BALL MILLING OF NON-NEWTONIAN DISPERSIONS

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ABSTRACT

This investigation attempts to obtain a better understanding of flow behavior of a pigmented dispersion in a ball mill.

A power-law fluid model was used to approximate the rheological behavior of the milled fluid. To correlate power data on each of 276 milled dispersions, this model was used together with power number, Reynolds number, Froude number, and some geometrical variables.

Laboratory size stainless steel jars containing stainless steel balls from 1/8 to 3/4 inches in diameter were used to mill a pigment/binder/solvent system at various speeds for different periods of time.

A correlation of power number as a function of a modified Reynolds number, Froude number, and mill diameter-to-ball diameter ratio was found.

APPROVAL OF THESIS

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INTRODUCTION

Ball milling is defined (10) as grinding of the mill base accomplished by rotating the mill and its contents about the mill's horizontal axis at a rate sufficient to lift the balls to one side and then cause them to roll, slide, and tumble to the lower side. There have been some papers written on the mechanics of a ball-mill (3, 16, 12), but a literature search revealed little attention given to establishing a power-fluid flow relationship.

This study is an attempt to establish a quantitative relation between power consumption and geometric, kinetic, and fluid property paramaters of non-Newtonian dispersions. Non-Newtonian dispersions were chosen for study since these occur more frequently in the industries using ball milling; Newtonian behavior will merely be considered as a special case of the non-Newtonian fluid model.

THEORY

A. Non-Newtonian Fluids

(1) General description. Ideal viscous bodies exhibit flow, with the flow rate being a function of stress. There are two types of flow that a viscous body can exhibit. One is flow in shear and second is the flow phenomenon which occurs when the volume is changed upon applying or relieving compression. The viscosity in shear is the commonly known one involved in flow phenomena, and it will be discussed here.

An ideal viscous body cannot sustain strains for long, since these are relieved by flow. Extremely viscous materials may exhibit elastic strain for a considerable time, which is short with respect to the time needed for appreciable flow.

The applied shearing stress to shear rate ratio for ideal viscous bodies is called the viscosity. See Fig. (1) for a diagrammatic definition of differential viscosity (13). Differential viscosity, μ_d, is the slope of the tangent to the curve at any point. The usual viscosity is derived from the slope of the line connecting the curve point with the origin. This is called the apparent viscosity μ_a (13).

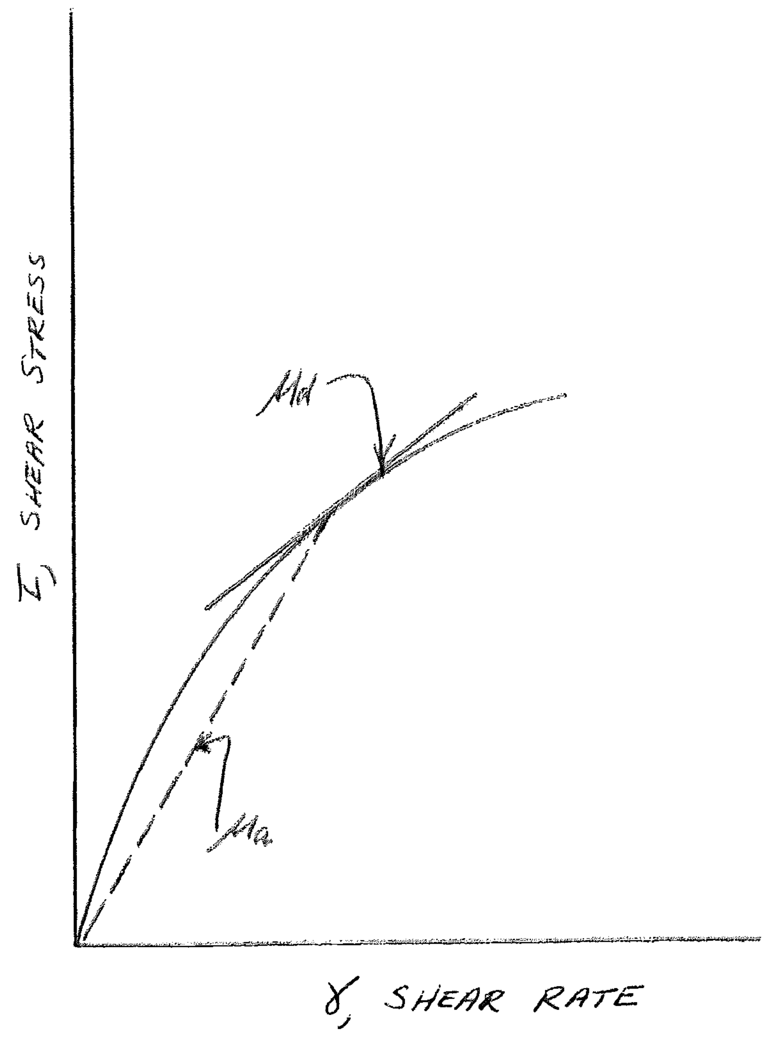

Figure 1. Definition of Differential and Apparent Viscosity

The best known ideal viscous body is the Newtonian fluid, for which the viscosity coefficient, μ , is a constant. The kinematic viscosity ν, which is directly observed in capillary tube viscometers where the stress comes from a fluid head on which the viscosity is being determined, equals the viscosity μ divided by the density.

Newtonian behavior is exhibited by fluids in which the dissipation of viscous energy is due to small molecular collisions. Most gases, liquids and solutions of low molecular weight come into this category. Notable exceptions are collodial suspensions and polymeric solutions where the molecular species are larger. These "non-Newtonian" fluids show marked deviations from Newtonian behavior.

(2) Pseudoplastic fluids. From the group of non-Newtonian time independent viscous fluids, concentration will be focused on pseudoplastic fluids since they best describe the behavior of the pigment/binder/solvent system.

Pseudoplastic fluids show no yield value and the typical flow curve for these materials indicates that the ratio of shear stress to shear rate falls progressively with shear rate and the flow curve becomes linear

The logarithmic plot of shear stress and shear rate for these materials is often found to be linear with a slope between zero and unity. As a result, an empirical functional relation known as the power law is widely used to characterize fluids of this type. This relation, which was originally proposed by Ostwald (9) and has since been fully described by Reiner (13), may be written as

$$\tau = K \gamma^{n} \qquad (1)$$

K is a measure of fluid consistency; n is a measure of the degree of non-Newtonian behavior.

The apparent viscosity for a power law fluid may be expressed as

$$\mu_a = \frac{K}{\gamma^{1-n}} \qquad (2)$$

This behavior is characteristic of suspensions of pigment particles and polymer solutions. The physical interpretation (5) of this phenomenon is probably that with increasing shear rates the pigment particles are progressively aligned; instead of the random intermingled state which exists when the fluid is at rest the major axes are brought into line with the flow direction.

Viscosity continues to decrease with increasing shear rate until no further alignment along the streamlines is possible and the flow curve then becomes linear.

Pseudoplastic fluids have been defined as time-independent fluids and this implies that the molecular alignment takes place instantaneously as the shear rate is increased or at any rate, so quickly that the time effect cannot be detected using ordinary viscometric techniques.

B. Viscometry

(1) General. A rotating body, immersed in a liquid, experiences a retarding force (drag). The drag's magnitude is a function of the body's rotational speed. In using viscosity equations, it makes no difference whether the body or container is rotated, the relationship between shear stress and rate is the same.

Theoretically, one movable and one stationary parallel plate system should serve as an ideal device for measuring viscosity (11). Practically this is difficult since thereare two problems to overcome; first, preventing the fluid from spilling out the sides; second, continually moving top plate from bottom plate in a practical manner. These two problems are overcome in a modified form in the band, rotating coaxial cylinders,

(2) Cone/plate viscometer. This instrument, depicted in Fig. (2), consists of a flat plate and a rotating cone with a very obtuse angle. The cone's apex just touches the plate surface and the fluid fills the narrow gap formed by the cone and plate. If the angle α is small $\sin\alpha = \alpha$, and the average gap width is correspondingly small $r\alpha$, the whole sample will be subjected to a constant shear rate and the end effects (sometimes significant in coaxial cylinder viscometers) will be negligible (17). This simplifies the analysis of non-Newtonian fluids because it gives the apparent viscosity as a function of shear rate directly as follows.

The linear velocity, V, of any point on the cone is proportional to the radius to that point, r is given by

$$V = r\omega \tag{3}$$

and the gap width at radius r is $r\alpha$, and the shear rate at radius r is

$$\gamma = \frac{\omega r}{r \tan\alpha} = \omega / \tan\alpha \tag{4}$$

This means that the shear rate is constant throughout the sample and independent of r.

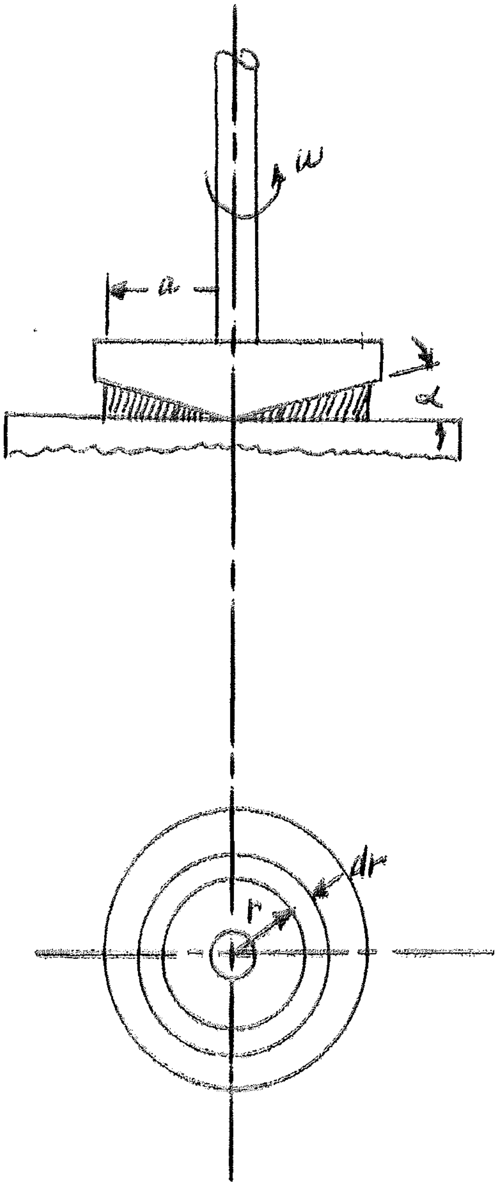

Figure 2. Cone/Plate Viscometer

Further

$$\gamma = \omega / \alpha$$

if α is small. (5)

Since $\gamma = f(t)$ and γ is constant we have that τ is constant.

Therefore

$$M = \int_0^{2\pi} \int_0^a \tau r^2 \, dr \, d\theta = \frac{\tau 2 \pi a^3}{3}$$

(6)

where M is the measured torque per unit height of liquid, hence

$$\tau = M \frac{3}{2 \pi a^3}$$

(7)

and

$$\mu_a = \frac{\tau}{\gamma} = \frac{M}{\omega} \frac{3\alpha}{2 \pi a^3}$$

(8)

Equation (8) gives the apparent viscosity at a shear rate given by ω/α. Alternatively the flow curve can be constructed by plotting the shear rate, γ , against the corresponding shear stress, τ , directly.

