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THE PLASTIC DESIGN OF A

STEEL COFFERDAM

BY

ROBERT Q DISQUE

A THESIS SUBMITTED TO THE FACULTY OF THE DEPARTMENTS OF CIVIL ENGINEERING OF NEWARK COLLEGE OF ENGINEERING AND DREXEL INSTITUTE OF TECHNOLOGY

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## SYMBOLS AND NOTATIONS

- A Area of Section
- $\mathbb{A}_{w}$  Area of web of wide-flange section
- A<sub>f</sub> Area of flange of wide-flange section
- d Depth of wide-flange section
- E Modulus of Elasticity
- 6 Unit strain
- I Moment of inertia
- M Moment
- M<sub>D</sub> Plastic Moment
- $M_{\rm DC}$  Modified plastic moment due to axial load
- $M_{\rm DS}$  Modified plastic moment due to shear
- My Moment producing yield stress
- P Axial load
- $\mathbf{P}_{\mathbf{V}}$  Axial load producing yield stress
- S Section modulus
- J Unit stress
- Ty Yield stress
- $\phi$  Angle of curvature
- $\Phi_{\mathcal{Y}}$  Angle of curvature corresponding to yield stress
- V Shear
- Vy Shear producing yield stress
- w Thickness of web in wide-flange section
- Z Plastic Modulus

#### CHAPTER I

#### INTRODUCTION

The fundamental purpose of this thesis is to employ the plastic theory in the design of a hypothetical steel sheet pile cofferdam. Prior to undertaking this work, the author had had no previous experience with the plastic theory. Therefore, a portion of this thesis is devoted to the development of the fundamentals necessary for the actual design computations.

The theory of plastic analysis has been used as far back as the 1920's in Hungary for the structural design of apartment buildings.<sup>1</sup> It is, however, only in recent years that the theory has been utilized to a significant degree in this country. Much progress has been made by J. F. Baker at Cambridge University, England and recently Lehigh University has been conducting many large scale tests of structural members and frames.<sup>1</sup> It is primarily through these endeavors that the use of the plastic theory is being stimulated in the United States.

Very little work has been done to date in the application of the plastic theory to structures subjected to soil pressures. It seems, however, that structures such as designed in this thesis are ideally suited to be designed by the plastic theory.

1 Lynn S. Beedle, <u>Plastic Strength</u> of <u>Steel Frames</u>, A.S.C.E. Paper 764, August, 1955, p. 1. The temporary nature of a cofferdam justifies the concept of designing for an ultimate load—deflections not being a consideration. Since the soil pressures are assumed to be triangular, there is very little possibility that the pressures would be greater than those assumed. It is, therefore, logical to design for the ultimate capacity of the structure. Nevertheless, a load factor is used to insure additional safety.

A solution to the problem of sheet pile penetration is presented in this thesis. As far as it is known, this is a new solution to the problem and would seem to be a significant contribution to the application of the plastic theory.

## CHAPTER II

#### BASIC PRINCIPLES

The fundamental difference between elastic and plastic design is the portion of the stress-strain curve with which the designer is working. In elastic design, only that portion below the yield point is considered, while in plastic design the entire curve may be utilized in localized areas. The structure is accordingly designed for its ultimate strength. This does not, however, mean a less safe structure. A factor to account for uncertainties is applied to the loads so that under these loads, the stresses will not be excessive.

# Resistance of Sections

As in the elastic theory, the basic equations of equilibrium must be satisfied. It is, therefore, necessary to determine the resistance of a section after that section has been stressed beyond the elastic limit.

## Moment

It is convenient to assume that the stress-strain curve is horizontal beyond the yield point. See Figure 1. In many materials this is close to actuality and any deviation is on the side of safety. The stress distribution of a section of a symmetrical beam under pure bending is shown in Figure 2 for various stages of bending.





Idealized Stress-Strain Curve









# Figure 3.

Integration Diagram for Plastic Modulus.

It is now necessary to determine the resisting moment capacity of the section under condition (c), Figure 2. In elastic design (condition (a)), this capacity is  $\sigma \frac{T}{c}$  or  $\sigma S$ . In the plastic condition (c) a similar relationship develops. For a rectangular section, Figure 3,  $M = 2 \int_{0}^{c} by dy$ .

Unlike the elastic theory, all the stress is in the plastic range and does not vary with "y". Therefore, we can easily integrate:  $M = 2 \int_{0}^{C} \frac{bc^{2}}{z^{2}} = \sigma bc^{2}$ .

The expression  $bc^2$  is called the plastic modulus (Z) and is seen to be: twice the static moment about the neutral axis of the half sectional area for a symmetrical section. Since both section modulus and plastic modulus are a function of the shape of the section, there exists a definite relationship between them. The ratio: of these moduli is called the "shape factor" and for most wide-flange sections, the approximate formula can be used: Z = 1.14S.

