HEAT TRANSFER BY NATURAL CONVECTION
IN HORIZONTAL FLUID LAYERS

A Statistical Correlation of Experimental Data

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A thesis submitted to the faculty of the Department of Chemical Engineering of Newark College of Engineering in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

Newark, N. J.
1959

APPROVAL OF THESIS

FOR DEPARTMENT OF CHEMICAL ENGINEERING
NEWARK COLLEGE OF ENGINEERING

FACULTY COMMITTEE

APPROVED ______________________

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NEWARK, N. J.
JUNE, 1959

SUMMARY

The rate of heat transfer due to natural convection in confined, horizontal, fluid layers can be expressed by equations involving the dimensionless parameters Nusselt No., Rayleigh No., and Prandtl No., where Rayleigh No. equals the product of Grashof and Prandtl Nos.

Statistical correlation by stepwise multiple regression of experimental data obtained from four different investigators, on both gases and liquids, yields three separate equations corresponding to three distinct regimes of convection. The total range spanned by the data is from the onset of convection to Rayleigh = 10^9.

The equations are

Regime	Ra Range	Equation	Std. Error of Estimate
Initial	1600-3000	$Nu = 0.00238(Ra)^{.816}$	± 4%
Laminar	$3000-10^5$	$Nu = 0.221(Ra)^{.256}$	± 7%
Turbulent	10^5-10^9	$Nu = 0.0891(Ra)^{.316}(Pr)^{.0853}$	± 12%

The standard error of estimate, which is the probable error in the prediction of Nu from a regression equation, is obtained for each regime. A very high degree of accuracy was obtained for all correlations.

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INTRODUCTION

Heat transfer by natural convection in confined, horizontal, fluid layers was first investigated formally by Benard (1)* in 1901 and analyzed by Lord Rayleigh in 1916 (2). Since that time a considerable amount of work has appeared in the literature consisting largely of mathematical analyses defining the conditions under which a heated layer first becomes unstable and begins to circulate. Prior to 1950 only a small amount of experimental work had been done. Since then, however, three papers have been published (12, 20, 21) reporting experimental heat transfer data over a wide range of the phenomenon after the onset of convection. Working independently, with different fluids, and in different ranges, the authors differed in their correlations and no comprehensive relationship was developed.

This thesis presents an engineering correlation by stepwise multiple regression of the available data. This method is a powerful statistical tool that provides a correlating equation for a multiplicity of parameters and, at the same time, yields an estimate of the accuracy of the correlation and the significance of the individual parameters. To the author's knowledge, this thesis is the first use of stepwise multiple regression in the analysis and correlation of heat transfer data.

Three of the four sets of data (12, 20, 21) were obtained directly from the authors in the form of the dimensionless parameters Nusselt, Rayleigh, Grashof, and Prandtl numbers. (These will be abbreviated Nu, Ra, Gr, Pr hereafter.) The other set (5) was calculated from raw data tabulated in the paper.

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* Numbers in parentheses refer to references listed in the bibliography.

THEORY

The System

When a temperature difference is applied across a horizontal layer of fluid that is infinite in extent and confined between conducting horizontal surfaces, a temperature gradient will be established across the layer and heat will flow by conduction (and, in the case of gases, by radiation) according to Fourier's law $Q = \frac{kA(T_1 - T_2)}{L}$ - 1 -

If the hot surface is beneath the layer, the fluid will tend to circulate ~~due~~ owing to a density gradient across the layer induced by the temperature gradient. This circulation - convection - results in an increase in the rate of heat transfer over that due to conduction alone. This thesis considers the magnitude of the heat transfer due to convection.

Onset of Convection

Lord Rayleigh (2) found by analysis that no convection would take place until a critical value of the Rayleigh number ($Ra = Gr \times Pr$) was exceeded. His analysis has been confirmed and extended by others (3, 4, 9, 11) and the critical Ra of 1707.8 calculated by Jeffries (3) has been verified experimentally (6, 7, 10, 12, 14, 20).

Heat Transfer - Dimensional Analysis

The Rayleigh - Jeffries analysis does not yield a functional relationship between the rate of heat transfer and the system variables and constants after convection has begun. There have been very few attempts in the literature to derive such a relationship chiefly because the differential equations derived from the basic flow equation do not yield to linearization (16, 19). However, Batcheler (16) has derived the following expressions for the case of convection in vertical layers:

Laminar Region - $Nu = F \text{ of } (Ra)^{.25}$ - 2 -

Turbulent Region - $Nu = F \text{ of } (Ra)^{.33}$ - 3 -

The Rayleigh - Jeffries criterion suggests that the rate of heat transfer is some function of Ra and such a function can be derived by dimensional analysis.

If we postulate that at steady-state conditions the overall rate of heat transfer per unit area H is a function of $\frac{KT}{L}$, ρ, L, μ, K, C, β, G, T where KT/L = heat transfer rate per unit area by conduction and

K = thermal conductivity of the fluid
T = temperature difference
L = thickness of the fluid layer
ρ = density of the fluid
μ = dynamic viscosity of the fluid
C = specific heat at constant pressure of the fluid
β = coefficient of thermal expansion of the fluid
G = gravitational constant

(All constants are evaluated at the mean temperature of the layer)

Then the following expression can be derived by dimensional analysis: (23) (See Appendix 1)

$$\frac{H}{KT/L} = F \text{ of } \left(\frac{\beta G T L^3 \rho^2}{\mu^2}\right)^N \left(\frac{\mu C}{K}\right)^P \text{ or } Nu = F \text{ of } (Gr)^N (Pr)^P \qquad -4-$$

$$\text{When } N = P,\ Nu = F \text{ of } (Ra)^P \qquad -5-$$

$\frac{H}{KT/L}$ is a form of Nusselt number and is the ratio of overall rate of heat transfer due to convection and conduction to the rate of heat transfer due to conduction alone. When it is equal to one, no heat is being transferred by convection.

$\frac{\beta G T L^3 \rho 2}{\mu^2}$ is the Grashof number. It can be regarded as the ratio of buoyant forces, ($\beta G T$), to the viscous forces (μ/ρ), of the fluid (22).

The significance of the critical Ra discovered by Lord Rayleigh then, is that the buoyant force due to the temperature gradient is balanced by the viscous forces in the layer until the buoyant force attains a value represented by the critical Ra. When this happens, flow ~~commences~~ begins.

$\frac{\mu C}{K}$ is the Prandtl number, which is the ratio of momentum diffusivity to thermal diffusivity in the fluid; i.e., the ratio of the fluid property governing the transfer of momentum by viscous forces to the fluid property governing transfer of heat by temperature difference.

The dimensional analysis assumes that heat transfer through the layer is not affected by the particular geometry of the flow patterns or the absolute magnitude of the temperature difference. In addition, it assumes that the physical constants can be evaluated at the mean temperature of the layer.

Modes of Convection

After the onset of convection, different modes of flow have been observed (6, 7, 8, 12, 20, 21). Initially the flow takes place in discreet cells that have cross sections of regular polygons. As Ra increases the mode changes as shown by changes in the shape and width of the cells. Finally, the orderly cells disappear altogether as the convection changes from laminar to turbulent flow.

A number of analyses have been published predicting the Ra at which mode transitions occur (3, 9, 14), and some experimenters have tried to correlate their heat transfer data with the changes in mode that they observed. However, Malkus (15, 17) reported that the heat transfer function changes only with the change from laminar to turbulent flow and does not correspond to changes in mode within these regimes.

Form of Correlation

This thesis presents an emperical correlation of Nu vs three dimensionless parameters - Ra, Pr, and A/D^2, where A/D^2 is the ratio of area to thickness squared of the apparatus used in obtaining the data.

These parameters were selected for the following reasons:

1. Ra is predicted by dimensional analysis.
2. Pr is reported (20, 21) to be an additional parameter with Ra in the turbulent region.
3. A/D^2 is designed to test the experimental data for agreement with the assumption of infinite extent of the layer.

EXPERIMENTAL DATA

Sources

The data were obtained from the four sources listed in Table I. In three cases the data were obtained directly from the authors. In the other case (Mull and Reiher) raw data were taken from a tabulation in the paper, and the parameters calculated from it.

TABLE I

Source and Range of Experimental Data

Source	Date Published	Fluid	Ra Range	No. of Data
DeGraaf and Van Der Held(12)	1952	Air	$10^3 - 10^5$	26
Globe and Dropkin(21)	1958	Water, Silicone oils, Mercury	$10^5 - 10^9$	56
Mull and Reiher(5)	1930	Air	$10^3 - 10^6$	17
Schmidt and Silveston(20)	1958	Water, Glycol, Heptane, Silicone Oils	$10^3 - 10^5$	106

Apparatus and Procedure

All of the sources of data used similar apparatus, differing principally in details of construction and methods of measuring the net rate of heat transfer. A comparison of the important features of their apparatus is made in Table II.* In general, the fluid layer was confined between two parallel metal plates and an enclosing wall of insulating material. Spacing was fixed either by small spacers between plates or by the enclosure. The lower plate was equipped with electrical heaters, while the upper plate was designed to accommodate a flow of cooling water or air. Temperature difference was measured by

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THERMOCOUPLES EMBEDDED IN THE PLATES. HEAT TRANSFER RATE WAS CALCULATED EITHER BY MEASURING THE POWER CONSUMPTION OF THE HEATERS OR THE HEAT ACQUIRED BY THE COOLING MEDIUM. THE PROPERTIES OF THE FLUID WERE TAKEN AT THE ARITHMETIC MEAN TEMPERATURE OF THE HOT AND COLD PLATES. IN THE EXPERIMENTS ON AIR, THE HEAT TRANSFER DUE TO RADIATION WAS DETERMINED OVER THE TEMPERATURE RANGE OF THE EXPERIMENTS BY INVERTING THE HOT AND COLD SURFACES; I.E., BY HEATING FROM THE TOP, AND CORRECTING FOR CONDUCTION BY CALCULATION. EACH OF THE EXPERIMENTAL RUNS WAS CORRECTED BY SUBTRACTING THE APPROPRIATE RADIATION VALUE FROM THE MEASURED TOTAL HEAT TRANSFER RATE.

THE EXPERIMENTAL PROCEDURE WAS THE SAME FOR ALL THE SOURCES. A TEMPERATURE DIFFERENCE WAS APPLIED TO THE FLUID LAYER, THE SYSTEM WAS ALLOWED TO REACH EQUILIBRIUM AND MEASUREMENTS OF T_1, T_2, AND Q (OR ELECTRICAL POWER CONSUMPTION) WERE MADE.