C. Dimensional Analysis

The motion of a fluid can be defined in terms of length (L), time (T), mass (M), and force (F); however, mass and force are related by length and time as in Newton's second law of motion thereby reducing the system to F-L-T.

In a ball mill fluid motion and properties can be expressed in the same dimensions; therefore, similar fluid motion in two different mills can be related to each other.

Similarity is the underlying principle of dimensional analysis. There are three types:

(1) geometric - similarity exists when correspinding dimensions have the same ratio.

(2) kinematic - similarity exists if the patterns of motion are alike, and if velocities at corresponding points have the same ratio as velocities at other corresponding points.

(3) dynamic - similarity exists if there is kinematic similarity and if the ratio of masses and forces are equal to those at corresponding points.

Variables which affect fluid motion in ball milling are:

(1) Linear dimensions such as mill diameter and length.

(2) Fluid properties such as density and viscosity.

(3) Kinematic and dynamic flow characteristics such as RPM and power input.

Buckingham's Pi theorem which states if one variable depends upon a number of independent variables, they may be expressed as

$$f(d, D, H, C, L, \rho, v, g, N, P,) = 0 \qquad (9)$$

The general equation becomes:[2] (15)

$$f'(\pi_1, \pi_2, \pi_3, \ldots\ldots) = 0 \qquad (10)$$

Using ball diameter d for the reference length, the mill rotational speed N, and ρ for the fluid density, the values of the π terms can be evaluated as follows:

$$\pi_1 \text{ (for mill diameter D)} = d^{x_1} N^{y_1} \rho^{z_1} L^{-1} \qquad (11)$$

$$\pi_1 : L^x \left(\frac{1}{T}\right)^y \left(\frac{FT^2}{L^4}\right)^z L^{-1} = L^0 T^0 F^0 \qquad (12)$$

Therefore

$$\begin{aligned} L&: \; x \qquad\quad - 4z - 1 = 0 \\ T&: \qquad -y + 2z \qquad = 0 \\ F&: \qquad\qquad\; z \qquad = 0 \qquad (13) \end{aligned}$$

Therefore

$$z = 0$$

$$y = 0$$

$$x = 1 \qquad (14)$$

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[2] Assume first form of infinite series adequately express data.

and

$$\pi_1 = d/D \tag{15}$$

Similarly π_2, π_3, and π_4 are

d/H, d/C, and d/L respecitvely. (15^1)

$$\pi_5 \text{ (for } \nu) = L^x \left(\frac{1}{T}\right)^y \left(\frac{FT^2}{L^4}\right)^z \left(\frac{L^2}{T}\right)^{-1} = L^0 T^0 F^0 \tag{16}$$

and

$$z = 0$$
$$y = 1$$
$$x = 2 \tag{17}$$

and $$\pi_5 = \frac{d^2 N}{\nu} \tag{18}$$

Similarly $\pi_6 = \dfrac{dN^2}{g}$, $\pi_7 = \dfrac{d^5 N^3 \rho}{P}$ (18^1)

Combining

$$f'\left(\frac{d}{D}, \frac{d}{H}, \frac{d}{C}, \frac{d}{L}, \frac{d^2 N}{\nu}, \frac{dN^2}{g}; \frac{d^5 N^3 \rho}{P}\right) = 0 \tag{19}$$

The term characterizing the force of viscosity is the Reynolds number(N_{Re}):

$$N_{Re} = \frac{d^2 N}{\nu} = \frac{d^2 N \rho}{\mu} \tag{20}$$

(since $\nu = \mu/\rho$)

This Reynolds number expression is for Newtonian fluids. in which the viscosity doesn't vary with shear rate; however, in the non-Newtonian case viscosity varies with shear rate and a different viscosity model is needed. It is assumed in this case that a power-law fluid model fits or

$$T = K\gamma^{n}$$

where n is the flow-behavior index (6) and characterizes the degree of non-Newtonian behavior and K is the fluid consistency index (6) applying this relationship along with Bird's (2) and Metzner's (6, 8) analagous Reynolds numbers for non-Newtonian pipe flow and agitation, a new ball mill Reynolds number is defined as

$$N_{Re} = \frac{d^{2} N^{2-n} \rho}{K g_c} \tag{21}$$

However, this now introduces the dimensionless number n as a new term. The term characterizing the gravity force is the Froude number (N_{Fr}):

$$N_{Fr} = \frac{d N^{2}}{g} \tag{22}$$

The term $d^5N^3\rho/P$ characterizes the basic flow pattern. Since ρ is expressed in mass not force and the power term is usually more useful in the numerator the power number can be written

$$N_{P} = \frac{P g_c}{\rho N^{3} d^{5}} \tag{23}$$

and the ball mill power equation is

$$N_P = A\left(N_{Re}\right)^{h}\left(N_{Fr}\right)^{i}(n)^{j}\left(\frac{D}{d}\right)^{k}\left(\frac{H}{d}\right)^{l}\left(\frac{C}{d}\right)^{m}\left(\frac{L}{d}\right)^{n} \tag{24}$$

COURSE PURSUED IN STUDY

A. Apparatus and Procedure

The apparatus used is shown schematically in Fig. (3). A one-half hp motor drove the rollers using the small and medium mills; the large mill had its own one-half hp motor. Speed was measured manually with a tachometer; torque was measured with a Chatillon dynammometer. The fluid density was determined with a hydrometer; viscosities were determined with a Ferranti-Shirley cone and plate viscometer.

Three sizes of mills using four ball diameters at two loadings varying rotational speeds through several grinding times were used in the experiments. The range was

Mill length = 0.46 = 0.83 feet
Mill diameter = 0.46 - 0.83 feet
Mill speed = 45 = 90 RPM
Mill loading = 25 = 50%
Ball size - 1/8 - 3/4 inches
Milling time - 15 - 3840 minutes

The power data were calculated as shown in appendix A.

The milled fluids viscosity was determined from shear

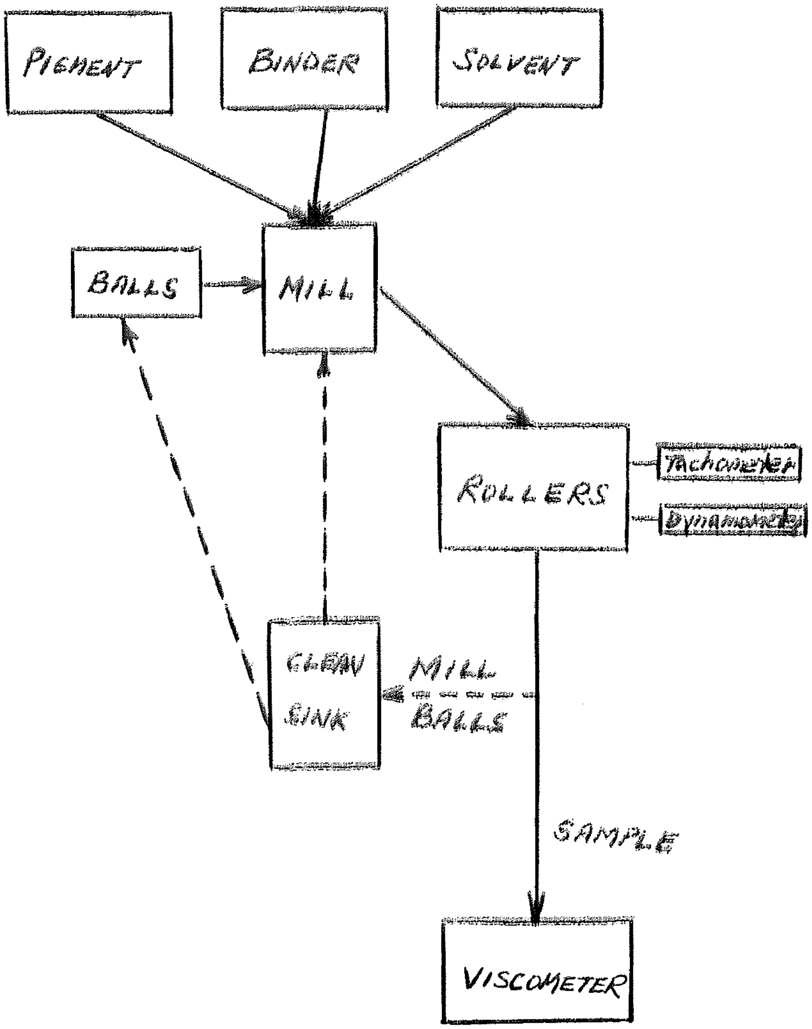

Figure 3. Schematic of Apparatus

stress and shear rate data gotten from the Ferranti-Shirley viscometer. See appendix A for details.

B. Method of Attack

The path chosen was assumption of a functional flow equation ($\tau = K\dot{\gamma}^n$) which is valid over the range of shear rates found in ball milling (100 - 20,000 sec^{-1}) and is the range over which τ and were determined. Then applying dimensional analysis as shown previously in order to develop a correlation. The limitations are as follows:

(1) Only one specific case is covered.

(2) It applies only over a defined shear rate range. But the correlation can probably be expanded to cover other cases and shear rate ranges, and it is a good example of a pigment/binder/solvent system used in many ball mills.

RESULTS

A. Equipment

The Ferranti-Shirley viscometer was checked with Newtonian[1] fluids to assure confidence in the non-Newtonian fluid measurements. This technique is discussed in appendix A and the results are in appendix C.

The average deviation in shear stress-shear rate curves for the experimental dispersion fluids was in the same range as the known Newtonian oils; that is from 1-5%.

B. Dispersion

The system of pigment/binder/solvent was chosen because it has wide applicability in the paint, coatings, finishes, and graphic arts industries. The components are pigment, Neo Spectra Mark I carbon black (Columbia Carbon Co.); binder, Elvacite 2008 acrylic resin (E. I. DuPont Co.); solvent, dimethyl ketone (acetone, Fisher Scientific).

As this system was milled under various conditions for different times the flow behavior and fluid consistency indices changed as shown by the data in appendix B. This

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[1] National Bureau of Standards oils with published viscometric data.

change in consistency curve during milling is certainly typical of preparing any dispersion which is non-Newtonian in character.

C. Data

Three hundred and sixteen experiments were planned but because of some encountered difficulties only two hundred and seventy-six were run. The following ranges were covered —

Ball diameter	1/8 - 3/4 inch
Mill diameter	$5\frac{1}{2}$ - 11 inches
L/D ratio	1
Mill speed	45 - 90 RPM
Charge loading	$\frac{1}{4}$ - $\frac{1}{2}$
Time	15 - 3840 minutes

D. Correlation

From an analysis of variance of equation (24) the model became

$$N_P = A\,(N_{Re})^{l}\,(N_{Fr})^{i}\,(D/d)^{j} \tag{25}$$

and using the data from the two hundred and seventy-six experiments, the constant and exponents were determined, therefore, the new relationship became

$$N_P = 19.17\,(N_{Re})^{-0.28}\,(N_{Fr})^{0.50}\,(D/d)^{5.20} \tag{26}$$

DISCUSSION OF RESULTS

This equation

$$N_P = 19.17 (N_{Re})^{-0.28} (N_{Fr})^{0.50} (D/d)^{5.20} \tag{26}$$

represents a relationship between power needed in a ball mill and the variables upon which it depends. Equation (26) can be put in general terms such as

$$\frac{N_P}{N_{Fr}^{0.50}} = 19.17 (N_{Re})^{-0.28} (D/d)^{5.20} \tag{27}$$

or

$$\frac{N_P}{N_{Fr}^{0.50}} = f(N_{Re}, D/d) \tag{28}$$

Therefore, a plot of log $\frac{N_P}{N_{Fr}^{0.50}}$ vs. log N_{Re} with D/d as a parameter should yield a group of parametric lines with a common slope. This is shown in Fig. (4,5,6).