# Moment and Axial Load

The ability of a section to resist moment plastically is modified under the influence of an axial load. It is, therefore, necessary to determine a relationship between the axial load and the modified plastic modulus. In the case of wide-flange sections this relationship is different when the neutral axis is in the flange

2. Lynn S. Beedle, Bruno Thurlmann and Robert L. Ketter, <u>Plastic Design in Structural Steel</u>, Lecture Notes, Summer Course, Sept., 1955 (Bethlehem, Penna., Lehigh University, 1957), p. 2.13. from when it is in the web. Appendix A includes the derivation of the equations and the corresponding curves are shown in Figure 4. <u>Shear</u>

The problem of shear is handled in a similar manner to that of an axial load, i.e., the plastic moment capacity of a section is modified by the influence of shear. It is reasonable, therefore, to expect to find a relationship of  $\frac{M_{P5}}{M_P}$  to  $\frac{V}{V_g}$ . Since moment is dimensionally equal to shear times a distance, a hypothetical moment arm is used instead of "V" and the thickness of the web (shear resisting capacity of a section) for "Vy".

It is assumed that yielding of ductile materials is caused by shearing stress. Since the material has been brought into the plastic range by the applied moment it cannot, therefore, develop additional shearing stresses to resist the shearing forces. All the shearing resistance of a section must be supplied by that portion still in an elastic condition. The remaining plastic portion, then, is available for plastic moment resistance. The equations used in the solution of this problem are presented in Appendix B and the resulting curves are shown in Figure 5.

#### Column Buckling

The equations and curves for determining the resistance to column buckling are based on the postulate that failure occurs when the rate of change of internal resisting moment with respect to curvature.is equal to the rate of change of external moment with respect to curvature.



Figure 4. Moment and Axial Load Curves.





Figure 5. Moment and Shear Curves.

When the loading is such as to produce the above condition before any stresses have reached the plastic range the column will buckle according to the Euler formula.

However, it is possible under certain loading conditions for portions of the column to be stressed in the plastic range at the instant of buckling. It is this condition that must be investigated in the plastic theory. The approach given in Appendix C is based on the work of Ketter<sup>3</sup> but is somewhat more direct. In addition, general equations have been derived for wide-flange shapes. Curves for the solution of this problem are in Figure 6.

#### External Moments and Shears

In the plastic theory the value of moments are of interest primarily at the point where a particular section has exceeded the elastic limit. When this occurs (assuming a horizontal stress-strain curve above the elastic limit) the member will rotate about that point with the value of the now plastic moment remaining constant. Other sections in the structure will in turn become plastic until the structure has acquired enough "hinges" to become unstable. At this point the structure has become a "mechanism" and is considered to have failed. This concept can be illustrated in a fixed-end beam shown in Figure 7.

3 R. L. Ketter, <u>Stability of Beam Columns Above the Elastic</u> <u>Limit</u>, A.S.C.E. Separate 692, May, 1955.



Figure G. Column Buckling Curves.



(a)(b)Elastic ConditionCollapse

Figure 7. Plastic Failure of a Fixed-end Beam.

In the elastic condition  $M_A$  and  $M_C$  equal  $\frac{WL}{12}^2$  and  $M_B$ equals  $\frac{WL}{24}^2$ . As the load, w, is increased to a certain value, points A and B will become plastic but will not exceed the plastic moment capacity of the section. At this point the beam acts as a simply supported structure and can support additional load until  $M_B$ becomes plastic. The structure now has three plastic hinges and is no longer stable. See Figure 4(b).

As can be seen from the above example, the problem is to determine the points on a structure which will become plastic and cause a mechanism to form under minimum loading. By using the fundamental principle that external work equals internal work, equations relating the plastic moment to the load can be written. Trial mechanisms are drawn and the work equations are applied to each configuration. The mechanism producing the lowest value of load as a function of the plastic moment is the correct one. A moment diagram is then drawn and if the correct mechanism has not been overlooked there will be no moments above the plastic moment. This last operation is known as a "plasticity check". An example of this method is shown in Appendix D.

# Deflections

The computation of deflections in plastic design is for the purpose of determining the maximum deflection prior to the collapse of the structure. Since under normal loading the stresses are in the elastic range, the deflections computed in the plastic theory are larger than those that would normally occur. The magnitude of the computed delections, however, are usually smaller than would result by assuming a simple beam between supports.<sup>4</sup>

Deflection computations are made by the slope-deflection equations and are computed at the point of failure, or just before the last plastic hinge has developed and the structure has turned into a mechanism.

The application of slope-deflection equations depend on the continuity of the beam. Since in plastic design the joints are assumed to rotate freely after the plastic moment has been reached, continuity no longer exists. If the beam under investigation has only one plastic moment the equation is applied to that point. Deflection, therefore, is determined immediately prior to that hinge developing. If, however, the beam has two or more plastic

4 Lynn S. Beedle, Bruno Thurlmann and Robert L. Ketter, <u>Plastic Design in Structural Steel</u>, Lecture Notes, Summer Course, Sept., 1955 (Betblehem, Penna., Lehigh University, 1957), p. 8.1. hinges, it is necessary to apply the equation to each hinge, the largest deflection computed being the correct one. A sample problem is shown in Appendix E.