CERTAIN DISCREPANCIES IN EXPERIMENTAL TECHNIQUE ARE WORTHY OF NOTE BECAUSE THEY BEAR ON THE ACCURACY OF THE DATA.

BOTH GLOBE AND DROPKIN AND SCHMIDT AND SILVESTON, WHO EXPERIMENTED WITH LIQUIDS, DETERMINED THE HEAT TRANSFER BY MEASURING ELECTRICAL POWER CONSUMPTION. THE APPARATUS OF THE LATTER WAS ELABORATELY INSULATED TO GUARD AGAINST HEAT LOSS, AND CORRECTIONS, WHERE NECESSARY, WERE APPLIED. HOWEVER, GLOBE AND DROPKIN, NEITHER INSULATED THEIR EQUIPMENT NOR CORRECTED FOR LOSS. AT LEAST THE AUTHORS ARE SILENT ON THIS POINT AND THEIR DIAGRAMS AND DISCUSSIONS REVEAL NO SUCH PRECAUTIONS. IF SUCH IS THE CASE, THEIR NU SHOULD TEND TO BE HIGHER AT PARTICULAR RA. THE APPARATUS OF DE GRAAF AND VAN DER HELD WAS NOT INSULATED EITHER, BUT THEY MEASURED THE HEAT TRANSFER RATE BY MEASURING THE HEAT ACQUIRED BY THE COOLING WATER. IN ADDITION, THEY CORRECTED FOR EXTRANEOUS LOSSES AT THE SAME TIME THAT THEY CORRECTED FOR RADIATION.

De Graaf and Van Der Held have pointed out (12, 13) that Mull and Reiher calculated the thermal coefficient of expansion for air incorrectly, using 1/273 instead of $^1/T$ (avg.) Therefore, the data of Mull and Reiher was recalculated before being used in this correlation. Appendix II contains the derivation of A = $^1/T_{avg}$. Appendix III describes the calculation of Mull and Reiher's Nusselt and Grashof numbers using the constants of De Graaf and Van Der Held and $^1/T_{avg}$.

The experimental data on air (5, 12) was plotted by the original authors in the form of Nu vs Gr, since Pr is nearly a constant for air over the temperature range of their experiments. Ra for these data was calculated from the original Gr (recalculated in the case of Mull and Reiher) using the Pr that are plotted versus temperature in Fig. 1. Although the variation with temperature is not large, this was done to reduce error in the correlation.

CORRELATION

Method

The data was analyzed statistically by the method of stepwise, linear, multiple, regression (25). Linear multiple regression consists of finding by the method of least squares the function of the form

$$Y = A + B_1X_1 + B_2X_2 + B_3X_3 \ldots\ldots$$

that best fits the data. Therefore, the hypothetical function $Nu = A\ (Ra)^{B1}\ (Pr)^{B2}\ (A/D^2)^{B3}$ was expressed in logarithmic form.

$$\text{Log } Nu = \text{Log } A + B_1 \text{ Log } Ra + B_2 \text{ Log } Pr + B_3 \text{ Log } A/D^2 \qquad - 6 -$$

The least squares best fit is that equation that results in the minimum standard deviation of the distribution of experimental data about the regression. A more detailed explanation of the method and the interpretation of the statistics is given in Appendix IV.

In stepwise multiple regression, the variables are added to the regression analysis one at a time in the order of their contribution to the goodness of fit. This procedure has several advantages. At each step it provides the following:

1. A regression equation for each of the variables included in the regression up to that step.
2. An estimate of the probable error in the prediction of Y from the regression equation.

3. An estimate of the probable error in the regression coefficients B_1, B_2, B_3 ......
4. An estimate of the significance of each variable in the regression.
5. Rejection of insignificant variables.

The calculations first were carried out manually to determine the regimes into which the data should be divided. Then, for accuracy, the data were analyzed on an I.B.M. 704 computer using a stepwise, linear, multiple regression program developed by the Computing Center of the Esso Research and Engineering Co. The results of the two analyses were in complete agreement. Table III* lists the experimental data and their logarithms, which were the input to the computer. All the data were assumed to have equal weight.

Results

The experimental data correlated in three distinct regimes of convection - initial, laminar, and turbulent. The range covered by each regime, the parametric equations for each regime, the standard error of estimate of Nu from each equation, and the standard error of the coefficients* are listed in Table IV.

*Coefficients throughout this paper refer to the coefficients of the logarithmic equation. These become exponents of the parameters in the power law equation.

* Table III - Pg. 18

TABLE IV

SUMMARY OF REGRESSION ANALYSIS

REGIME		EQUATION	STD. ERROR OF COEF. RA	STD. ERROR OF COEF. PR	STD. ERROR OF ESTIMATE %
INITIAL	1600-3000	$Nu = 0.00238(Ra)^{.816}$	±.042		+4.32 -4.08
LAMINAR	$3000-10^5$	$Nu = 0.221(Ra)^{.256}$	±.006		+7.19 -6.73
TURBULENT	10^5-10^9	$Nu = 0.0891(Ra)^{.316}(Pr)^{.0853}$	±.007	±.0044	+13.6 -11.9

THE FOLLOWING CONCLUSIONS CAN BE DRAWN FROM THE ANALYSIS:

1. CONVECTIVE HEAT TRANSFER IN HORIZONTAL LAYERS CAN BE EXPRESSED ACCURATELY BY THREE DISTINCT EQUATIONS.
2. NU CAN BE EXPRESSED AS A SIMPLE POWER LAW FUNCTION OF RA, AND, IN THE TURBULENT REGIME, OF RA AND PR.
3. A/D^2 IS NOT A SIGNIFICANT CORRELATING PARAMETER AND THE ASSUMPTION OF LAYERS OF INFINITE HORIZONTAL EXTENT IS JUSTIFIED.

THE VALUE OF THE CRITICAL RA CALCULATED FROM THE EQUATION OF THE INITIAL REGIME IS 1640. THIS IS IN REASONABLE AGREEMENT WITH THE ANALYTICAL VALUE OF 1707.8 (3) AND WITH EXPERIMENTAL VALUES REPORTED IN THE LITERATURE. (6, 7, 20, 21)

IN FIG. 3, NU VS RA OF THE EXPERIMENTAL DATA IS PLOTTED TOGETHER WITH THE EQUATIONS OF THE THREE REGIMES. THE TURBULENT EQUATION IS PLOTTED WITH VARIOUS PR AS PARAMETERS. IN FIG. 5 THE TURBULENT REGIME IS COMBINED INTO ONE EQUATION BY PLOTTING $Nu/Pr^{.0853}$ VS RA. IT CAN BE

SEEN FROM THESE FIGURES THAT THREE REGIMES COMPLETELY DESCRIBE THE DATA. CHANGES IN MODE OF CONVECTION WITHIN THESE REGIMES APPARENTLY DO NOT AFFECT THE HEAT TRANSFER FUNCTION.

IN FIG. 4, THE EQUATIONS OF THE THREE REGIMES ARE PLOTTED AS NU VS RA, SIMILAR TO FIG. 3, EXCEPT THAT THE EXPECTED TRANSITIONS BETWEEN LAMINAR AND TURBULENT REGIMES ARE SHOWN AS DASHED LINES. IT CAN BE SEEN THAT THE TRANSITION FROM INITIAL TO LAMINAR REGIME IS SHARP, WHILE THE TRANSITION FROM LAMINAR TO TURBULENT REGIME DEPENDS ON THE PR OF THE FLUID. PREVIOUS INVESTIGATORS HAVE NOT REALIZED THIS. FLUIDS HAVING A HIGH PR UNDERGO A SHARP TRANSITION, WHILE THOSE WITH SMALL PR GO THROUGH A MORE GRADUAL TRANSITION. THE DATA, ALTHOUGH SPARSE IN THIS REGION, SUPPORTS THIS CONCLUSION EXCEPT FOR MERCURY, WHICH INEXPLICABLY DEVIATES FROM ITS EXPECTED PATTERN IN THE DIRECTION OF LOWER NU.

IN FIG. 5 IT CAN BE SEEN THAT THE DATA IN THE TRANSITION REGION CORRELATES WELL WITH THE REST OF THE REGIME. EVEN THE LOWER MERCURY POINTS FALL WITHIN THREE SIGMA LIMITS AND, THEREFORE, CANNOT BE EXCLUDED ON STATISTICAL GROUNDS. THEREFORE, THE TURBULENT REGRESSION EQUATION CAN BE USED TO PREDICT NU IN THE TRANSITION REGION.

DISCUSSION OF THE ANALYSIS OF DATA

TABLE V* SUMMARIZES THE COMPUTER PRINT-OUT OF THE STEPWISE REGRESSION ANALYSIS. AN EXPLANATION OF THE STATISTICS IS GIVEN IN APPENDIX IV.

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Significance of the Correlating Parameters

The computer analysis showed that all of the correlating parameters were statistically significant in all of the regimes; i.e., the best correlation included all three parameters. However, the important statistical measure is the degree of significance of the parameters, which is given by the coefficients of determination. These are tabulated in Table VI* at each step of the regression. The coefficient of determination (equal to the square of the multiple correlation coefficient) measures the per cent of the total variance in Nu accounted for by the regression at each step; i.e., as each parameter is added to the regression analysis. The residual represents the variance in Nu that is not accounted for by the regression (scatter of data about the regression line) and is assumed to be random error. It can be seen from Table VI that Ra is very highly significant while, except for one case, Pr and A/d^2 are less than the residual. We conclude, therefore, that Pr and A/d^2 are not significant physically and that the correlation found by the regression is spurious; i.e., due to chance. The variance reduction attributed to Pr and A/d^2 is regarded as error and the proper regression is the result of step one, involving only Ra. The exception is Pr in the turbulent regime. Its significance is also shown by the substantial reduction of the standard error of estimate after Pr is added to the regression (Table V). For this regime we conclude that the proper correlating equation is the result of step two, involving Ra and Pr.

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THE MULTIPLE CORRELATION COEFFICIENTS FOR ALL THREE REGIMES REFLECT VERY HIGH DEGREES OF CORRELATION. THEREFORE, WE CONCLUDE THAT THE FUNCTIONAL RELATIONSHIP NU = F OF (RA) OR F OF (RA)(PR) ACCURATELY DESCRIBES THE PHYSICAL PHENOMENON. THERE IS NO EVIDENCE IN THE GRAPHS OF THE DATA TO SUGGEST THAT THERE ARE MORE REGIMES THAN THE THREE ALREADY DESCRIBED.