Equation (25) and Fig.(4,5,6) represent the end product of this investigation. Some factors did not appear in the final correlation as were pre-supposed in the dimensional analysis model. This and other points will be discussed in the subsequent parts of the discussion.

Figure 4. log $\frac{N_p}{N_{Fr}^{0.5}}$ vs. log N_{Re} (0.5 gal. mill)

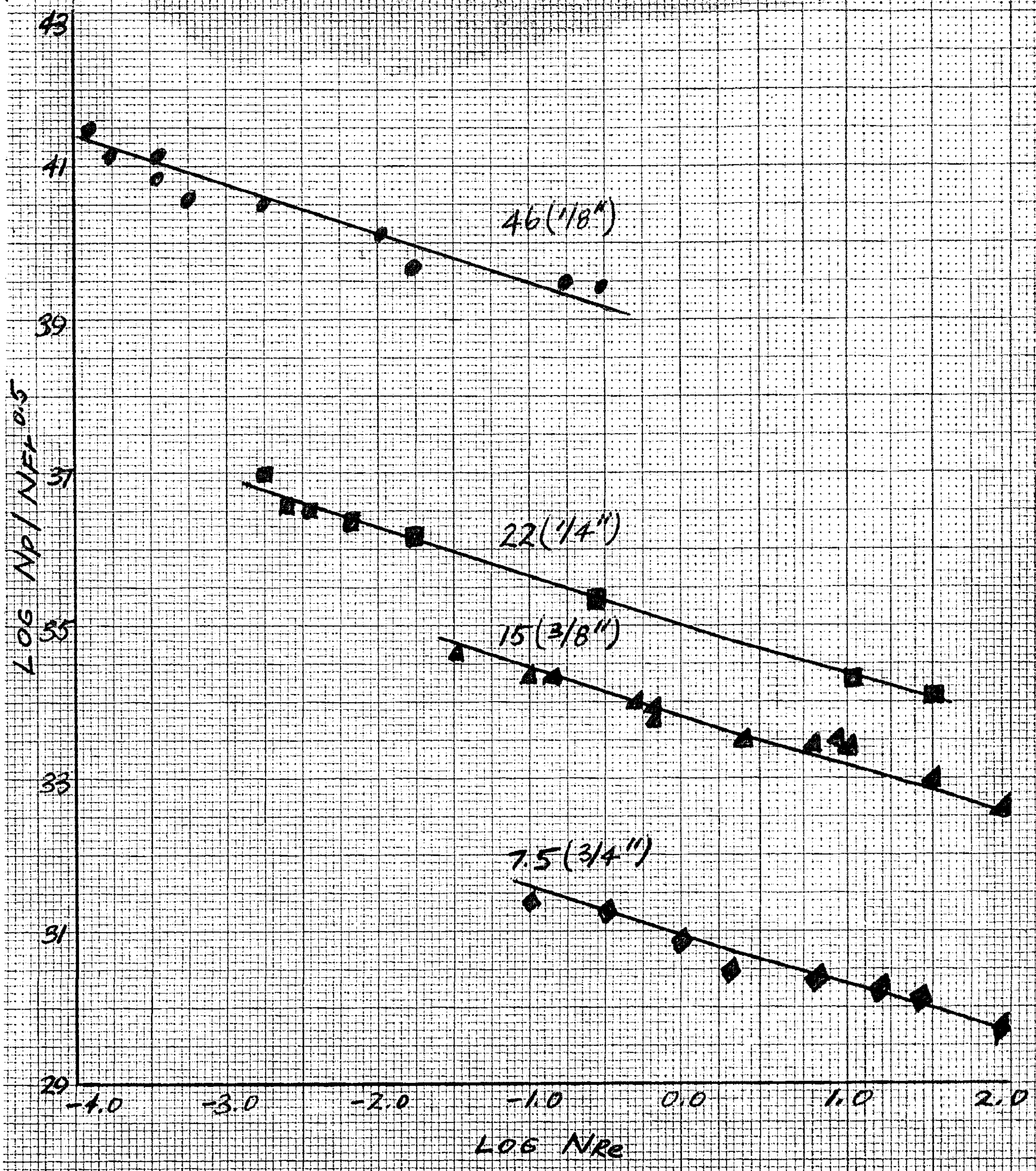


Figure 5. $\log \frac{N_p}{N_{Fr}^{0.5}}$ vs. $\log N_{Re}$ (1.1 gal mill)

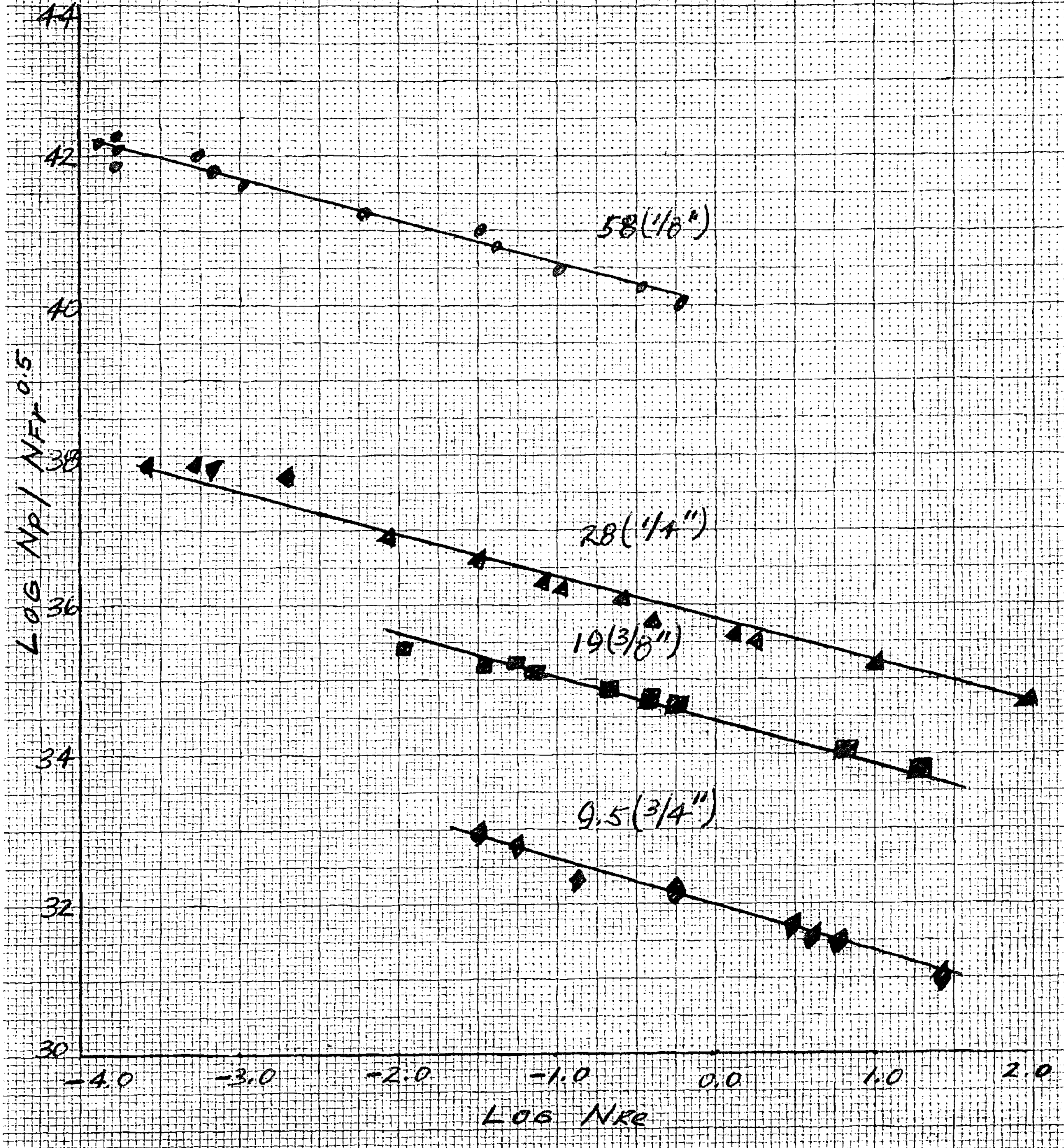
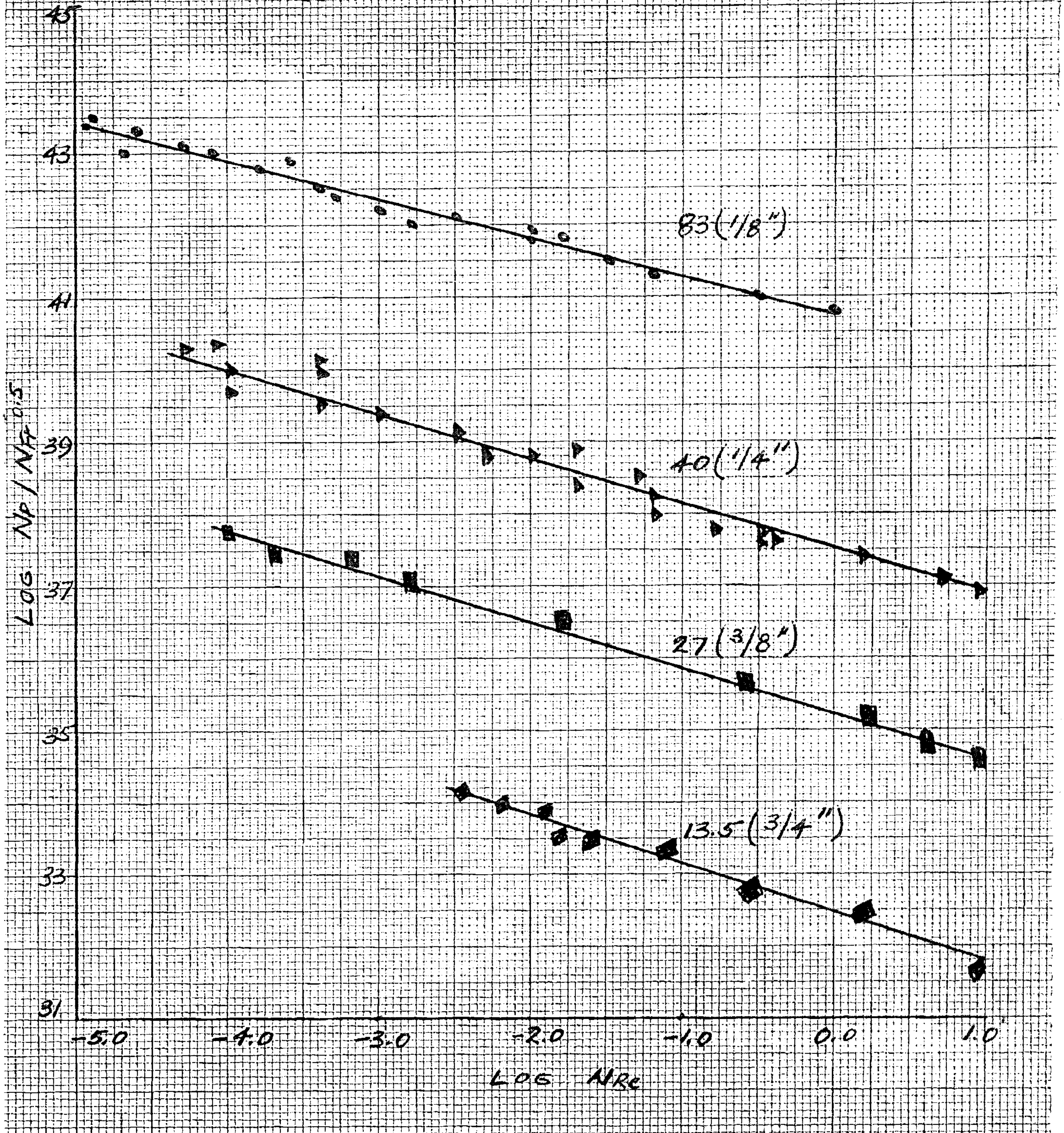


Figure 6. log $\frac{N_p}{N_{Fr}\ 0.5}$ vs. log N_{Re} (1.5 gal. mill)



This correlation is, of course, based on only one pigment/binder/solvent system, and other data is needed to support it as a general relationship. But it is certainly of much value that even a specific system in a ball mill can be described in this manner. Unfortunately only laboratory size equipment was used and many geometric[1] and mill speed variables were overlooked.

