 10 = .224 \frac{SCI}{PSI}$$

Total Transfer Function.

$$\frac{P_2}{P_1} = \frac{1}{RCs+1} = \frac{1}{(.224)(6.3 \times 10^{-3})s+1}$$

$$\frac{P_2}{P_1} = \frac{1}{1.41 \times 10^{-3} s+1}$$

It can be seen that the above time constant is negligible compared to the kettle and thermowell and, therefore, was neglected.

Frequency Response Testing for Kettle Only

While the overall system was being subjected to the frequency response test a bare wire thermocouple was placed in the kettle and recorded independently of the thermowell. This test was performed to determine whether or not the mathematical system derived for the kettle element was correct.

Although the time constant for the mathematical and frequency response test of the kettle deviates by 15 minutes, the general shape of the curve at least indicates that a first order system assumption for the kettle is valid. The time constant is not accurate for the frequency response data because a slight change in how it is determined from the graph can vary the number significantly.

Table 3 gives the actual frequency response test data and Figures 18 and 19 give the Bode Plot resulting from this data.

TABLE 3

AMPLITUDE RATIO AND PHASE ANGLE FOR FREQUENCY

RESPONSE TESTING OF THE KETTLE ONLY

ω (cycles/min.)	Amplitude Ratio (db)	Phase Angle
.01465	0	-12.3
.0294	- 5.04	-32.0
.0586	-10.5	-36.0
.1175	-16.5	-40.6
.2340	-24.5	-40.5
.4690	-30.5	-45.0