STANDARD ERROR ESTIMATE

THE STANDARD ERROR OF ESTIMATE IS THE STANDARD DEVIATION OF THE DISTRIBUTION OF EXPERIMENTAL VALUES OF NU ABOUT THE REGRESSION. IT HAS THE USUAL STATISTICAL SIGNIFICANCE; I.E., APPROXIMATELY 2/3 OF THE DATA IS EXPECTED TO FALL WITHIN ± ONE STANDARD ERROR OF ESTIMATE (SIGMA) OF THE VALUE PREDICTED BY THE REGRESSION. IN THIS CASE, THE STANDARD ERROR OF ESTIMATE IS GIVEN IN LOGARITHMIC FORM BY THE REGRESSION ANALYSIS. IN ORDER TO EXPRESS IT IN TERMS OF NU IT MUST BE STATED AS A PERCENTAGE. THIS ACCOUNTS FOR THE FACT THAT ITS POSITIVE AND NEGATIVE NUMERICAL VALUES ARE DIFFERENT. IF IT IS DESIRED TO CALCULATE TWO OR THREE SIGMA LIMITS, THEN THE LOGARITHMIC VALUE MUST BE USED TO CALCULATE THE APPROPRIATE PERCENTAGE.

ERROR OF THE REGRESSION

THE STANDARD ERROR OF A COEFFICIENT IS A MEASURE OF THE PROBABLE ERROR (± ONE SIGMA) OF THE COEFFICIENT ON THE AVERAGE. HOWEVER, IT IS A SPECIFIC PROPERTY OF REGRESSIONS THAT THIS ERROR IS NEGLIGIBLE AT THE CENTER OF THE RANGE COVERED BY THE EXPERIMENTAL DATA AND LARGE AT EITHER END. (SEE APPENDIX IV) THEREFORE, THE REGRESSION EQUATION SHOULD NOT BE EXTRAPOLATED BEYOND THE RANGE OF THE DATA WITHOUT FURTHER SUBSTANTIATION.

TABLE II

COMPARISON OF THE APPARATUS USED BY THE SOURCES OF THE DATA

	DE GRAAF AND VAN DER HELD	MULL AND REIHER	GLOBE AND DROPKIN	SCHMIDT AND SILVESTON
SHAPE OF FLUID CHAMBER	SQUARE PRISM	RECTANGULAR PRISM	CYLINDER	CYLINDER
AREA OF FLUID CHAMBER, M^2	0.142(APPROX.)	0.617	0.013,0.014	0.031
DIAM. OR LENGTH/WIDTH, MM	430/430	1010/612	127,134	199
SPACING, MM	6.9-22.9	12-196	35-66	1.45-13.0
CONTAINER WALL MATERIAL	GLASS	WOOD	"PLEXIGLAS", "PYREX"	"PLEXIGLAS"
WAS IT INSULATED?	NO	YES	NO	YES
WAS THERE A GUARD HEATER?	NO	YES	NO	YES
METHOD OF HEATING	ELECTRICAL	ELECTRICAL	ELECTRICAL	ELECTRICAL
METHOD OF COOLING	WATER	WATER	AIR, WATER	WATER
MAX. HOT TEMP., C	146	146	92	70 (EST.)
MAX. TEMP., DIFFERENCE, C	100	29	27	50
WERE LIQUIDS DEGASSED?	-	-	YES	YES
METHOD OF PLATE TEMP.MEASUREMENT	3 THERMOCOUPLES IN HOT PLATE; MEAN COOLING WATER TEMP.	15 THERMOCOUPLES IN HOT PLATE; 6 THERMOCOUPLES IN COLD PLATE	1 THERMOCOUPLE IN EACH PLATE	3 THERMOCOUPLES IN EACH PLATE
METHOD OF HEAT TRANSFER MEASUREMENT	HEAT ACQUIRED BY COOLING WATER	ELECTRICAL POWER CONSUMPTION	ELECTRICAL POWER CONSUMPTION	ELECTRICAL POWER CONSUMPTION
RADIATION CORRECTION	YES	YES	-	-

TABLE III

DATA

SOURCES:
MR = MULL AND REIHER (5)
SS = SCHMIDT AND SILVESTON (20)
GD = GLOBE AND DROPKIN (21)
DV = DE GRAAF AND VAN DER HELD (12)

TURBULENT REGIME: RA 10^5 TO 10^9

SOURCE	FLUID	RA x 10^{-5}	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
MR	AIR	6.32	.699	69.4	6.67	5.8007	-.1555	1.8414	.8241
		5.03	.704	69.4	5.75	5.7016	-.1524	"	.7597
		6.4	.693	69.4	6.80	5.8062	-.1593	"	.8325
		6.09	.689	69.4	6.55	5.7846	-.1618	"	.8162
		52.6	.700	16.2	14.15	6.7210	-.1549	1.2095	1.1492
		25.8	.706	16.2	10.83	6.4116	-.1512	"	1.0334
SS	WATER	2.47	6.07	306.	5.50	5.3927	.7832	2.4857	.7404
		3.80	5.43	306.	6.59	5.5798	.7348	"	.8189
	HEPTANE	1.25	6.26	3290.	4.23	5.0969	.7966	3.5172	.6263
	SILICONE OIL AK3	1.86	35.0	1224.	5.30	5.2695	1.5441	3.0878	.7243
GD	1000 C.S. SILICONE OIL	1.51	8750.	4.91	8.38	5.1790	3.9420	.6911	.9232
		2.01	8500.	"	9.53	5.3010	3.9294	"	.9791
		6.20	7360.	"	10.90	5.7924	3.8669	"	1.0374
	150 C.S. SILICONE OIL	2110	16.	"	48.8	8.3243	1.2041	"	1.6884
		6760	11.	"	66.9	8.8300	1.0414	"	1.8254
		4920	12.5	"	59.1	8.6920	1.0969	"	1.7716
		1810	15.	"	42.9	8.2577	1.1761	"	1.6325

TABLE III (CONT'D)

TURBULENT REGIME: RA 10^5 TO 10^9

SOURCE	FLUID	RA x 10^{-5}	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
GD	150 C.S.			4.91				.6911	
	SILICONE OIL	1140	15.5	"	34.3	8.0569	1.1903	"	1.5353
		391	17.	"	32.7	7.5922	1.2304	"	1.5146
		280	17.	"	30.4	7.4472	1.2304	"	1.4829
		495	17.	"	34.3	7.6946	1.2304	"	1.5353
		760	15.5	"	41.0	7.8808	1.1903	"	1.6128
		954	15.	"	39.8	7.9796	1.1761	"	1.5999
		805	17.	"	41.6	7.9058	1.2304	"	1.6191
		1512	15.	"	46.5	8.1796	1.1761	"	1.6674
		5527	12.	"	65.0	8.7425	1.0792	"	1.8129
	WATER	2490	2.3	"	33.2	8.3962	.3617	"	1.5211
		1934	2.5	"	30.9	8.2865	.3979	"	1.4900
		1384	2.5	"	29.2	8.1411	.3979	"	1.4654
		890	3.5	"	28.4	7.9494	.5441	"	1.4533
		579	3.8	"	27.1	7.7627	.5798	"	1.4330
		388	4.3	"	24.3	7.5888	.6335	"	1.3856
		183	4.9	"	19.3	7.2624	.6902	"	1.2856
		80.4	5.7	"	17.8	6.9053	.7559	"	1.2504
		42.8	5.5	"	15.4	6.6314	.7404	"	1.1875
		116.8	5.7	"	19.0	7.0674	.7559	"	1.2788
	50 C.S.								
	SILICONE OIL	112.	299.	"	24.1	7.0492	2.4757	"	1.3820
		191	245.	"	26.5	7.2810	2.3892	"	1.4232
		40.7	364.	"	16.7	6.6096	2.5611	"	1.2227
		11.8	402.	"	12.8	6.0719	2.6042	"	1.1072

TABLE III (CONT'D)

TURBULENT REGIME: RA 10^5 TO 10^9

SOURCE	FLUID	RA x 10^{-5}	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
GD	MERCURY	9.87	.0232	3.19	4.91	5.9943	-1.6345	.5038	.6911
		35.	.0218	"	6.63	6.5441	-1.6615	"	.8215
		3.76	.0235	"	3.07	5.5752	-1.6289	"	.4871
		6.14	.0233	"	3.87	5.7882	-1.6326	"	.5877
		16.5	.0227	"	5.61	6.2175	-1.6440	"	.7490
		9.57	.0229	"	5.22	5.9809	-1.6402	"	.7177
		7.92	.0231	"	4.38	5.8987	-1.6364	"	.6415
		4.67	.0232	"	3.67	5.6693	-1.6345	"	.5647
		13.2	.0228	"	6.20	6.1206	-1.6421	"	.7924
		22.	.0222	"	7.11	6.3424	-1.6536	"	.8519
		34.3	.0214	"	8.08	6.5353	-1.6696	"	.9074
		35.8	.0212	"	7.87	6.5539	-1.6757	"	.8960
		36.2	.0212	"	7.64	6.5587	-1.6757	"	.8831
		51.	.0203	"	8.32	6.7076	-1.6925	"	.9201
		9.05	.0231	"	5.56	5.9566	-1.6383	"	.7451
		5.47	.0233	"	4.50	5.7380	-1.6326	"	.6532
		3.73	.0235	"	3.25	5.5717	-1.6289	"	.5119
		1.63	.0236	"	1.87	5.2122	-1.6271	"	.2718
		2.02	.0236	"	2.55	5.3054	-1.6271	"	.4065
		24.7	.0227	11.28	7.68	6.3927	-1.6440	1.0523	.8854
		51.6	.0200	"	9.72	6.7126	-1.6990	"	.9877
		13.0	.0238	"	6.02	6.1139	-1.6234	"	.7796
		56.0	.0235	"	9.59	6.7482	-1.6289	"	.9818
		92.8	.0232	"	10.91	6.9676	-1.6345	"	1.0378
		168.9	.0222	"	13.86	7.2276	-1.6536	"	1.1418
		333.7	.0219	"	16.88	7.5234	-1.6596	"	1.2274

TABLE III (CONT'D)