A. Correlation Analysis

Since an analysis of variance, as part of the regression stastics computed, shows that about 60 percent of the correlation is from the Reynolds number, and about 20 percent each from the Froude number and D/d ratio, the other dimensionless terms were insignificant and omitted.

Also, an F test showed that the means of the data for the respective factors was different on the 99 percent confidence level.

The residual standard error was 0.42 which is well within the 2.0 range needed to be sure (95% confident) that the correlation did not occur by chance.

A correlation, not described in the dimensional analysis section, tried to incorporate a time factor in

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[1] Since L/D ratio was one for all the mills used, the L/d represents both L/d and D/D.

the form of $(Nt)^W$ along with the Reynolds and Froude numbers and the other geometric ratios. But an analysis of variance of the correlation showed that the $(Nt)^W$ term was insignificant as well as the other factors previously determined insignificant. Also the residual standard error was higher, but the constant for the equations as well as the Reynolds, Froude, and D/d exponents varied by less than 5%.

Since no previous correlation exists for ball mills there is no data to check the correlation within this case.

B. Reynolds Number

Many forms of the Reynolds number have been proposed and used in equations for boundary layers, fluidization, form drag, agitation, classification of solid particles and now ball milling.

As can be seen in Fig. (4) the Reynolds number varies from about $(10)^{-3}$ to $(10)^{2}$ which is an appreciable range; however, no regimes such as laminar, transition, and turbulent appear in this case. There are not many points lying in the $>$ 100 range and perhaps if there were the flow regimes might be recognized.

Since this Reynolds number was composed for a ball mill there is no basis to compare with to determine if and when the three regimes exist. It has been shown that in pipe flow a Reynolds number of < 2100 represented the laminar region, and similarly in agitation a Reynolds number of 20-25 (7) represented laminar flow. Perhaps further work is needed in ball milling to determine where the three regimes exist; as it is possible that most of the data gathered here falls into transition range where interpretation is difficult.

C. Froude Number

This group represents a ratio of the kinetic energy forces to the gravity forces; it accounts for the gravity force's part in determining fluid motion. Since ball milling is a constant tumbling action of balls being lifted against gravity and then falling over each other as it breaks-up the charge, it was felt that a Froude number would probably be significant in correlating power and fluid flow data.

The range of Froude numbers is from about $(10)^{-4}$ to $(10)^{-2}$, which mean they are small numbers for the ball milling case. The numbers are in this range because the ball diameter is small and the mill speed is relatively slow (as compared to agitation stirrers). But, nevertheless, the Froude number is an important factor in depicting a ball mill power-fluid flow relation.

D. Flow Behavior Index, n

This index, n, was used in the dimensional analyis model because it is dimensionless and was hoped it would prove to be an important parameter. However, it showed to be of little significance. The reason is since n is contained in the Reynolds number and it has such an important role in the correlation; n's independent effect has probably been reduced. It may be possible to separate the effect of n into its independent and Reynolds number part, but it was not tried in this investigation.

E. Geometric Variables

It was found that the mill diameter and/or mill length to ball diameter ratio is a significant factor in correlating power and fluid flow data. The mill diameter's and length's independent effect could not be determined because mills of L/D ratio one were the only ones used. In a more complete study L/D ratios of other than one should be chosen to separate each variable's effect.

The effects of H/d and C/d proved to be negligible in this correlation. Both H and C are dependent upon charge loading since only ¼ and ½ loadings were tried their effect proved to be unappreciable here. But if a higher and/or broader range was chosen their effects might become significant.

F. Dispersion Quality

Pigments, binders and solvents are milled together to form a dispersion since they are not all soluble in each other. But the suspension formed has certain properties which make it desirable or indesirable. As an example, if this dispersion is milled for 60 minutes a covering power[1] of 0.80 is measured, but if milled for 240 minutes its covering power increases to 2.00 which is the specification then this milling time is required. However, the viscosity of this dispersion is lower than required then some alteration is needed in either milling variables or formulation to change this relationship. Also, the suspension may not be stable under the milling conditions and again some alteration in milling or formulating is needed to bring this property within specification.

The above examples show that quantitative and qualitative numbers are difficult to put on dispersions as

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[1] Covering power is defined here as the optical transmission density of a 1 mil thick sample of dispersion measured with a yellow light source on a diffuse densitometer.

a measure of their quality. In the samples covering power varied from 0.40 to 2.50, and stability[2] was in the $<$ 5 minutes to $>$ 30 minutes range. The flow behavior and fluid consistency indices varied from 0.21 to 0.90 and 0.06 to 73.20 respectively. But no correlation was found with the milling variables.

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2. Stability is defined as the time (minutes) required to break the suspension into a residue and supernatant on a centrifuge at 175 RPM.

CONCLUSIONS

Based on the work presented here, it is concluded that:

1. The assumption that a pigmented dispersion can be described by a simple power-law model has led to a power-fluid flow correlation for ball milling.

2. The effect of mill diameter (or length) to ball diameter ratio is significant in describing the power-fluid flow relationship.

3. Mill charge loading, flow behavior index, and time factor were not significant in effecting the power-fluid flow relationship in this study.

3. No distinct regimes of flow were found in this study's correlation.

RECOMMENDATIONS

Since there is no previous work in this area of ball milling I hope others will elaborate on this study and try

1. A different pigment/binder/solvent system.

2. Different shape and size equipment.

3. To obtain higher Reynolds numbers and determine if there exists three regime flow.

4. To separate the effect of n into its independent and Reynolds number part.

5. Dilatant fluids.

6. To correlate the milling process with dispersion quality.

NOMENCLATURE

a	Radial distance, (ft.)
C	Height from mill wall to charge, (ft.)
D	Mill Diameter, (ft.)
d	Ball diameter, (ft.)
F	Force (lb. force)
f, f^1	Unspecified functions
g	Gravitional acceleration, (ft./sec^2)
gc	Conversion factor, (lb. mass-ft/lb. force-sec.^{2})
H	Charge depth, (ft.)
h,i,j,k	Unspecified exponents
K	Fluid consistency, (lb. force-sec.^{n}/ft.^{2})
l,m,n	Unspecified exponents
L	Mill length, (ft.)
M	Measured torque/unit height of liquid, (lb. force-ft.)
N	Rotational speed, (rev./min.)
Np	Power number, (dimensionless)
N_{Re}	Reynolds number, (dimensionless)
N_{Fr}	Froude number, (dimensionless)
n	Flow behavior index, (dimensionless)
P	Power, (ft.-lbs. force/min.)
r	Radial distance, (ft.)

T	Shear stress, (lb. force/ft.^{2})
t	Time, (min.)
v	Linear velocity, (ft./sec.)
w, x, y, z	Unspecified exponents

Greek Letters

α	Cone/plate angle, (radians)
γ	Shear rate, (sec.$^{-1}$)
μ, μ_a, μ_d	Viscosity, (lb. mass/ft.-sec.)
ρ	Density, (lb. mass/ft.^{3})
τ	Shear stress, (lb. force/ft.^{2})
ω	Angular velocity, (rad./sec.)
ν	Kinematic viscosity, (ft.^{2}/sec.)

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APPENDIX A. DETAILS OF APPARATUS, PROCEDURE AND CALCULATIONS

A. General

A dynammometer was used to measure milling power requirements, a rotational viscometer was used to determine viscosity characteristics, and a Hydrometer was used to measure fluid density. This section includes a description of equipment, methods, and calculations used in this work.

B. Milling System

Three stainless steel ball mills of 5½, 7, and 11 inches in diameter were used[1]. The two smaller mills were run on driven rollers whereas the larger mill had its drive and variable speed system.

The milling balls were Type 430 stainless steel and had diameters of 1/8, ¼, 3/8 and 3/4 inches.

C. Milling Equipment

(1) Description and procedure. Fig. (7) illustrates the mounting of the mills on the rollers, and also the larger mill's compact unit.

[1] These mills were designed with a length/diameter ratio of 1, and correspond to 0.5, 1.0, and 1.5 gallon sizes respectively.

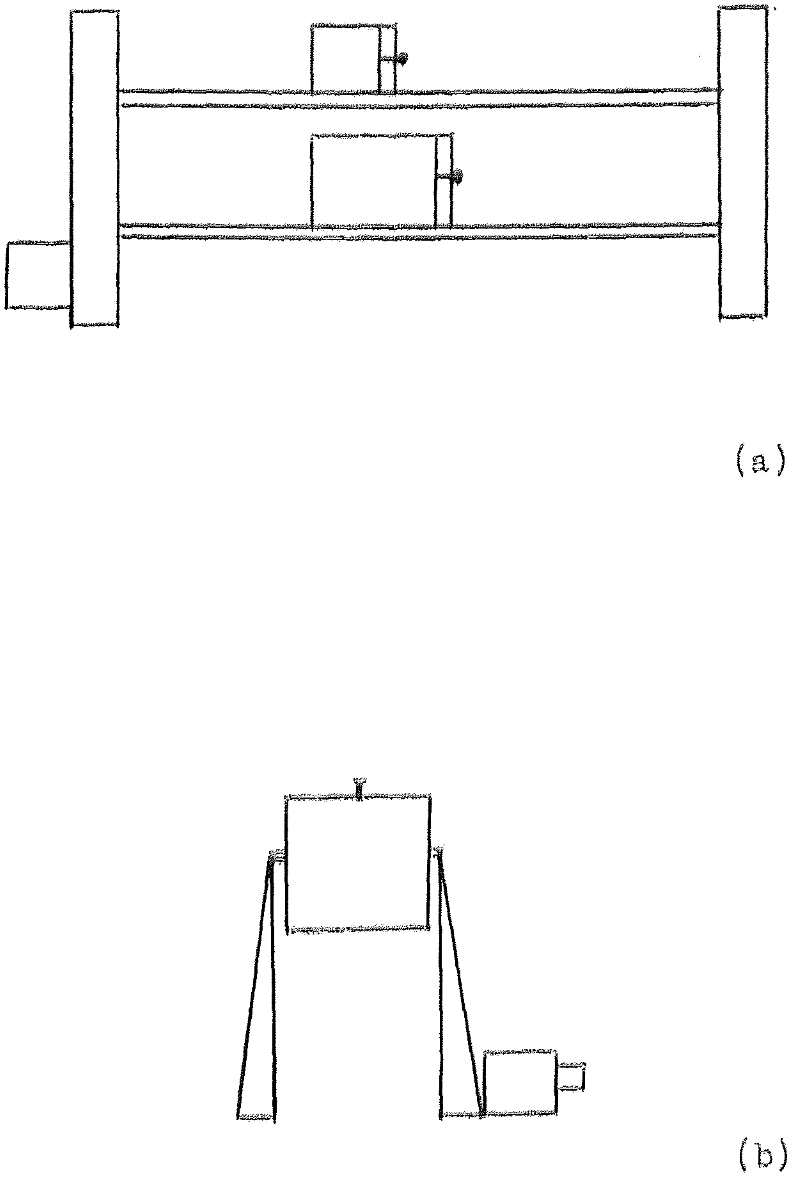

Figure 7. a) 0.5 and 1.0 gal. Mills on Roller

b) 1.5 gal. Mill Unit

In order to measure the torque exerted on the mill charge a steel thread was attached from the roller shaft over a pulley to a chatillon dynammometer so that the force could be measured. The forces were measured with the difference taken to determine the net force.