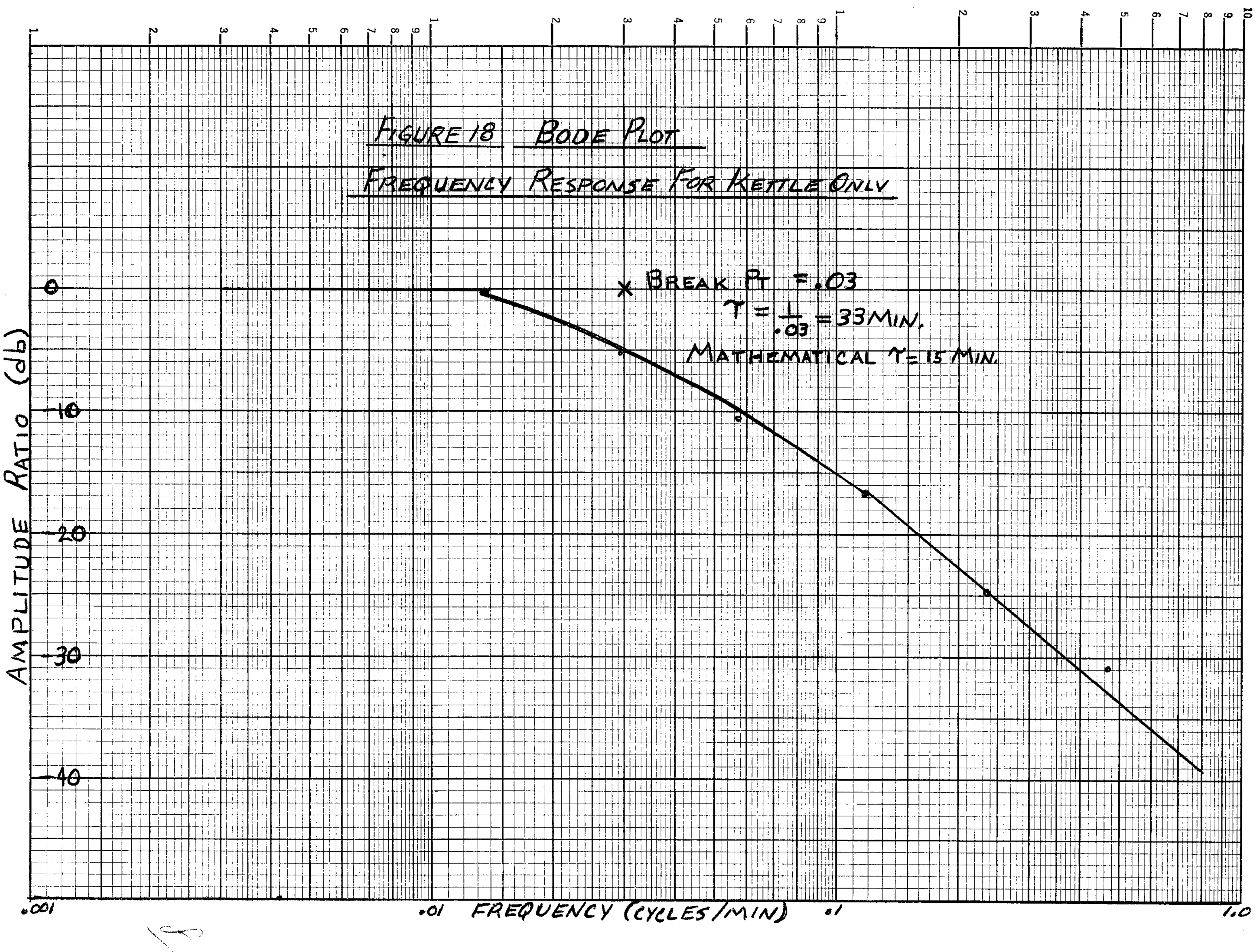


FIGURE 18 BODE PLOT
FREQUENCY RESPONSE FOR KETTLE ONLY
× BREAK PT = .03
τ = 1/.03 = 33 MIN.
MATHEMATICAL τ = 15 MIN.
AMPLITUDE RATIO (db)
0
-10
-20
-30
-40
.001
.01
.1
1.0
FREQUENCY (CYCLES/MIN)

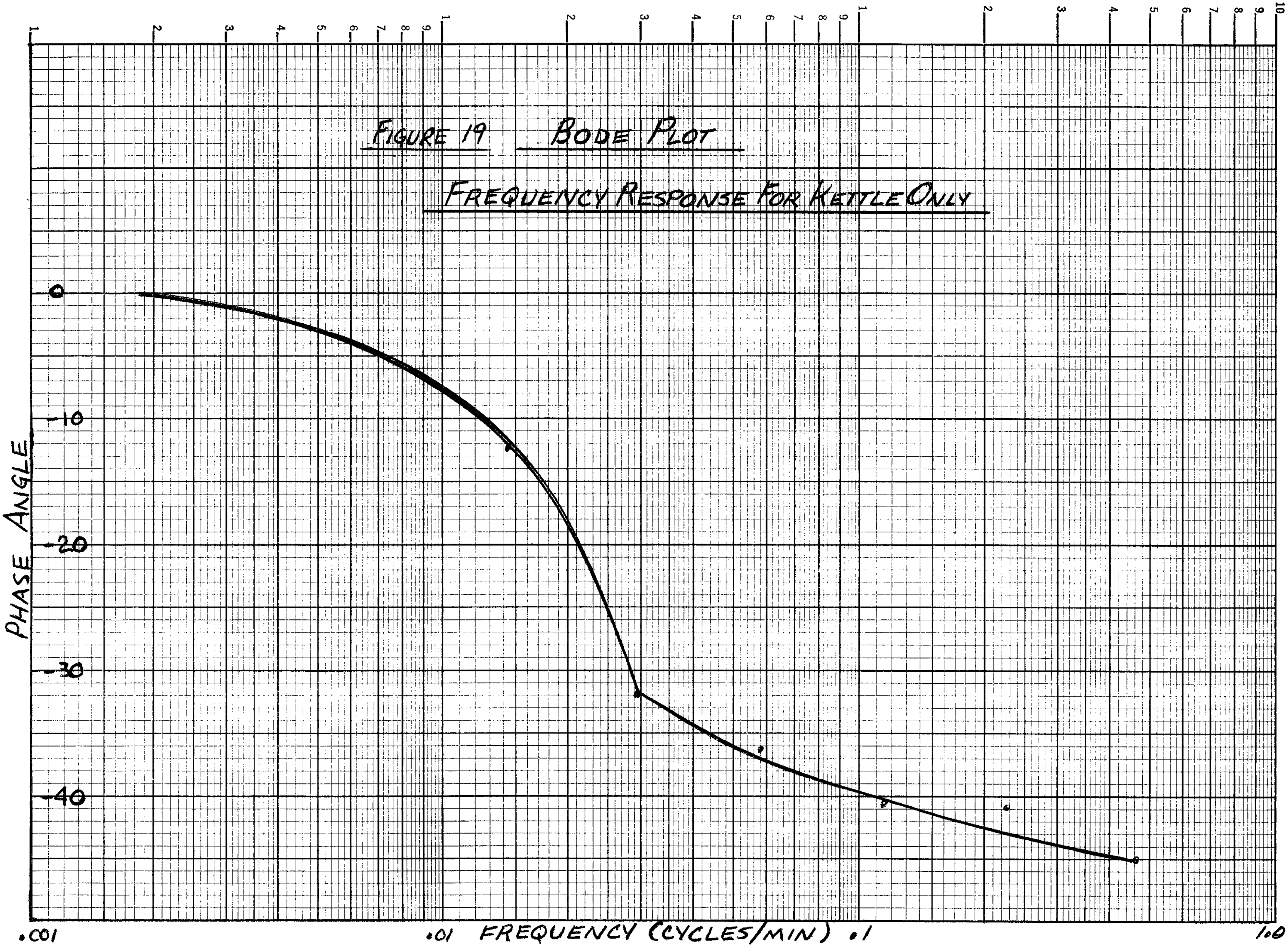


FIGURE 19
BODE PLOT
FREQUENCY RESPONSE FOR KETTLE ONLY
PHASE ANGLE
0
-10
-20
-30
-40
.001
.01
.1
1.0
FREQUENCY (CYCLES/MIN)

LIST OF EQUIPMENT

- Ultra Low Frequency Sinusoidal Signal Generator, Model SG-101P, Procedyne Associates Inc., New Brunswick, New Jersey.