LAMINAR REGIME: RA 3000 TO 10^5

SOURCE	FLUID	RA	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
DV	AIR	3560	.710	761.	1.52	3.5514	-.1487	2.8814	.1818
		6490	.707	761.	2.19	3.8122	-.1506	2.8814	.3404
		8280	.705	761.	2.26	3.9180	-.1518	2.8814	.3541
		9420	.703	761.	2.50	3.9740	-.1530	2.8814	.3979
		4000	.709	761.	1.63	3.6021	-.1494	2.8814	.2122
		5280	.708	761.	1.80	3.7226	-.1500	2.8814	.2553
		7070	.707	761.	2.02	3.8494	-.1506	2.8814	.3054
		8720	.704	761.	2.25	3.9405	-.1524	2.8814	.3522
		10100	.702	761.	2.37	4.0043	-.1537	2.8814	.3748
		11000	.699	761.	2.36	4.0414	-.1552	2.8814	.3729
		11100	.698	761.	2.38	4.0453	-.1561	2.8814	.3766
		23400	.709	231.	3.33	4.3692	-.1494	2.3636	.5224
		40600	.706	231.	3.72	4.6085	-.1512	"	.5705
		52000	.703	231.	4.01	4.7160	-.1530	"	.6031
		58100	.702	231.	3.81	4.7642	-.1537	"	.5809
		63200	.700	231.	3.88	4.8007	-.1549	"	.5888
		66600	.697	231.	4.06	4.8235	-.1568	"	.6085
		68100	.696	231.	4.02	4.8332	-.1574	"	.6042
		10600	.701	761.	2.34	4.0253	-.1543	2.8814	.3692
		16600	.710	231.	2.82	4.2201	-.1487	2.3636	.4502
		23700	.709	231.	3.48	4.3748	-.1494	"	.5416
		32600	.708	231.	3.57	4.5527	-.1500	"	.5527
		44100	.706	231.	3.55	4.6444	-.1520	"	.5502
		54700	.703	231.	3.93	4.7380	-.1530	"	.5944
		65100	.700	231.	3.91	4.8136	-.1549	"	.5922
MR	AIR	9660	.695	1090.	2.15	3.9850	-.1580	3.0374	.3324
		9640	.695	543.	2.13	3.9841	-.1580	2.7348	.3284
		9430	.695	271.	2.23	3.9745	-.1580	2.4330	.3483
		9470	.695	136.	2.23	3.9764	-.1580	2.1335	.3483
		84500	.701	259.	3.59	4.9269	-.1543	2.4133	.5551
		85400	.688	259.	3.64	4.9315	-.1625	"	.5611
		88700	.691	259.	3.85	4.9479	-.1605	"	.5855
		91200	.695	259.	3.81	4.9600	-.1580	"	.5866

TABLE III (CONT'D)

LAMINAR REGIME: RA 3000 TO 10^5

SOURCE	FLUID	RA	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
MR	AIR	45500	.706	259.	3.44	4.6580	-.1512	2.4133	.5366
		86000	.695	134.	3.76	4.9345	-.1580	2.1271	.5752
		80500	.688	134.	3.47	4.9058	-.1625	"	.5403
SS	WATER	3510	6.05	3290	1.74	3.5453	.7818	3.5172	.2406
		3430	6.06	3290	1.75	3.5353	.7825	3.5172	.2430
		5330	5.51	3290	2.03	3.7267	.7412	3.5172	.3075
		3310	3.07	3290	1.82	3.5198	.4871	3.5172	.2601
		6050	3.79	3290	2.20	3.7818	.5786	3.5172	.3424
		9380	3.59	3290	2.31	3.9722	.5551	3.5172	.3636
		9380	3.59	3290	2.30	3.9722	.5551	3.5172	.3617
		4740	6.57	1224	2.00	3.6758	.8176	3.0878	.3010
		6520	6.46	1224	2.18	3.8142	.8102	3.0878	.3385
		9280	6.40	1224	2.45	3.9676	.8062	3.0878	.3892
		5390	4.34	1224	2.03	3.7316	.6375	3.0878	.3075
		8000	4.33	1224	2.15	3.9031	.6365	3.0878	.3324
		13100	4.98	3290	2.42	4.1173	.6972	3.5172	.3838
		13000	4.99	3290	2.42	4.1139	.6981	3.5172	.3838
		15400	6.30	1224	2.60	4.1875	.7993	3.0878	.4150
		23000	6.16	1224	2.74	4.3617	.7896	3.0878	.4378
		31200	5.94	1224	2.90	4.4942	.7738	3.0878	.4624
		41800	5.63	1224	3.17	4.6212	.7505	3.0878	.5011
		66000	5.16	1224	3.59	4.8195	.7126	3.0878	.5551
		17100	4.37	1224	2.58	4.2330	.6405	3.0878	.4116
		24500	4.41	1224	2.71	4.3892	.6444	3.0878	.4330
		36500	4.39	1224	3.03	4.5623	.6425	3.0878	.4814
		56300	4.30	1224	3.48	4.7505	.6335	3.0878	.5416
		78600	4.44	1224	3.75	4.8954	.6474	3.0878	.5740
		39600	6.13	306	3.25	4.5977	.7875	2.4857	.5119
		79700	6.17	306	3.66	4.9015	.7903	2.4857	.5635
		97800	6.24	306	4.18	4.9903	.7952	2.4857	.6212

TABLE III (Cont'd)

Laminar Regime: Ra 3000 to 10^5

Source	Fluid	Ra	Pr	A/D^2	Nu	Log Ra	Log Pr	Log A/D^2	Log Nu.
SS	Heptane	3210	6.23	14570	1.61	3.5065	.7945	4.1635	.2068
		3230	6.23	14570	1.60	3.5092	.7945	4.1635	.2041
		3830	6.24	14570	1.73	3.5832	.7952	4.1635	.2380
		4470	6.23	14570	1.82	3.6503	.7945	4.1635	.2601
		6040	6.26	14570	1.97	3.7810	.7966	4.1635	.2945
		9460	6.28	14570	2.21	3.9759	.7980	4.1635	.3444
		14800	6.18	14570	2.38	4.1703	.7910	4.1635	.3766
		23500	6.22	3290	2.52	4.3711	.7938	3.5172	.4014
		29400	6.23	3290	2.66	4.4684	.7945	3.5172	.4249
		35900	6.23	3290	2.84	4.5551	.7945	3.5172	.4533
		50400	6.20	3290	3.23	4.7024	.7924	3.5172	.5092
		83700	6.26	3290	3.70	4.9227	.7966	3.5172	.5682
SS	Silicone Oil AK3	3360	35.8	13600	1.73	3.5263	1.5539	4.1342	.2380
		6140	35.2	3290	2.21	3.7882	1.5465	3.5172	.3444
		8020	35.7	3290	2.30	3.9042	1.5527	3.5172	.3617
		9750	35.8	3290	2.41	3.9890	1.5539	3.5172	.3820
		19500	35.4	3290	2.89	4.2900	1.5490	3.5172	.4609
		28200	35.2	3290	3.15	4.4502	1.5453	3.5172	.4983
		31300	35.5	1224	3.15	4.4955	1.5502	3.0878	.4983
		49600	35.7	1224	3.57	4.6955	1.5527	3.0878	.5527
		93900	35.2	1224	4.39	4.9727	1.5465	3.0878	.6425
SS	Silicone Oil AK 350	3200	2630.	628	1.60	3.5052	3.4200	2.7980	.2041
		5160	2580.	628	2.00	3.7126	3.4116	2.7980	.3010
		4500	3650.	306	1.96	3.6532	3.5623	2.4857	.2923
		6490	3560.	306	2.23	3.8122	3.5514	2.4857	.3483
		9140	3400.	306	2.45	3.9610	3.5315	2.4857	.3892
		7310	3080.	181.6	2.35	3.8639	3.4886	2.2591	.3711
		12100	3050.	181.6	2.63	4.0828	3.4843	2.2591	.4200
		17000	3060.	181.6	2.95	4.2304	3.4857	2.2591	.4698
		22800	3120.	181.6	3.15	4.3579	3.4942	2.2591	.4983
		29200	3050.	181.6	3.39	4.4654	3.4843	2.2591	.5302

TABLE III (Cont'd)

Laminar Regime: Ra 3000 to 10^5

Source	Fluid	Ra	Pr	A/D^2	Nu	Log Ra	Log Pr	Log A/D^2	Log Nu.
SS	Ethylene-Glycol	3940	126.	3290	1.83	3.5955	2.1004	3.5172	.2648
		3183	52.5	3290	1.68	3.5024	1.7202	3.5172	.2253
		4460	53.5	3290	1.94	3.6593	1.7284	3.5172	.2878
		7960	53.0	3290	2.28	3.9009	1.7243	3.5172	.3579
		4080	136.	1224	1.86	3.6107	2.1335	3.0878	.2695
		4590	133.	1224	2.03	3.6618	2.1238	3.0878	.3075
		6620	120.	1224	2.24	3.8209	2.0792	3.0878	.3502
		12100	54.0	3290	2.61	4.0828	1.7324	3.5172	.4166
		13000	134.	1224	2.65	4.1139	2.1271	3.0878	.4232
		21500	133.	306	3.05	4.3324	2.1238	2.4857	.4843
		29600	130.	306	3.28	4.4713	2.1139	2.4857	.5159
		43300	134.	306	3.64	4.6365	2.1271	2.4857	.5611
		80600	132.	306	4.23	4.9063	2.1206	2.4857	.6263

TABLE III (CONT'D)

INITIAL REGIME: RA 1600 TO RA 3000

SOURCE	FLUID	RA	PR	A/D²	NU	LOG RA	LOG PR	LOG A/D²	LOG NU.
DV	AIR	1590	.699	2540.	1.13	3.2014	-.1555	3.4048	.0531
SS	WATER	1630	4.36	14570.	1.02	3.2122	.6395	4.1635	.0086
		1770	4.37	"	1.04	3.2480	.6405	"	.0170
		1870	4.37	"	1.06	3.2718	.6405	"	.0253
		1940	4.37	"	1.09	3.2878	.6405	"	.0374
		2090	4.35	"	1.17	3.3202	.6385	"	.0682
		2230	4.41	"	1.24	3.3483	.6444	"	.0934
		1700	6.54	3290.	1.05	3.2304	.8156	3.5172	.0212
		2260	4.16	"	1.37	3.3541	.6191	"	.1367
		2650	6.03	"	1.55	3.4232	.7803	"	.1903
		2660	4.14	"	1.58	3.4249	.6170	"	.1987
	ETHYLENE GLYCOL	1680	133.	3290	1.01	3.2253	2.1238	3.5172	.0043
		1820	130.	"	1.06	3.2601	2.1139	"	.0253
		1900	134.	"	1.10	3.2788	2.1271	"	.0414
		2180	130.	"	1.22	3.3385	2.1139	"	.0864
		2240	133.	"	1.31	3.3502	2.1238	"	.1173
		2670	129.	"	1.50	3.4265	2.1106	"	.1761
		1860	52.5	"	1.10	3.2695	1.7202	"	.0414
		2480	53.0	"	1.41	3.3944	1.7243	"	.1492
		2780	138.	1224	1.53	3.4440	2.1399	3.0878	.1847
	HEPTANE	2240	6.21	14570	1.29	3.3502	.7931	4.1635	.1106
		2720	6.23	"	1.47	3.4346	.7945	"	.1673
	SILICONE OIL AK3	1730	35.6	13600	1.05	3.2380	1.5514	4.1342	.0212
		1960	35.7	"	1.16	3.2923	1.5527	"	.0645
		2380	36.0	"	1.38	3.3766	1.5563	"	.1399