A compact tachometer (JACQUET) was used to determine the speed during the milling cycles. No drifting in speed was noted over the milling range except at start and finish.

(2) Calculations. The results obtained from the preceding procedure were used to calculate the power numbers.

The power transferred to the mill charge is related to speed and torque by the following:

$$P = 2\pi \tau N \tag{A1}$$

The torque (τ), however, is the product of the force (W) exerted on the dynammometer scale times the radius of the roller shaft (R). Thus,

$$P = 2\pi RWN \tag{A2}$$

The power number is

$$N_P = \frac{P g_c}{d^5 N^3 \rho} = \frac{2\pi R W g_c}{d^5 N^2 \rho} \qquad \text{(A3)}$$

Therefore, with the dynammometer reading, roller shaft speed and radius, milling ball diameter, and fluid density one can calculate the power numbers.[2]

D. Ferranti-Shirley Rotational Viscometer

(1) Description and Procedure. The rotational viscometer used to measure fluid properties was a cone/plate (Ferranti-Shirley) type. A complete description is contained in (18).

A water bath with a temperature control was attached to the cone and plate of the viscometer and left circulating for 15 minutes before using. A sample is placed on the plate in the radius of the solvent inclosure ring. The plate is then raised until it just touches the cone; this has been pre-determined by running the viscometer without any fluid. The gear lever is placed into the high range

2. Power numbers in this work were calculated on the IBM-1620 Computer.

and the speed selector is moved to 10 RPM. A response is seen on the meter scale and recorded; this repeated through 1000 RPM range.

(2) Calculations. The raw data is the speed in RPM and the scale readings which can be converted into shear rate and stress as follows[3]:

$$\tau \text{ (shear stress)} = 25.185 \text{ (scale reading), Dynes/am}^2$$

$$\gamma \text{ (shear rate)} = 17.25 \text{ (RPM), sec}^{-1}$$

Using the power-law model of

$$\tau = K\gamma^n \qquad (A4)$$

and taking logarithms

$$\log \tau = \log K + n \log \gamma \qquad (A5)$$

Thus a plot of τ vs. γ on log scales would give a slope of n and an intercept of K. This was done by feeding the raw data into the IBM-1620 computer using the following program:

---

[3] These constants have been pre-determined for the particular cone and plate by the manufacturer. Using a National Bureau of Standards oil merely verified this part.

POWER-LAW FLUID CONSTANTS

```
  1  Read 100, A1, A2, B, B1, S
100  Format (2A3, 3F 10.7)
101  Format (2A3, F13.5, F11.5, F12.5)
102  Format (7H Run No., 7X 1HK, 12X, 1HN, 7X, 8H RESIDUAL 1)
     Print 102
     BZ  EXPF (B)   .000005
     B1  B1   .000005
     S   S    .000005
     Print 101, A1, A2, BZ, B1, S
     Go to 1
     End
```

And the values of K and n were gotten. The average deviations being from 1-5%.

APPENDIX B. RAW DATA FOR CORRELATION

APPENDIX B. TABLE B

Run	P	N	d	p	n	K	D,L	H	C
1	115.1	55	.010	52.6	.60	1.13	.58	.05	.43
2				58.3	.26	33.30			
3				58.1	.47	3.32			
4				54.9	.63	0.33			
5				50.8	.49	1.30			
6	114.7		.021	53.0	.62	1.05			
7				56.0	.68	0.60			
8				58.0	.61	1.24			
9				52.3	.58	0.82			
10				51.3	.78	0.13			
11	114.1		.031	56.3	.50	5.23			
12				56.5	.52	2.82			
13				56.0	.53	3.12			
14				55.0	.64	0.97			
15				54.6	.71	0.30			
16	111.0		.062	50.5	.52	1.71		.06	
17				52.5	.81	0.20			
18				59.3	.70	0.61			
19				59.6	.56	2.38			
20				59.6	.58	1.13			

Run	P	N	d	ρ	n	K	D,L	H	C
21	147.0	70	.010	56.3	.73	0.39	.58	.05	.43
22									
23				53.2	.74	0.31			
24									
25				55.6	.61	1.61			
26	146.3		.021	54.2	.63	1.74			
27									
28				54.3	.61	1.30			
29									
30				50.6	.78	0.17			
31	146.0		.031	51.2	.76	0.31			
32				51.6	.74	0.41			
33									
34									
35				51.9	.79	0.40			
36	142.0		.062	50.8	.48	5.41		.06	
37				52.4	.78	0.32			
38				57.4	.65	0.74			
39				59.6	.46	4.82			
40				59.6	.47	3.28			

Run	P	N	d	ρ	n	K	D,L	H	C
41	94.6	45	.010	56.2	.49	5.36	.58	.05	.43
42									
43				54.2	.52	1.53			
44									
45				55.1	.56	1.11			
46	94.3		.021	59.3	.21	56.87			
47									
48				57.0	.56	3.06			
49									
50				52.3	.83	0.31			
51	94.0		.031	57.6	.47	6.38			
52									
53				57.2	.54	3.11			
54									
55				56.1	.58	4.62			
56	91.4		.062	59.6	.28	58.77		.06	
57									
58				59.5	.31	36.33			
59									
60				59.5	.27	41.92			

Run	P	N	d	ρ	n	K	D,L	H	C
61	153.8	55	.010	56.1	.47	3.24	.58	.10	.29
62				57.5	.41	6.51			
63				54.0	.49	1.65			
64				52.9	.46	1.80			
65				53.0	.43	2.91			
66	153.0		.021	54.0	.46	4.44			
67				56.5	.51	3.05			
68				58.9	.55	1.91			
69				52.6	.57	0.79			
70				50.7	.63	0.33			
71	152.5		.031	52.2	.64	1.33			
72				55.0	.55	2.00			
73				50.9	.64	0.97			
74				52.0	.55	1.41			
75				50.8	.71	0.18			
76	148.4		.062					.12	
77									
78									
79									
80				56.1	.56	1.60			

Run	P	N	d	ρ	n	K	D,L	H	σ
81	196.0	70	.010	54.6	.73	0.42	.58	.10	.29
82									
83				56.3	.75	0.29			
84									
85				57.0	.76	0.19			
86	195.0		.021	53.8	.65	1.39			
87									
88				53.9	.73	0.43			
89				51.5	.64	0.31			
90									
91	194.4		.031	52.2	.64	1.30			
92									
93				52.4	.66	1.06			
94									
95				52.4	.71	0.36			
96	189.0		.062	50.9	.77	0.19		.12	
97									
98				51.3	.86	0.18			
99									
100				56.4	.58	1.06			

Run	P	N	d	ρ	n	K	D,L	H	C
101	126.0	45	.010	59.6	.29	42.81	.58	.10	.29
102									
103				58.1	.50	3.31			
104									
105				57.2	.61	1.06			
106	125.4		.021	59.2	.31	41.11			
107									
108				57.0	.50	4.11			
109									
110				54.2	.68	1.02			
111	125.0		.031	56.2	.52	7.10			
112									
113				54.2	.56	4.61			
114									
115				54.4	.51	4.92			
116	121.7		.062	59.6	.27	50.19		.12	
117									
118				59.5	.31	39.83			
119									
120				59.4	.28	49.66			

Run	P	N	d	*p*	n	K	D,L	H	C
121	55.3	75	.010	53.1	.41	9.60	.46	.04	.34
122				56.0	.50	4.15			
123				59.0	.37	10.40			
124				56.2	.45	4.00			
125				51.7	.46	1.77			
126	55.0		.021	52.8	.57	2.13			
127				58.3	.55	2.13			
128				59.6	.43	10.24			
129				52.0	.70	0.32			
130				50.8	.63	0.52			
131	53.5		.031	58.8	.50	6.50			
132				58.0	.60	1.42			
133				58.6	.52	2.92			
134				55.8	.67	0.69			
135				54.0	.64	0.48			
136	52.0		.062	50.7	.55	3.19		.05	
137				52.6	.79	0.23			
138				56.5	.71	0.44			
139				59.6	.53	3.43			
140				59.6	.46	5.16			

Run	P	N	d	ρ	n	K	D,L	H	C
141	70.0	95	.010	54.5	.67	0.90	.46	.04	.34
142									
143				54.1	.61	1.37			
144									
145				56.2	.54	4.23			
146	69.6		.021	53.0	.69	1.09			
147									
148				56.1	.72	3.61			
149				51.2	.45	11.19			
150				50.6	.87	0.06			
151	67.5		.031	51.0	.85	0.13			
152				52.1	.79	0.39			
153									
154				52.7	.44	11.80			
155				52.0	.75	0.39			
156	65.9		.062	50.8	.57	2.07		.05	
157				52.1	.80	0.21			
158				56.7	.71	0.46			
159				59.6	.51	2.98			
160				59.6	.48	3.22			

Run	P	N	d	p	n	K	D,L	H	C
161	44.3	60	.010	57.2	.49	4.30	.46	.04	.34
162				55.3	.51	1.90			
163				51.8	.53	.97			
164	44.0		.021	59.6	.21	73.20			
165				56.2	.53	2.01			
166				51.4	.73	0.24			
167	42.8		.031	58.2	.47	5.40			
168				56.2	.54	2.23			
169				57.0	.45	3.44			
170	41.6		.062	59.7	.28	59.62		.05	
171				58.9	.32	36.41			
172				59.2	.28	41.83			
173	73.8	75	.010	55.7	.43	5.43		108	.23
174				59.5	.37	10.61			
175				54.0	.49	1.65			
176				53.2	.53	1.19			
177				51.2	.51	1.14			
178	73.3		.021	54.6	.61	1.10			
179				56.8	.53	2.37			
180				59.3	.40	8.90			