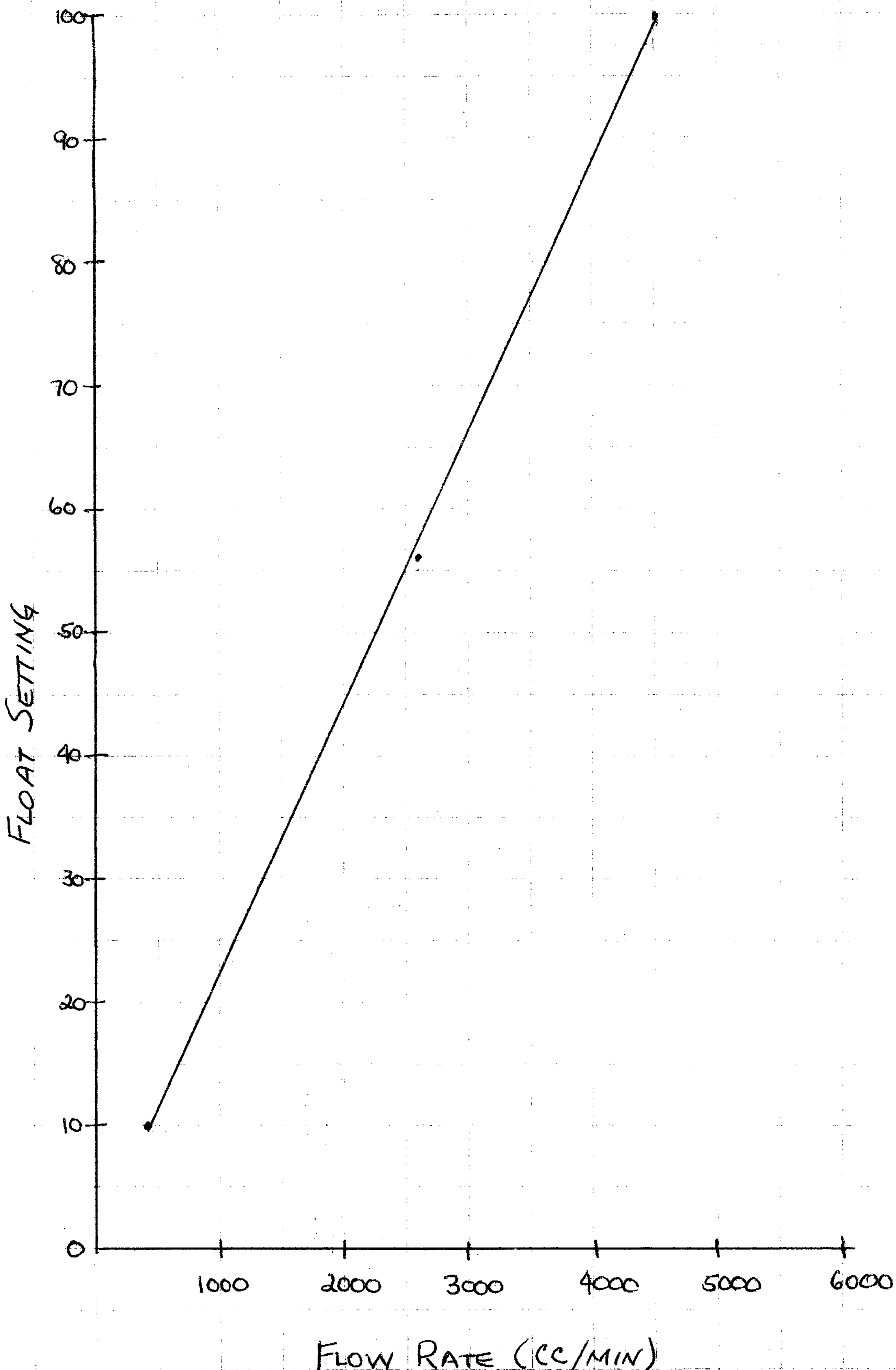
- T/C Pneumatic Transmitter, Model 410T, Transmation Inc., Rochester, New York.

- Consotrol Controller, Model 58, Foxboro Co., Foxboro, Massachesetts.

- Control Valve, Model #37-24681, Mason Neilon Inc., Norwood, Massachusetts.

- Rotameter, Model #10A2735A - ½ - GSVT-48, Fischer-Porter, Warminster, Pennsylvania.

FIGURE 20
ROTAMETER CALIBRATION CURVE



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