TABLE III (CONT'D)

INITIAL REGIME: RA 1600 TO RA 3000

SOURCE	FLUID	RA	PR	A/D^2	NU	LOG RA	LOG PR	LOG A/D^2	LOG NU.
SS	SILICONE OIL AK 350	1880	3660.	628.	1.09	3.2742	3.5635	2.7980	.0374
		2170	3640.	"	1.26	3.3365	3.5611	"	.1004
		2680	3520.	"	1.49	3.4281	3.5465	"	.1732
		1760	2630.	"	.99	3.2455	3.4200	"	.0000
		2460	2630.	"	1.31	3.3909	3.4200	"	.1173
		1890	3640.	306.	1.09	3.2765	3.5611	2.4857	.0374
		2910	3670.	"	1.59	3.4639	3.5647	"	.2014

TABLE V

Step	Variable Entering Regression	Multiple Correlation Coefficient	S_Y	F_X at Entry	Regression Coefficients			Std. Error of Regression Coefficients			Constant
					B_1	B_2	B_3	B_1	B_2	B_3	Log A
					Initial Regime (N = 32)						
0			.0659								
1	Ra	.9624	.0182	376.7	.816			.042			-2.624
2	Pr	.9690	.0137	6.1	.835	-.007		.040	.003		-2.675
3	A/D^2	.9803	.0137	15.8	.821	-.019	-.031	.032	.004	.008	-2.498
					Laminar Regime (N = 107						
0			.1219								
1	Ra	.9691	.0302	1619.1	.256			.006			- .656
2	A/D^2	.9736	.0281	17.8	.241		-.024	.007		.006	- .524
3	Pr	.9738	.0270	9.1	.247	.008	-.022	.007	.002	.006	-.559
					Turbulent Regime (N = 66)						
0			.3913								
1	Ra	.9320	.1447	411.0	.348			.017			-1.260
2	Pr	.9920	.0554	373.3	.316	.085		.007	.004		-1.050
3	A/D^2	.9908	.0541	4.1	.322	.083	.024	.007	.004	.012	-1.110

S_Y = Std. error of estimate of Nu from regression.

F_X = Ratio of remaining variance in Nu due to variable entering regression, to residual variance.

N = Number of observations (data) in each regime.

TABLE VI

Coefficients of Multiple Correlation and Determination

Regime	Step	Variable Entering	Coef. of Mult. Correl.	Coef. of Mult. Determ. %	% due to Entering Variable
Initial	1	Ra	.9624	92.6	92.6
	2	Pr	.9690	93.9	1.3
	3	A/d^2	.9803	96.1	2.2
		Residual		3.9	3.9
Laminar	1	Ra	.9691	93.9	93.9
	2	A/d^2	.9736	94.8	0.9
	3	Pr	.9758	95.2	0.4
		Residual		4.8	4.8
Turbulent	1	Ra	.9302	86.3	86.3
	2	Pr	.9902	98.0	11.7
	3	A/d^2	.9908	98.2	0.2
		Residual		1.8	1.8

FIG. 1

Prandtl No. for Air vs Temperature

Reference (24)

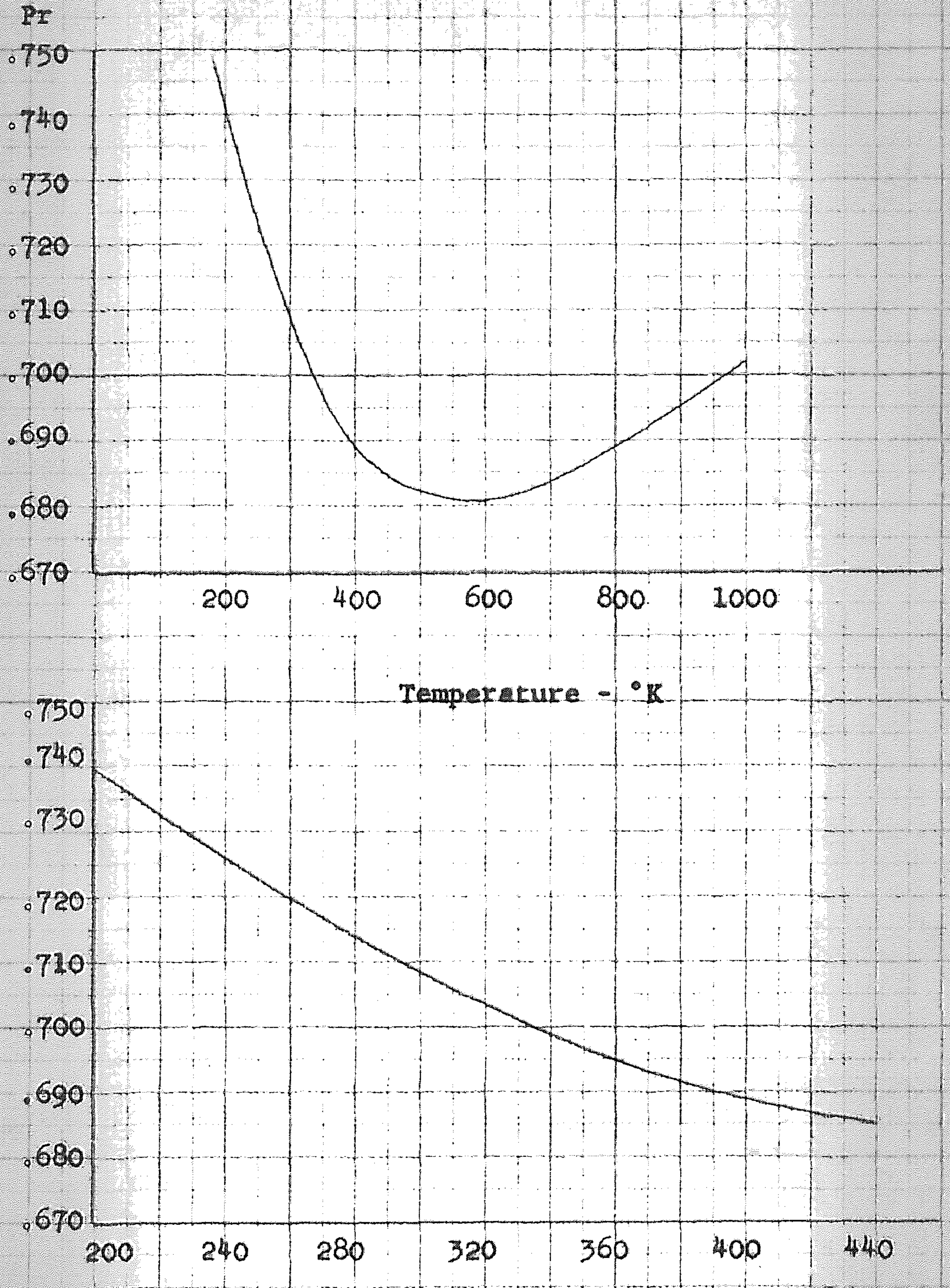
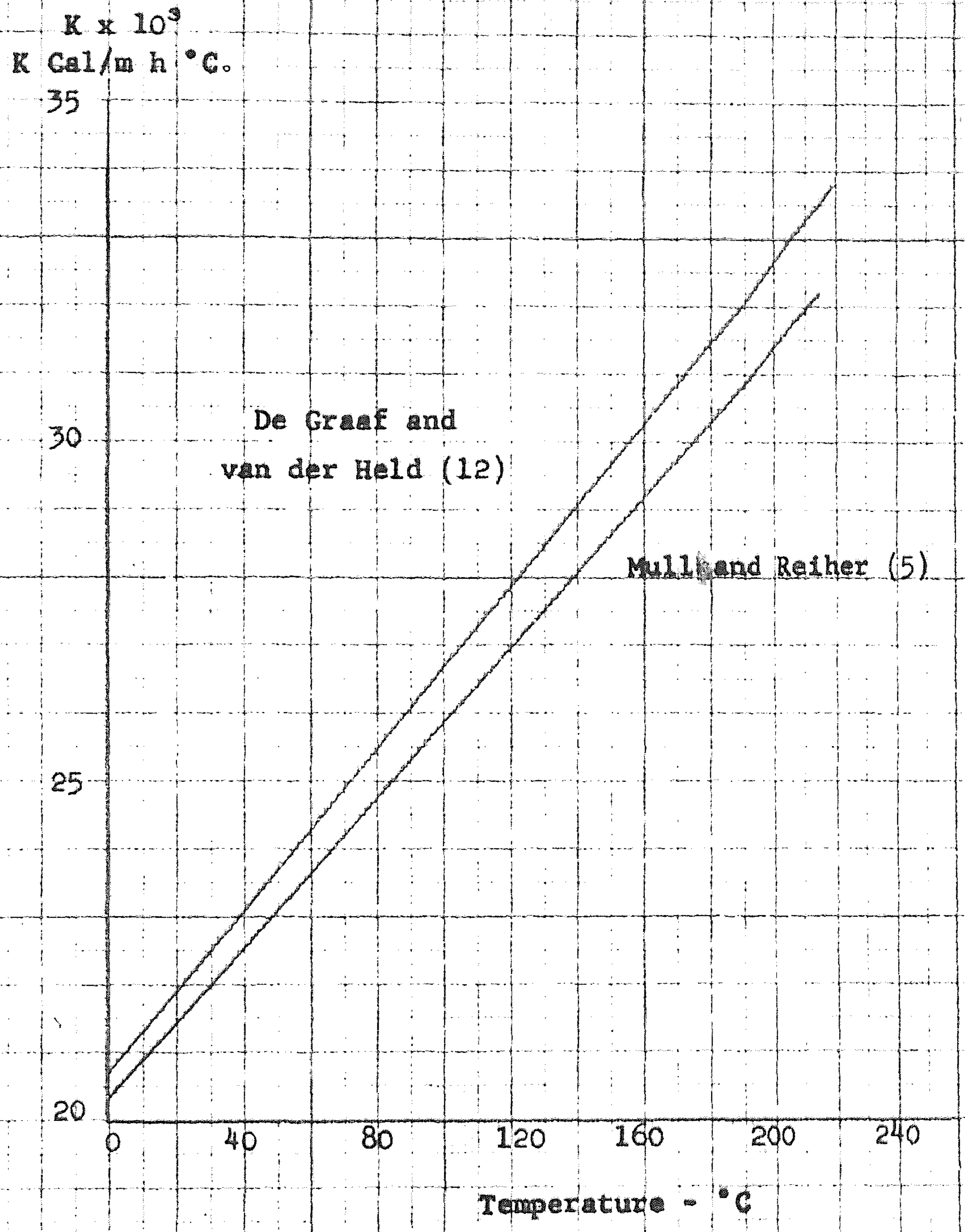