Run	P	N	d	ρ	n	K	D,L	H	C
181	73.3	75	.021	53.8	.64	0.67	.46	.08	.23
182				50.9	.45	4.00			
183	71.5		.031	52.0	.57	2.74			
184				54.6	.60	1.42			
185				56.5	.60	1.36			
186				52.3	.68	0.34			
187				51.1	.65	0.38			
188	69.4		.062	50.9	.75	0.28		.10	
189				51.7	.80	0.32			
190				58.6	.60	1.64			
191				57.9	.47	4.42			
192				57.3	.59	1.37			
193	93.3	95	.010	58.0	.61	1.38		.08	.23
194				55.7	.76	0.33			
195				56.5	.75	0.29			
196				53.1	.68	1.11			
197				51.4	.80	0.41			
198	92.7	95	.021	53.8	.52	6.67	.46	.08	.23
199				54.8	.71	0.56			
200				53.8	.73	0.43			

Run	P	N	d	p	n	K	D,L	H	G
201	92.7	95	.021	52.0	.70	0.28	.46	.08	.23
202				50.9	.71	0.56			
203	90.4		.031	51.7	.66	1.38			
204				52.8	.69	0.29			
205				54.0	.62	0.40			
206				51.9	.71	0.28			
207									
208	87.9		.062	51.4	.75	0.22		.10	
209				50.9	.74	0.26			
210				55.1	.60	1.30			
211				58.7	.61	1.09			
212				58.9	.53	1.53			
213	59.0	60	.010	59.2	.28	42.53		.08	
214									
215				57.3	.50	3.03			
216									
217				51.8	.55	0.84			
218	58.6		.021	59.1	.29	40.81			
219				57.4	.50	4.98			
220				53.8	.55	1.63			

Run	P	N	d	ρ	n	K	D,L	H	C
221	57.1	60	.031	56.4	.49	6.11	.46	.08	.23
222				56.8	.59	3.91			
223				56.0	.46	4.81			
224	55.5		.062	59.6	.28	55.11		.10	
225				57.5	.31	39.83			
226				59.0	.28	49.62			
227	376.0	60	.010	54.2	.70	0.59	.83	.145	.41
228				58.1	.60	1.00			
229				58.9	.68	0.51			
230				53.5	.70	0.52			
231				53.0	.85	0.08			
232	373.5		.021	53.5	.71	0.60			
233				57.5	.61	1.25			
234				58.6	.69	0.52			
235				53.0	.70	0.54			
236				52.5	.86	0.09			
237	372.0		.031	53.3	.69	1.32			
238				57.2	.73	1.56			
239				58.1	.72	0.62			
240				51.3	.70	0.62			

Run	P	N	d	ρ	n	K	D,L	H	C
241	372.0	60	.031	52.4	.90	0.09	.83	.145	.41
242	369.7		.062	51.4	.69	1.76		.160	
243				56.0	.76	1.60			
244				56.5	.76	0.71			
245				51.3	.71	0.59			
246				50.8	.86	0.09			
247	313.5	50	.010	56.9	.65	1.02		.145	
248				57.9	.70	0.76			
249				56.5	.45	7.52			
250				55.0	.35	13.60			
251				54.0	.31	18.30			
252	311.0		.021	56.5	.66	1.05			
253				57.7	.71	0.67			
254				56.3	.46	7.65			
255				54.2	.35	13.68			
256				53.2	.31	18.36			
257	310.0		.031	56.1	.71	1.18			
258				57.5	.73	0.69			
259				56.0	.46	8.32			
260				54.0	.35	13.98			

Run	P	N	d	ρ	n	K	D,L	H	C
261	310.0	50	.031	52.9	.31	19.62	.83	.145	.41
262	308.0		.062	54.2	.71	1.32		.160	
263				56.2	.73	0.73			
264				54.7	.48	8.60			
265				53.1	.37	14.63			
266				52.0	.33	19.98			
267	469.0	75	.010	56.8	.42	25.46		.145	
268				54.5	.26	37.11			
269				53.1	.51	5.62			
270				51.8	.31	16.11			
271				51.1	.33	14.81			
272	466.0		.021	56.5	.32	27.31			
273				53.8	.28	33.86			
274				52.6	.50	4.81			
275				51.5	.31	15.21			
276				50.9	.32	14.91			
277	464.0		.031	56.2	.30	29.83			
278				53.5	.30	30.11			
279				52.4	.48	4.62			
280				51.4	.41	14.81			

Run	P	N	d	p	n	K	D,L	H	C
281	464.0	75	.031	50.8	.31	13.76	.83	.145	.41
282	461.5		.062	55.0	.31	28.36		.160	
283				53.0	.32	29.12			
284				51.2	.50	3.91			
285				51.0	.31	13.11			
286				50.6	.37	12.91			
287	235.0	50	.010	58.2	.52	3.24		.73	.62
288				56.3	.37	9.42			
289				55.7	.50	1.50			
290	233.8		.021	58.0	.53	3.61			
291				56.0	.38	9.73			
292				55.3	.50	1.52			
293			.031						
294									
295									
296	231.0		.062	57.3	.54	6.22		.80	
297				55.0	.39	10.36			
298				54.1	.51	1.60			
299	284.0	60	.010	54.3	.41	18.62		.73	
300				58.6	.41	4.82			

Run	P	N	d	ρ	n	K	D,L	H	C
301	284.0	60	.010	53.9	.31	9.83	.83	.73	.62
302	281.2		.021	54.0	.38	16.21			
303				58.3	.49	4.96			
304				53.7	.34	10.76			
305	280.0		.062	52.8	.33	15.23		.80	
306				57.0	.47	4.04			
307				52.0	.37	9.51			
308	377.0	75	.010	56.8	.39	20.11		.73	
309				55.0	.48	4.00			
310				51.9	.31	16.1			
311	373.5		.021	56.3	.32	28.61			
312				54.9	.51	3.91			
313				51.6	.32	15.83			
314	370.8		.062	55.1	.32	25.45		.80	
315				51.9	.50	3.74			
316				51.3	.33	14.77			

APPENDIX C. RAW DATA FOR DETERMINING K AND n

Three National Bureau of Standards' oils were used; the raw data is shown below. The listed viscosities of these oils at 25°C are

oil D - 0.025 poise

oil H - 0.067 poise

oil J - 0.184 poise

TABLE C-1

OIL	SCALE READINGS AT RPM OF											
	10	50	100	200	300	400	500	600	700	800	900	1000
D	0.3	0.8	1.4	3.2	4.5	6.0	7.1	9.2	10.8	11.3	12.7	13.6
H	0.8	2.2	4.6	9.9	13.9	18.2	22.3	27.0	33.0	34.5	39.0	42.0
J	2.3	6.4	13.0	28.9	40.7	52.8	63.1	80.1	90.6	103.2	113.8	121.4

TABLE G-2

Run	10	100	200	300	500	700	900	1000
1	1.4	3.1	5.0	8.5	11.0	13.4	16.3	17.8
2	5.8	8.5	9.0	10.3	13.4	16.0	18.1	19.0
3	1.7	3.6	5.4	7.4	9.6	10.2	11.9	12.1
4	0.5	1.0	1.5	2.7	4.3	5.8	6.4	6.6
5	0.8	1.8	2.1	3.0	4.5	5.8	6.7	7.0
6	1.4	3.3	5.5	7.8	10.3	15.6	17.7	18.0
7	2.4	4.3	5.2	7.1	10.4	14.2	17.4	18.9
8	2.1	4.4	6.1	7.3	10.7	15.2	17.7	18.4
9	1.0	1.9	2.7	3.8	6.3	7.9	9.4	10.4
10	1.0	1.7	2.5	3.6	6.0	8.5	10.4	11.6
11	2.7	5.2	7.6	9.4	13.1	16.3	18.6	19.7
12	2.8	5.2	7.0	8.9	11.9	14.2	17.3	19.7
13	3.1	6.1	8.5	10.7	15.0	18.3	21.0	22.3
14	2.9	4.2	6.0	8.1	13.1	16.4	19.6	21.8
15	1.0	2.5	3.6	4.9	7.7	10.3	12.8	13.1
16	1.7	3.7	4.8	5.8	7.4	9.0	11.0	13.0
17	1.8	3.5	5.3	7.6	12.1	17.5	22.8	24.9
18	2.8	4.3	6.1	8.5	12.9	18.6	23.0	25.0
19	3.7	7.0	8.0	9.4	13.7	19.0	26.0	29.0
20	2.6	4.3	5.6	7.1	9.3	11.2	12.7	13.2

Run	10	100	200	300	500	700	900	1000
21	2.0	4.2	6.1	8.1	12.0	15.8	20.0	22.6
22	2.2	3.7	4.7	5.9	8.7	12.8	17.2	19.0
23 NR								
24 NR								
25	3.1	5.9	7.7	9.3	12.9	17.2	21.0	22.1
26	5.7	8.3	11.7	14.8	19.9	25.0	32.9	35.8
27 NR								
28	3.8	5.8	7.5	9.4	13.0	17.0	21.1	22.3
29 NR								
30	1.5	2.3	3.2	4.5	7.5	10.4	13.2	15.0
31	2.8	4.0	6.1	8.3	12.3	16.3	20.1	22.0
32	2.9	4.2	6.4	8.3	12.5	16.4	20.0	22.4
33 NR								
34 NR								
35	1.9	3.3	4.5	5.6	9.4	14.3	19.3	21.0
36	5.0	8.0	8.0	9.1	14.3	20.6	26.0	27.9
37	3.6	6.2	8.2	10.9	15.0	19.1	21.4	22.0
38	1.9	2.7	3.9	5.1	10.2	13.9	17.0	18.1
39	4.3	5.9	7.1	9.0	11.5	14.9	18.3	19.0
40	2.7	4.2	5.3	6.7	9.0	10.9	12.6	13.2

Run	10	100	200	300	500	700	900	1000
41	3.9	8.2	11.0	13.3	18.1	22.2	26.0	27.0
42 NR								
43	2.1	3.0	4.0	5.0	7.4	9.3	10.4	10.5
44 NR								
45	2.9	4.3	6.1	7.6	10.3	12.7	14.7	15.2
46	9.9	17.0	20.0	23.0	27.0	30.6	37.1	38.1
47 NR								
48	4.6	7.1	9.9	11.9	17.6	22.5	27.7	29.6
49 NR								
50	3.1	4.6	6.2	8.2	12.1	17.1	22.7	24.9
51	6.7	9.1	11.4	15.3	19.5	23.9	27.4	29.2
52 NR								
53	5.0	7.1	9.2	11.0	15.1	20.1	25.4	28.1
54 NR								
55	5.3	7.8	10.0	11.1	15.4	20.4	26.0	28.2
56	10.7	17.3	20.1	23.3	27.0	30.8	36.4	37.9
57 NR								
58	8.4	13.6	15.1	18.2	22.1	28.6	31.7	32.8
59 NR								
60	6.5	12.9	14.2	15.0	19.1	22.5	23.1	23.2

Run	10	100	200	300	500	700	900	1000
61	1.7	3.6	5.4	6.8	9.7	10.9	11.9	12.1
62	6.4	8.9	12.9	15.2	21.3	22.5	23.1	23.4
63	1.1	2.1	2.9	4.4	6.1	7.4	8.1	8.8
64	1.0	1.8	2.2	4.0	5.5	6.1	6.7	7.0
65	2.1	3.5	4.9	5.8	8.1	10.3	11.8	12.6
66	2.4	4.6	7.1	9.0	11.8	14.1	16.4	17.8
67	1.9	4.6	5.2	6.7	8.6	11.2	14.3	15.7
68	1.5	4.5	5.7	7.2	10.8	13.9	16.1	17.0
69	0.9	1.7	2.5	3.6	5.9	7.7	9.4	10.0
70	1.1	2.1	3.0	3.8	4.8	5.4	6.0	6.4
71	2.9	5.8	7.8	11.0	16.1	21.0	26.6	29.0
72	2.4	4.7	6.0	8.8	11.5	15.0	17.8	19.0
73	2.8	4.5	6.6	8.3	12.3	16.0	19.1	20.0
74	1.6	3.2	4.1	5.6	7.9	10.3	12.0	12.9
75	1.0	2.0	3.1	4.0	5.8	6.3	7.0	7.3
76 NR								
77 NR								
78 NR								
79 NR								
80	2.6	4.4	6.0	7.6	10.2	12.7	14.7	15.2

Run	10	100	200	300	500	700	900	1000
81	2.8	4.5	6.2	8.1	12.1	16.3	20.1	23.2
82 NR								
83	1.9	3.7	4.7	5.9	8.6	12.9	17.1	19.0
84 NR								
85	1.5	3.2	4.3	5.5	7.9	10.2	12.1	12.4
86	5.1	8.3	10.4	12.3	19.0	25.6	31.0	34.3
87 NR								
88	2.4	4.1	6.5	8.6	12.4	16.8	20.0	21.1
89	1.0	1.9	3.1	4.1	5.7	7.2	8.1	8.4
90 NR								
91	5.0	8.1	10.3	12.1	19.0	25.4	31.1	34.0
92 NR								
93	4.3	5.8	8.1	11.0	15.4	20.0	24.0	24.4
94 NR								
95	2.6	4.6	6.0	7.5	10.6	13.1	14.3	14.5
96	1.3	3.9	5.6	7.0	9.5	11.1	12.2	12.4
97 NR								
98	2.1	4.5	7.0	9.9	15.7	23.2	32.4	35.1
99 NR								
100	2.0	3.1	4.2	5.5	8.2	10.4	12.1	12.9

Run	10	100	200	300	500	700	900	1000
101	8.1	13.0	16.0	17.8	19.9	23.0	29.2	31.5
102 NR								
103	2.2	4.0	5.6	7.8	10.7	14.0	16.0	16.3
104 NR								
105	3.3	6.1	7.7	9.1	12.7	17.0	18.6	19.9
106	7.2	13.0	15.5	16.4	19.4	22.7	26.2	29.0
107 NR								
108	4.7	6.1	7.0	8.8	11.1	14.3	17.9	20.4
109 NR								
110	3.9	5.8	8.0	10.6	15.0	19.8	24.1	25.4
111	6.0	14.4	18.3	22.1	28.2	37.6	44.4	47.8
112 NR								
113	5.0	7.0	8.3	10.1	13.0	16.7	18.4	19.6
114 NR								
115	5.4	8.1	10.9	13.4	18.2	22.8	26.8	27.4
116	10.6	17.0	20.2	23.1	27.0	30.1	36.0	37.1
117 NR								
118	8.3	13.7	15.2	18.3	22.4	28.7	31.9	32.4
119 NR								
120	8.4	13.2	16.1	17.9	19.9	23.3	29.9	31.8

Run	10	100	200	300	500	700	900	1000
121	3.5	7.2	10.0	11.7	14.4	17.1	20.8	22.6
122	2.9	5.7	8.0	10.4	14.0	18.1	23.0	24.2
123	3.1	5.7	7.0	8.1	11.1	12.8	17.1	19.4
124	2.7	4.0	5.0	6.3	10.0	14.5	19.3	22.0
125	1.0	1.7	2.2	3.1	4.8	5.7	6.7	6.9
126	2.0	5.2	8.1	10.7	14.8	19.7	23.8	26.5
127	1.8	5.1	7.0	9.1	12.9	15.8	18.7	19.2
128	5.2	9.7	11.5	13.7	18.9	24.1	28.0	29.7
129	1.2	2.2	3.5	4.5	7.2	9.7	11.4	12.1
130	1.0	2.4	3.6	4.7	6.1	8.0	10.0	10.4
131	5.8	8.7	11.2	14.8	19.0	23.2	27.1	28.7
132	1.9	5.0	7.4	9.6	13.0	17.0	20.0	21.0
133	2.4	5.0	6.1	7.9	10.7	14.1	17.2	18.1
134	1.7	4.0	5.5	7.1	11.0	15.2	19.8	21.5
135	1.2	2.3	3.2	4.4	6.8	8.2	9.2	9.4
136	5.6	7.4	8.3	10.5	17.0	23.2	28.0	30.1
137	1.9	3.2	5.0	7.0	11.8	16.3	21.0	23.1
138	1.6	3.3	4.8	6.1	10.0	14.0	17.2	18.6
139	4.8	7.0	9.1	11.0	15.1	20.1	25.3	28.1
140	3.6	6.0	8.1	10.0	12.7	15.4	17.1	17.6

Run	10	100	200	300	500	700	900	1000
141	3.2	5.7	8.1	10.7	15.0	19.7	24.2	25.4
142 NR								
143	3.9	5.8	7.5	9.3	12.9	17.1	21.0	22.1
144 NR								
145	4.8	6.1	8.4	10.2	15.0	20.1	24.8	27.6
146	5.9	8.3	12.0	15.2	23.2	29.2	36.3	39.8
147 NR								
148	8.1	20.2	25.6	29.2	37.2	44.2	57.8	61.4
149	8.9	13.6	17.4	21.0	26.2	30.7	36.7	38.2
150	1.7	2.4	3.3	4.4	7.4	11.4	14.2	16.8
151	1.1	2.9	5.0	7.3	11.6	15.6	18.2	19.9
153 NR								
154	8.5	13.0	16.2	19.8	25.8	30.3	33.0	33.5
155	2.2	4.0	5.9	8.4	12.5	17.0	21.9	24.3
156	1.8	5.0	8.0	10.8	14.1	19.0	23.8	26.5
157	1.9	3.0	4.4	6.1	10.4	15.8	20.6	22.2
158	1.7	3.7	5.1	7.0	11.1	15.3	18.1	19.2
159	2.9	5.0	6.8	8.8	12.1	15.0	18.0	19.4
160	2.7	4.3	5.5	7.0	9.5	11.8	13.7	14.5

Run	10	100	200	300	500	700	900	1000
161	2.5	6.0	7.8	9.2	13.1	17.8	21.0	22.6
162	1.5	2.8	4.0	5.1	8.0	10.3	11.9	12.2
163	0.8	1.6	2.2	3.1	4.9	6.8	7.1	7.2
164	9.1	14.4	16.0	17.6	19.6	22.2	24.0	24.8
165	1.6	3.3	4.7	6.1	10.0	13.2	15.6	16.4
166	0.8	1.2	2.4	4.0	7.6	10.6	14.0	15.0
167	3.0	6.0	9.0	10.2	14.7	18.8	21.0	21.6
168	2.0	3.9	5.8	7.6	11.8	15.6	17.6	17.9
169	1.8	3.5	4.8	5.7	8.0	10.1	11.7	12.5
170	10.7	17.2	20.0	23.3	27.0	30.7	35.9	37.6
171	8.3	13.6	15.0	18.2	22.1	28.7	31.6	32.7
172	6.5	12.9	14.1	15.0	19.0	22.0	23.1	23.3
173	2.1	5.1	7.1	9.0	11.0	12.8	13.5	13.8
174	3.5	5.5	7.1	9.2	12.0	14.7	16.8	17.0
175	1.1	2.1	3.1	4.2	6.3	7.8	8.6	9.1
176	0.9	2.1	3.1	4.4	6.1	7.4	8.4	8.8
177	0.8	1.7	2.2	3.2	4.4	5.7	6.6	7.1
178	1.2	3.8	6.0	8.4	12.0	14.4	16.4	17.8
179	2.0	3.9	6.3	8.7	12.4	15.4	17.8	18.8
180	3.1	6.1	8.0	10.0	13.2	15.4	16.7	17.0

Run	10	100	200	300	500	700	900	1000
181	1.0	2.5	3.9	4.9	7.9	11.7	15.4	17.0
182	1.8	4.3	5.1	6.5	8.3	11.0	14.0	15.2
183	4.8	7.0	9.8	11.9	17.3	22.5	27.7	29.6
184	2.0	4.7	6.8	9.0	12.7	15.7	18.1	19.2
185	2.2	4.5	6.6	8.3	12.3	16.0	19.1	20.0
186	1.0	2.1	3.5	4.4	6.9	8.3	9.8	10.1
187	0.8	1.9	3.0	4.0	5.5	7.0	7.9	8.1
188	1.9	3.7	4.7	5.9	8.6	12.9	17.1	19.0
189	2.3	3.9	6.0	8.1	12.0	17.0	22.4	24.8
190	3.0	5.4	7.7	9.9	14.8	19.9	20.2	20.6
191	5.0	5.9	7.1	8.8	11.9	15.3	18.1	20.3
192	3.6	4.7	6.3	7.9	11.0	13.9	16.5	16.9
193	3.7	5.8	7.5	9.3	12.9	17.1	21.0	22.1
194	3.0	4.0	6.0	8.2	12.3	16.3	20.2	22.0
195	3.2	3.6	4.1	6.5	8.6	12.4	16.8	21.1
196	7.0	8.3	12.0	15.1	23.0	29.0	36.1	39.5
197	2.1	3.5	4.3	5.6	8.4	12.7	16.7	18.7
198	6.2	14.5	18.6	22.3	28.1	37.5	44.0	47.3
199	1.9	4.5	7.0	9.4	13.1	17.4	20.9	22.7
200	1.6	4.1	6.5	8.6	12.4	16.8	20.0	21.1

Run	10	100	200	300	500	700	900	1000
201	1.0	2.1	3.4	4.5	7.1	9.6	11.4	12.1
202	1.7	4.3	7.0	9.3	13.0	17.4	20.8	22.6
203	6.2	8.1	10.6	13.7	21.0	26.7	32.4	34.6
204	0.8	2.0	3.0	4.0	6.1	8.0	9.6	10.0
205	0.6	1.1	1.4	2.7	4.7	5.7	6.4	6.8
206	0.8	2.3	3.2	4.3	6.7	9.0	11.2	12.1
207 NR								
208	0.9	2.3	4.1	5.9	8.2	10.5	12.3	12.5
209	0.7	2.8	3.9	5.0	7.9	10.4	14.3	15.5
210	2.9	4.3	5.8	7.3	11.0	16.9	17.3	17.9
211	2.5	4.1	6.0	7.6	10.5	14.9	18.6	19.9
212	1.9	3.0	4.0	5.0	7.4	9.3	10.4	10.5
213	8.0	13.0	16.0	17.6	19.7	23.0	29.0	31.4
214 NR								
215	2.0	4.1	5.7	7.9	10.9	14.2	16.0	16.4
216 NR								
217	0.8	1.6	2.2	3.2	4.9	6.3	6.9	7.1
218	6.4	12.8	14.0	15.0	19.0	22.4	23.2	23.4
219	2.9	5.7	8.0	10.1	14.2	18.8	23.0	24.2
220	1.1	2.1	2.9	4.2	6.2	7.6	8.3	8.8