FIG. 2

Thermal Conductivity of Air vs Temperature
Used by Sources of Data



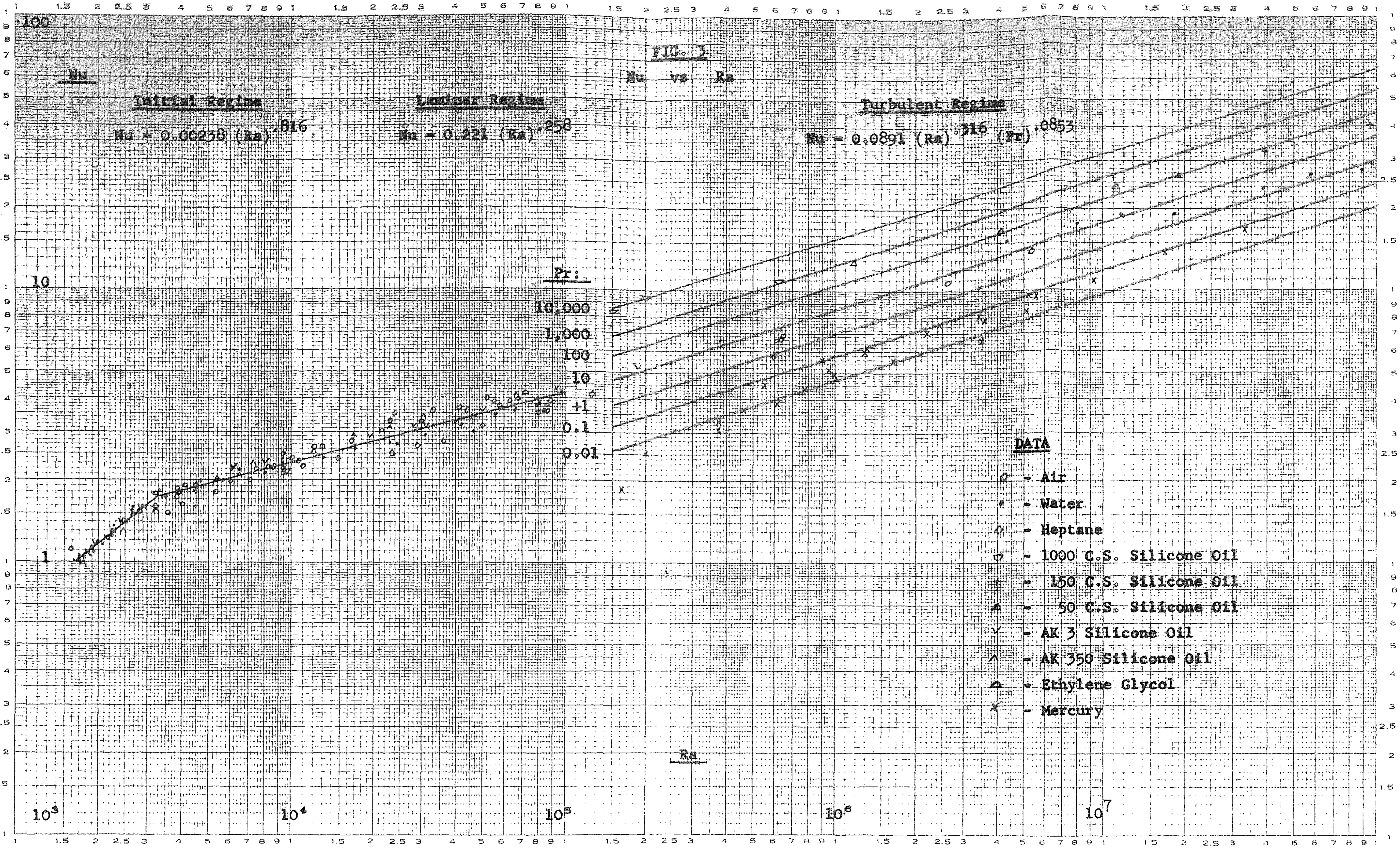


FIG. 3

Nu vs Ra

Nu
Ra
Initial Regime
Nu = 0.00238 (Ra)^.816
Laminar Regime
Nu = 0.221 (Ra)^.258
Turbulent Regime
Nu = 0.0891 (Ra)^.316 (Pr)^.0853
100
10
1
10^3
10^4
10^5
10^6
10^7
Pr:
10,000
1,000
100
10
1
0.1
0.01
DATA
- Air
- Water
- Heptane
- 1000 C.S. Silicone Oil
- 150 C.S. Silicone Oil
- 50 C.S. Silicone Oil
- AK 3 Silicone Oil
- AK 350 Silicone Oil
- Ethylene Glycol
- Mercury

FIG. 4

Nu vs Ra

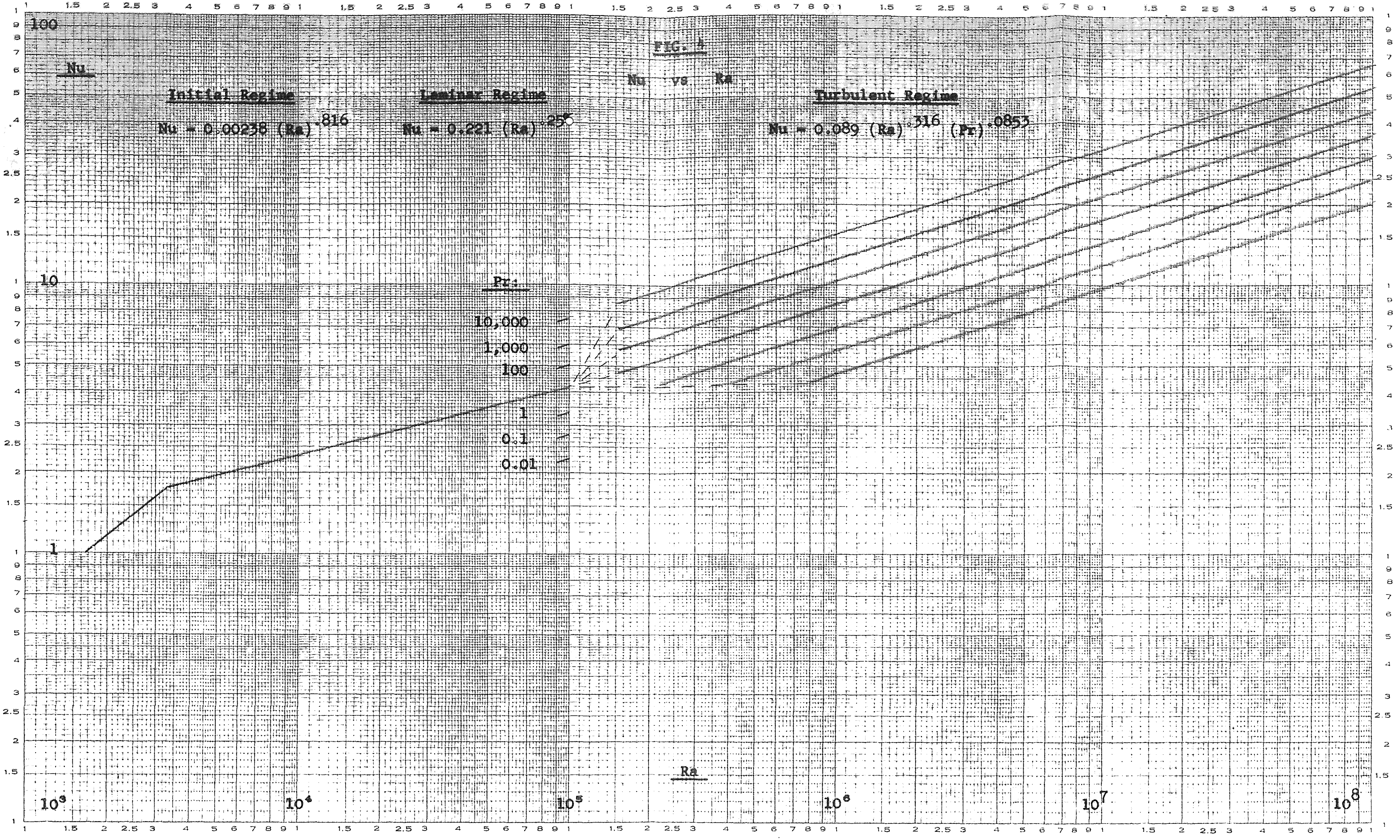
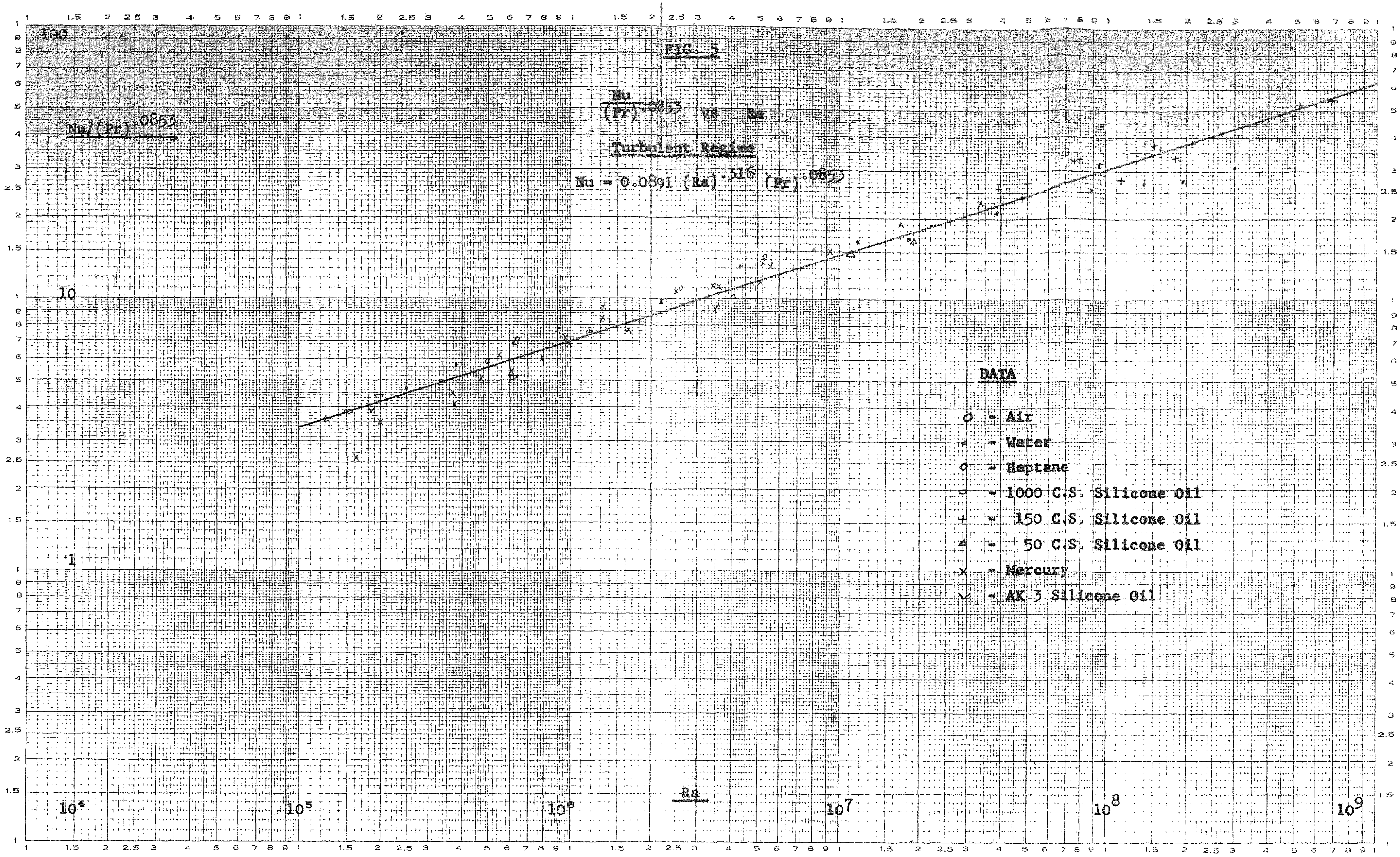

Nu
Initial Regime
$Nu = 0.00238\ (Ra)^{.816}$
Laminar Regime
$Nu = 0.221\ (Ra)^{.25}$
Turbulent Regime
$Nu = 0.089\ (Ra)^{.316}\ (Pr)^{.0853}$
Pr:
10,000
1,000
100
1
0.1
0.01
100
10
1
Ra
10^3
10^4
10^5
10^6
10^7
10^8

FIG. 5


100
10
1
Nu/(Pr).0853
Nu / (Pr).0853 vs Ra
Turbulent Regime
Nu = 0.0891 (Ra).316 (Pr).0853
DATA
O - Air
- Water
◇ - Heptane
▭ - 1000 C.S. Silicone Oil
+ - 150 C.S. Silicone Oil
△ - 50 C.S. Silicone Oil
X - Mercury
✓ - AK 3 Silicone Oil
Ra
10⁴
10⁵
10⁶
10⁷
10⁸
10⁹

APPENDIX I

DIMENSIONAL ANALYSIS OF HEAT TRANSFER BY CONDUCTION AND CONVECTION ACROSS A HORIZONTAL FLUID LAYER, BOUNDED BY INFINITE, RIGID, CONDUCTING SURFACES AND HEATED FROM BELOW.

ASSUMPTION:

IF THERE IS NO CONVECTION, HEAT WILL BE TRANSFERRED ONLY BY CONDUCTION AND THE HEAT TRANSFER RATE PER UNIT AREA CAN BE EXPRESSED BY $H_C = KT/L$.

AT STEADY-STATE CONDITIONS WHERE HEAT IS BEING TRANSFERRED BY BOTH CONDUCTION AND CONVECTION THE NET HEAT TRANSFER RATE PER UNIT AREA, H_{CC}, WILL BE LARGER THAN H_C SO THAT

$$\frac{H_{CC}}{H_C} = \frac{H_{CC}}{KST/L} = \text{A FUNCTION OF } T, L, \text{ AND THE FLUID CONSTANTS.}$$

THE FOLLOWING WILL BE THE BASIC UNITS:

TIME	-	T	LENGTH	-	L	HEAT QUANTITY	-	Q
MASS	-	M	TEMPERATURE	-	t			

THEREFORE THE PHYSICAL CONSTANTS AND VARIABLES WILL HAVE THE FOLLOWING MEASURE FORMULAE:

K	=	THERMAL CONDUCTIVITY	Q/tLT
t	=	TEMPERATURE DIFFERENCE	t
D	=	DENSITY	M/L^3
M	=	DYNAMIC VISCOSITY	M/LT
C	=	SPECIFIC HEAT (AT CONSTANT PRESSURE)	Q/tM
A	=	COEFFICIENT OF THERMAL EXPANSION	$1/t$
G	=	ACCELERATION DUE TO GRAVITY	L/T^2
(AGt)	=	FORCE OF BUOYANCY PER UNIT MASS	L/T^2

$(AG\tau)$ represents the buoyancy force in the fluid under the influence of a temperature-induced density gradient and it is convenient to treat it as an entity having the units L/T^2.

If we postulate that

$$H_{cc} = F \text{ of } (D^a, L^b, M^c, K^d, C^e, (AG\tau)^f)$$

then the expression in parentheses must be dimensionless, that is

$$Q^0 \tau^0 M^0 L^0 T^0 \qquad (M/L^3)^a (L)^b (M/LT)^c (Q/\tau LT)^d (Q/\tau M)^e (L/T^2)^f$$

Therefore, equating the sum of the exponents of individual dimensions on the right to zero, we obtain the following indicial equations:

$$d + e = 0 \qquad -3a + b - c - d + f = 0$$

$$-d - e = 0 \qquad -c - d - 2f = 0$$

$$a + c - e = 0$$

Retaining e and f we can express the remaining indices as follows:

$$a = 2f \qquad c = e - 2f$$

$$b = 3f \qquad d = -e$$

Therefore

$$H_{cc} = F \text{ of } \left(\frac{AG\tau L^3 D^2}{M^2}\right)^f \left(\frac{MC}{K}\right)^e$$

or $Nu = F \text{ of } (G_R)^f (P_R)^e$

APPENDIX II

FOR GASES, THE THERMAL COEFFICIENT OF EXPANSION IS EQUAL TO $^1/T_o$, WHERE T_o = MEAN ABSOLUTE TEMPERATURE.

CONSIDER A CONFINED HORIZONTAL LAYER OF GAS, AT ATMOSPHERIC PRESSURE, ACROSS WHICH A TEMPERATURE GRADIENT $T_2 - T_1$ HAS BEEN APPLIED.

T_2 = ABSOLUTE TEMPERATURE OF THE HOT SURFACE
T_1 = " " " " COLD SURFACE
T_o = MEAN ABSOLUTE TEMPERATURE
= COEFFICIENT OF THERMAL EXPANSION
$\tau = T_2 - T_o = T_o - T_1 = (T_2 - T_1)/2$

ASSUMING CONSTANT PRESSURE THROUGH THE LAYER AND VALIDITY OF THE PERFECT GAS LAW, THEN

$$V_1 = V_o\,(1 + \beta\,\tau) \qquad V_2 = V_o\,(1 - \beta\,\tau)$$

$$V_1 = \frac{N\,R\,T_1}{P} \qquad V_2 = \frac{N\,R\,T_2}{P}$$

$$V_2 - V_1 = \frac{NR}{P}\,(T_2 - T_1) = \frac{V_o}{T_o}\,(T_2 - T_1)$$

$$V_2 - V_1 = V_o(1 + \beta\tau) - V_o(1 - \beta\tau) = V_o(2\beta\tau) = V_o\beta\,(T_2 - T_1)$$

THEREFORE $V_o\beta\,(T_2 - T_1) = \dfrac{V_o}{T_o}\,(T_2 - T_1)$

$$\beta = \frac{1}{T_o}$$

APPENDIX II (CONT'D)

THE EXPRESSION $V = V_0 (1 + \beta T)$ IS BASED ON THE ASSUMPTION THAT β IS A CONSTANT. THIS IS VALID ONLY OVER SMALL INCREMENTS OF TEMPERATURE. TAKING β AT THE MEAN TEMPERATURE INSTEAD OF T_1 OR T_2 IS MORE ACCURATE BECAUSE IT REDUCES THE INTERVAL OVER WHICH IS ASSUMED CONSTANT, AND BECAUSE ERRORS TEND TO CANCEL OUT.

APPENDIX III

CORRECTION OF THE DATA OF MULL AND REIHER (5)

THE DATA OF BOTH MULL AND REIHER (5) AND DE GRAAF AND VAN DER HELD (COMMUNICATION FROM THE AUTHORS) CONTAINS TABULATIONS OF THE FOLLOWING QUANTITIES: (ALL DATA FOR AIR)

GRASHOF NO. AND NUSSELT NO.;
THERMAL CONDUCTIVITY OF AIR AT DIFFERENT TEMPERATURES;
T_1 AND T_2, THE TEMPERATURE GRADIENT ACROSS THE LAYER;
L, THE LAYER THICKNESS.

DE GRAAF AND VAN DER HELD HAVE SHOWN BY RECALCULATION OF MULL AND REIHER'S DATA (13) THAT THESE AUTHORS USED $^1/273$ AS THE COEFFICIENT OF THERMAL EXPANSION FOR AIR AND THAT THEY USED DIFFERENT VALUES FOR THE CONSTANTS IN GR AND NU. A MORE ACCURATE VALUE FOR THE EXPANSION COEFFICIENT IS $^1/T_{AVG}$ (SEE APPENDIX II) WHERE TAVG IS THE MEAN ABSOLUTE TEMPERATURE IN THE LAYER.

IN ORDER TO PUT ALL THE AIR DATA ON THE SAME BASIS, THE DATA OF MULL AND REIHER WAS RECALCULATED USING $^1/T_{AVG}$ AND THE CONSTANTS (THERMAL CONDUCTIVITY, DENSITY, AND VISCOSITY) OF DE GRAAF AND VAN DER HELD, SINCE THEIR WORK WAS MUCH MORE RECENT THAN THAT OF MULL AND REIHER.