Run	10	100	200	300	500	700	900	1000
221	5.0	9.0	11.4	15.0	19.3	23.8	27.2	29.0
222	6.0	7.2	8.1	10.0	16.5	22.5	28.0	30.1
223	4.4	5.9	7.1	9.0	11.5	14.9	18.3	19.0
224	10.4	17.0	20.0	23.3	27.0	30.7	36.4	37.0
225	8.1	13.0	16.0	17.7	19.7	23.0	29.0	31.4
226	10.6	17.8	20.2	23.6	27.6	31.4	37.1	38.4
227	2.7	4.3	6.1	8.5	12.9	18.6	23.0	25.0
228	1.9	3.1	4.2	5.5	8.2	10.4	12.1	12.9
229	2.5	4.2	5.1	7.0	10.1	13.9	17.1	18.5
230	2.2	4.5	5.9	7.8	11.6	14.8	20.1	21.0
231	2.0	2.7	3.7	4.8	7.8	12.1	15.7	17.4
232	2.7	4.3	6.1	8.5	12.9	18.6	23.0	25.0
233	2.2	4.7	7.0	9.0	12.0	15.3	17.8	18.7
234	2.5	4.1	5.1	7.1	10.0	14.0	17.0	18.6
235	3.0	4.4	5.8	7.8	11.7	14.7	20.2	21.2
236	1.8	2.8	3.7	4.7	7.7	12.0	15.5	17.3
237	6.0	8.2	10.7	13.8	21.0	26.8	32.3	34.6
238	4.3	8.1	12.0	15.1	23.0	29.1	36.2	39.7
239	2.3	5.1	7.9	9.9	12.0	15.3	17.8	18.7
240	2.7	4.3	6.1	8.5	12.9	18.6	23.0	23.0

Run	10	100	200	300	500	700	900	1000
241	1.8	2.6	3.7	4.9	7.7	12.0	15.7	17.3
242	5.7	7.9	10.4	13.2	21.0	26.2	32.0	34.4
243	4.2	8.0	11.7	15.0	22.8	29.0	35.8	39.2
244	2.0	5.3	7.8	10.1	12.2	15.6	17.9	18.8
245	3.2	7.0	9.4	11.2	15.5	19.2	22.7	23.6
246	2.0	2.7	3.7	4.8	7.8	12.1	15.7	17.4
247	4.4	5.9	8.2	11.0	15.5	20.1	24.1	24.7
248	6.8	9.4	11.5	13.8	18.0	21.0	22.2	22.8
249	6.3	8.9	12.8	15.0	21.1	21.7	22.7	23.0
250	3.3	11.0	7.2	8.0	10.4	14.4	18.3	19.7
251	4.3	5.9	6.7	8.4	11.0	14.1	17.4	20.2
252	4.2	6.0	8.0	11.0	15.7	20.0	24.0	25.0
253	7.0	9.7	11.5	13.8	18.2	21.0	22.3	22.9
254	6.5	8.2	12.8	15.1	21.2	22.4	23.0	23.2
255	3.6	6.2	7.0	8.1	10.6	14.7	18.4	19.8
256	4.0	6.2	6.9	8.3	11.3	14.0	17.0	20.0
257	4.4	7.7	11.2	14.9	21.9	29.0	34.6	39.0
258	2.1	5.7	7.6	10.0	12.0	15.4	17.8	18.2
259	6.3	8.8	12.7	15.0	21.1	22.3	23.0	23.3
260	3.3	6.8	7.1	8.3	10.4	16.4	19.7	20.4

Run	10	100	200	300	500	700	900	1000
261	3.8	6.0	7.0	8.2	11.5	14.2	17.3	20.2
262	4.5	7.3	11.0	15.0	21.5	28.7	35.0	39.2
263	2.6	5.9	8.2	10.1	12.8	15.7	17.9	18.9
264	6.1	9.0	12.9	15.2	21.3	23.0	23.8	24.1
265	3.1	6.7	7.0	8.0	10.1	16.1	19.4	20.1
266	3.6	5.9	6.8	8.1	11.2	14.0	17.0	19.7
267	9.1	14.4	16.0	17.6	19.6	22.2	24.0	24.8
268	5.8	8.5	9.0	11.2	13.2	16.4	18.5	19.0
269	4.7	8.1	10.9	13.2	18.0	22.2	26.1	27.0
270	4.3	6.0	7.0	8.5	11.8	17.0	18.6	19.5
271	4.0	6.0	6.9	8.0	10.2	13.7	16.2	16.8
272	7.0	9.7	10.8	11.3	14.4	22.3	27.3	30.2
273	6.2	10.9	13.7	15.2	20.8	24.3	30.0	33.6
274	4.2	5.9	7.1	9.0	11.5	14.9	18.3	19.0
275	4.5	6.0	7.0	8.7	11.0	14.2	17.8	20.0
276	4.4	5.9	7.2	8.4	11.3	14.0	17.3	19.8
277	6.8	9.6	10.4	11.2	15.4	22.3	27.8	30.1
278	6.7	9.4	10.1	11.0	15.9	21.9	27.3	29.2
279	4.4	5.7	7.2	8.6	11.8	15.1	17.7	19.8
280	4.0	6.1	6.8	8.2	10.1	13.6	16.1	16.7

Run	10	100	200	300	500	700	900	1000
281	3.4	11.0	7.0	8.1	10.3	14.4	18.3	20.2
282	6.8	9.6	10.4	11.3	15.5	22.4	27.8	30.1
283	6.7	9.4	10.7	11.1	15.3	21.9	27.0	29.8
284	2.7	5.6	8.2	10.1	14.2	18.7	23.1	24.0
285	3.6	6.0	7.0	8.1	10.3	13.6	16.1	16.9
286	3.4	6.2	7.3	8.4	10.7	14.7	18.3	19.9
287	2.7	5.0	7.1	9.1	13.0	18.0	24.5	26.5
288	3.0	5.1	7.0	8.8	11.3	12.5	13.7	14.1
289	1.0	1.9	2.9	4.0	5.6	7.0	7.7	7.8
290	4.6	7.1	9.1	11.2	15.1	20.2	25.4	28.0
291	3.1	5.1	6.0	7.2	10.2	12.8	14.9	15.8
292	1.1	2.0	3.0	4.0	5.7	7.2	7.7	7.9
293 NR								
294 NR								
295 NR								
296	4.7	9.2	11.5	15.0	19.2	23.7	27.1	29.0
297	3.4	5.5	7.0	8.8	11.4	15.2	16.7	17.3
298	1.3	3.7	4.8	5.8	7.3	8.9	11.1	13.0
299	3.0	7.0	9.3	11.2	13.2	16.1	20.0	21.0
300	4.4	5.9	7.1	9.0	11.5	14.9	18.3	19.0

Run	10	100	200	300	500	700	900	1000
301	3.3	7.0	10.1	11.4	14.4	17.3	21.0	22.4
302	4.3	6.0	7.0	8.5	10.8	14.0	17.5	19.3
303	4.0	5.7	7.1	9.0	11.4	14.7	18.0	18.9
304	3.2	5.4	7.0	9.3	11.5	14.2	16.3	16.9
305	4.5	6.0	7.0	8.7	11.0	14.2	17.8	20.0
306	2.8	4.0	5.1	6.3	10.0	14.5	19.3	22.0
307	3.2	5.1	6.0	7.1	10.0	12.7	14.9	15.7
308	3.7	5.2	6.9	8.3	10.4	14.0	17.0	19.4
309	2.3	5.8	7.7	9.0	13.0	17.5	20.4	22.0
310	4.2	5.9	7.2	8.4	11.0	14.1	16.9	19.0
311	6.7	9.4	10.1	11.0	15.4	21.9	27.0	29.1
312	2.6	5.4	8.0	10.0	14.0	17.9	22.7	23.8
313	4.3	5.9	6.8	8.2	10.8	14.0	17.7	19.6
314	7.0	9.7	10.8	11.4	15.5	22.6	27.6	30.0
315	2.8	5.0	6.2	8.3	12.1	17.7	24.2	27.0
316	4.0	6.0	6.9	8.0	10.2	13.7	16.2	16.5

APPENDIX D. MECHANICS OF A BALL MILL

The forces responsible for grinding in a ball mill are grinding (12)

by abrasion due to balls rolling over each other with the grind in between them.

by application of pressure due to the ball's weight; however, this is included in abrasion factor.

due to balls impacting each other and the mill shell with the grind between them.

At slow speeds grinding by abrasion is most prominent whereas grinding by impact increases in importance as mill speed increases; therefore, a much different dispersion should be gotten at the slowest and fastest speeds with everything else remaining constant.

Theoretically (16), the pattern of ball and grind movement in the mill may be followed from Fig. (8). When the mill lining is rough and its speed is close to critical, balls and grind rise from the lower mill part up along a circular path; then, from points located on curve AA^1, they drop along a parabolic path. In their fall they pass through points located on curve BB^1.

From the grind movement pattern in a mill rotating at close to critical speed, it's evident that grinding

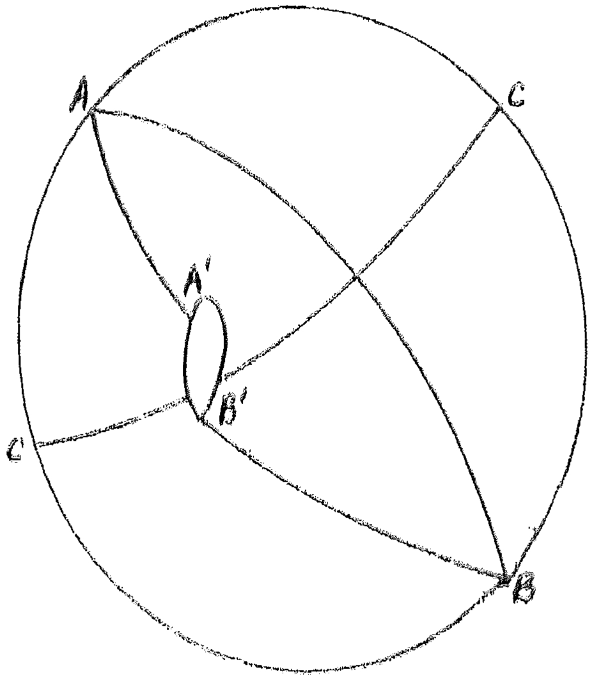

Figure 8. Line Diagram of Ball Path

action occurs because of grind and ball collisions within their dropping zones.

At slower speeds, the balls may be considered as rolling one on the other. This action is depicted by path CC^1 in which the balls are gradually lifted up on one mill side to a higher and higher level until a point is reached in which it has no support from below. At this point the balls cascade falling and tumbling over each other along the sloping surface CC^1 to the lower mill side. The grinding action occurs because of abrasion due to rolling action of the balls.

APPENDIX E. PIGMENTS IN BINDERS

In ball milling pigments in binders three individual processes (4) take place

- pigment wetting with the binder
- agglomerate break-up
- dispersion stabilization.

If dispersion depended on wetting power solely, it would be sufficient to grind pigments with solvents; however in organic solvents the pigments are flocculated. Even though the original agglomerates would be destroyed, floccules would build up during milling.

Using Rohrer's (14) determination for optimum grind composition as a guideline a system of 10% pigment, 5% binder, and 85% solvent was selected. Since no commercial application was intended for the dispersion the selection of the grind composition did not play a major role in the experimentation.