THE CORRECTIONS WERE MADE AS FOLLOWS:

NUSSELT NO.

$$Nu = \frac{H}{KT/L}$$

WHERE H = MEASURED RATE OF HEAT TRANSFER PER UNIT AREA DUE TO CONVECTION AND CONDUCTION

APPENDIX III (Cont'd)

K = thermal conductivity

T = temperature difference across the layer

L = layer thickness

Fig. 2 shows the thermal conductivities vs temperature used by both Mull and Reiher and De Graaf and Van Der Held. The curves are plotted from tabulations in the respective author's data. Correction was made as follows:

$$\text{Corrected Nu} = \text{Original Nu} \times \frac{\text{K of Mull and Reiher}}{\text{K of De Graaf \& Van Der Held}}$$

Grashof No.

$$\text{Gr} = \frac{\beta G T L^3 \rho^2}{\mu^2}$$

where β = thermal coefficient of expansion

G = gravitational constant

T = temperature difference across the layer

L = layer thickness

ρ = density

μ = viscosity

Tavg = mean absolute temperature

Correction was made as follows:

$$\text{Gr of De Graaf and Van Der Held} = \frac{\beta G \rho^2}{\mu^2} = Z$$

Z was plotted against Tavg. Then, for each observation of Mull and Reiher, Z was taken from the curve at the appropriate Tavg and

$$TL^3 \text{ of Mull and Reiher} \times Z = \text{Corrected Gr}$$

APPENDIX IV

STEPWISE MULTIPLE LINEAR REGRESSION

REFERENCES: 25, 26, 27, 28, 29

MULTIPLE LINEAR REGRESSION

MULTIPLE LINEAR REGRESSION BY THE METHOD OF LEAST SQUARES CONSISTS OF FINDING THE CONSTANT AND COEFFICIENTS OF AN EQUATION OF THE FORM

$$Y = A + B_1 X_1 + B_2 X_2 + B_3 X_3 \ldots\ldots$$

WHERE Y = PREDICTED VALUE OF THE DEPENDENT VARIABLE

Y = MEASURED VALUE " " " "

X = ARBITRARY VALUES OF THE INDEPENDENT VARIABLES

X = MEASURED VALUES " " " "

A = CONSTANT = Y INTERCEPT

B = COEFFICIENTS OF THE INDEPENDENT VARIABLES

$\overline{Y}$ = AVERAGE OF MEASURED DEPENDENT VARIABLE

$\overline{X}$ = AVERAGE OF MEASURED INDEPENDENT VARIABLE

S = STANDARD DEVIATION

V = VARIANCE = S^2

N = NUMBER OF OBSERVATIONS OR PIECES OF DATA

SO THAT THE SUM OF SQUARES OF THE DEVIATIONS $(Y - Y)$ IS A MINIMUM.

FOR THE CASE OF SIMPLE REGRESSION, THE CALCULATION PROCEDURE CAN BE EXPRESSED AS FOLLOWS:

$$(Y_1 - Y_1) = Y_1 - (A + B X_1) = Y_1 - A - B X_1$$

$$\sum (Y - Y)^2 = \sum (Y - A - BX)^2$$

APPENDIX IV (CONT'D)

TO SOLVE FOR A AND B WHEN $\Sigma(Y - Y)^2$ IS A MINIMUM

$$\frac{\partial}{\partial A} \Sigma(Y - A - BX)^2 = 0$$

$$\frac{\partial}{\partial B} \Sigma(Y - A - BX)^2 = 0$$

THESE EQUATIONS REDUCE TO THE FOLLOWING INVOLVING SIMPLE SUMMATIONS ON Y, X, X^2, AND XY:

$$-\Sigma(Y) + AN + B\Sigma(X) = 0$$

$$-\Sigma(XY) + A\Sigma(X) + B\Sigma(X^2) = 0$$

THE SUMMATION EQUATIONS CAN BE SOLVED SIMULTANEOUSLY FOR A AND B, BUT IN PRACTICE THESE EQUATIONS ARE COMBINED, ELIMINATING A, SOLVED FOR B, AND A IS DETERMINED FROM

$$\bar{Y} = A + B X.$$

IN MULTIPLE REGRESSION, THERE ARE ADDITIONAL EQUATIONS FOR EACH VARIABLE ADDED.

THESE ARE SOLVED SIMULTANEOUSLY FOR B_1, B_2, B_3 ..... AND A IS FOUND FROM

$$\bar{Y} = A + B_1 \bar{X}_1 + B_2 \bar{X}_2 + B_3 \bar{X}_3 \ldots.$$

IN STEPWISE PROCEDURE THE SIMULTANEOUS EQUATIONS ARE SOLVED FOR ONE VARIABLE AT A TIME BY MATRIX ALGEBRA, WHICH SIMPLIFIES THE PROCEDURE (25). AT EACH STEP STATISTICAL INFORMATION IS OBTAINED THAT PERMITS EVALUATION OF THE SIGNIFICANCE OF THE CORRELATING PARAMETERS.

APPENDIX IV (CONT'D)

STANDARD ERROR OF ESTIMATE AND CORRELATION COEFFICIENT

THE BASIC APPROACH OF LINEAR REGRESSION MAY BE DESCRIBED AS FOLLOWS:

IF THERE IS NO CORRELATION BETWEEN THE DEPENDENT AND AN INDEPENDENT VARIABLE, THEN THE BEST ESTIMATE OF Y IS $\bar{Y}$ AND THE PROBABLE ERROR OF ESTIMATE IS S_Y, THE STANDARD DEVIATION. IF THERE IS CORRELATION BETWEEN Y AND X, THEN THE REGRESSION CAN BE REGARDED AS ACCOUNTING FOR SOME OF THE VARIANCE (EQUAL TO S_Y^2) IN Y. THE RESIDUAL VARIANCE WHICH THE REGRESSION DOES NOT ACCOUNT FOR IS REGARDED AS ERROR OR MAY BE DUE TO OTHER PARAMETERS, SUCH AS PR IN THE TURBULENT REGIME.

THE MEASURE OF THE RESIDUAL ERROR VARIANCE IS

$$\frac{(Y - Y)^2}{DF} = V = \text{MEAN SQUARE OF RESIDUALS}$$

WHERE DF = DEGREES OF FREEDOM

= N - NO. OF CONSTANTS IN THE REGRESSION EQUATION.

$\sqrt{V} = S_Y$, THE STANDARD DEVIATION OF THE RESIDUALS OR STANDARD ERROR OF ESTIMATE.

THEREFORE,

TOTAL VARIANCE IN Y - ERROR VARIANCE = VARIANCE CONTRIBUTION OF X.

IF ALL THE EXPERIMENTAL DATA WERE TO FALL ON THE REGRESSION LINE, THEN ALL OF THE VARIANCE IN Y WOULD BE ACCOUNTED FOR BY THE REGRESSION OF Y UPON X AND THE RATIO

$$\frac{S_Y}{S_X} = \frac{V_Y}{V_X} = 1$$

APPENDIX IV (CONT'D)

WHERE S_X AND V_X REFER TO THE Y VARIANCE CONTRIBUTIONS OF X.

$\frac{S_X}{S_Y}$ IS THE COEFFICIENT OF MULTIPLE CORRELATION. IT WILL VARY BETWEEN 0 AND ONE DEPENDING ON THE SIGNIFICANCE OF THE CORRELATION.

$$\frac{V_X}{V_Y} = \left(\frac{S_X}{S_Y}\right)^2 = \text{COEFFICIENT OF MULTIPLE DETERMINATION.}$$

IT IS A MORE ACCURATE ESTIMATE OF SIGNIFICANCE, SINCE VARIANCES ARE ADDITIVE.

ERROR OF THE REGRESSION

ASSUMING THAT PERFECT CORRELATION HAS NOT BEEN ACHIEVED, THE COEFFICIENTS OF THE REGRESSION ARE SUBJECT TO UNCERTAINTY AS MEASURED BY THE STANDARD ERROR OF THE COEFFICIENTS. IT CAN BE REGARDED AS THE STANDARD DEVIATION OF THE POPULATION OF REGRESSION LINES THAT CAN BE DRAWN THROUGH INDIVIDUAL EXPERIMENTAL DATA AND THE POINT $\bar{X}$, $\bar{Y}$ SINCE $\bar{X}$, $\bar{Y}$ MUST BE A POINT ON THE REGRESSION LINE. THEREFORE, THE UNCERTAINTY IN THE COEFFICIENT WILL BE ZERO AT $\bar{X}$, $\bar{Y}$ (ASSUMED TO BE NEAR THE CENTER OF THE RANGE OF DATA) AND LARGE AT EITHER END. THAT IS, THE REGRESSION LINE CAN BE VISUALIZED AS PIVOTING ABOUT $\bar{X}$, $\bar{Y}$ AS FAR AS ITS PROBABLE ERROR IS CONCERNED. THE EFFECT OF THIS PROPERTY OF REGRESSIONS IS TO INCREASE THE PROBABLE ERROR OF THE REGRESSION AT THE EXTREMES OF THE RANGE OF DATA AND TO MAKE EXTRAPOLATION BEYOND THAT RANGE HAZARDOUS. EQUATIONS FOR ESTIMATING PROBABLE ERROR AT ANY POINT ON A REGRESSION ARE GIVEN IN THE REFERENCES.

SYMBOLS

T = ABSOLUTE TEMPERATURE

T_o = ABSOLUTE MEAN TEMPERATURE OF THE LAYER

$T_1 - T_2$, T = TEMPERATURE DIFFERENCE

K = THERMAL CONDUCTIVITY

μ = VISCOSITY

β = THERMAL COEFFICIENT OF EXPANSION

ρ = DENSITY

L, D = LAYER THICKNESS

A = Area

c = SPECIFIC HEAT

Q = HEAT QUANTITY

H = RATE OF HEAT TRANSFER PER UNIT AREA

V = VOLUME

S = STANDARD DEVIATION

S_Y = STANDARD ERROR OF ESTIMATE

S_B = STANDARD ERROR OF THE COEFFICIENT

B = COEFFICIENT OF LOGARITHMIC EQUATION AND EXPONENT OF POWER LAW EQUATION

Log A = CONSTANT OF LOGARITHMIC EQUATION

Nu = Nusselt No.

Ra = Rayleigh No. = Gr x Pr

Gr = Grashof No.

Pr = Prandtl No.

A/D^2 = RATIO OF AREA TO THICKNESS SQUARED OF EXPERIMENTAL APPARATUS

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