# Load Factor

Since the theory of plastic design considers the behavior of a structure after the elastic limit of the material is reached, it is necessary to apply a factor to the loads.

As shown in Chapter II, a wide-flange section can resist a moment 1.14 times that which will produce yield in the outer fibers before becoming completely plastic. If the yield stress is assumed to be 33,000 psi and the working stress, 20,000 psi, the ration between the two is 1.65. Therefore, the factor 1.88 (1.14 times 1.65) must be applied to the loads to obtain the same load factor of safety against ultimate strength that the simple beam now has when designed according to the A.I.S.C. Specification.<sup>5</sup>

5 Lynn S. Beedle, Bruno Thurlmann and Robert L. Ketter, <u>Plastic Design in Structural Steel</u>, Lecture Notes, Summer Course, Sept., 1955 (Lehigh University, Bethlehem, Penna.) p. 11.2.

## CHAPTER III

## DISTRIBUTED LOADS AND SOIL PRESSURES

It is often convenient in the plastic theory to replace a distributed load with equivalent concentrated loads. As in the elastic theory, this method of handling distributed loads is always on the conservative side and is often used in the plastic theory.

In order to design with more economy, there is worked out in appendix F equations for the determination of plastic moments for trapazoidal loading. The resulting curve is plotted in Figure 8. With this aid it is possible to design with rapidity and ease for trapazoidal, triangular and rectangular loading.

An adaptation of passive soil pressure to the plastic theory is presented in Appendix G. The basic difference between this approach and the universally accepted theories used in elastic design, is the inclusion of the effect of lateral movement on the passive pressures developed. Based on Terzaghi<sup>6</sup> it is assumed that passive soil pressure is a function of lateral displacement of the soil as well as the depth. This is, of course, close to reality in a cohesionless soil and is readily adaptable to the plastic theory.

6 K. Terzaghi, <u>Theoretical Soil Mechanics</u> (New York: John Wiley & Sons, Inc., 1944), p. 346





Figure 8. Hinge Location for Trapazoidal Loading.

Because of mathematical complications, the concept of passive pressure as a function of displacement is not considered in the usual methods of determining sheet pile penetration. The method presented here more closely follows the physical behavior of the soil and it is considered to be superior to the procedures used in the elastic theory.

There are two possible failure mechanisms for an anchored sheet pile retaining wall. The governing mechanism is determined by the proximity of the lowest support to the excavation surface. For the case where this support is relatively near the excavation, the piling will fail by a plastic negative moment at the support as shown in Figure 9(a). The second mechanism is produced by the development of a positive moment at a point below the support and by a negative moment at the support. These two mechanisms are illustrated in Figure 9.



Figure 9. Sheet Pile Failure Mechanisms

Case (b) in Figure 9 involves the simultaneous solution of five extremely complicated algebraic equations which are derived in Appendix G. Because of their complexity no solution is presented in this thesis.

Case (a) in Figure 9 is the more usual method of failure and a complete solution is presented in Appendix G. It is interesting to note that the necessary penetration is a function only of the distance from the lowest support to the excavation surface and is in all cases 55% of this distance.

## CHAPTER IV

## SHEET PILE COFFERDAM

## Introduction

The structure designed in this section is a hypothetical steel sheet pile cofferdam. The overall dimentions were chosen to be 40 feet long, 13 feet wide and 48 feet deep. The soil was assumed to be a drained, cohesionless sand. The plastic design computations resulted in the selection of MP115 steel sheet piling and four waler frames composed of wide-flange shapes.

A problem similar to sheet pile penetration as described in Chapter III exists if each successive excavation is carried below the design elevation of the waler frame prior to the installation of the frame itself. In this thesis it is assumed that each waler frame will be installed when excavation has proceeded not more than four feet below the lowest waler. When the excavation has been carried one foot below the design elevation, the waler is lowered and wedged into position. It is, therefore, permissible to design the sheet piling for the final position of the waler frame.



$$\frac{Soil}{\phi} = \frac{Properties}{\phi} = \frac{30^{\circ}}{\sigma}, \ \delta = 105^{\#}/Fr^{2}, \ mh = \frac{25}{B} Tons/Fr^{4}, \ K_{A} = ton^{2}(45^{\circ} + \frac{\phi}{2}) \\ K_{A} = ton^{2}(45^{\circ} + \frac{\phi}{2}) \\ K_{A} = 0.334 \\ w' = 0.334 \\ 105 = 35.1^{\#}/Fr^{3}. \\ \frac{Load}{Factor} \\ Load \ Factor = \frac{O_{5}}{O_{5}} = \frac{[33,000\,Psi]}{[24,000\,Psi]} \times 1.14 = 1.57 \\ Shooting$$

Sheeting  
Use MP115  

$$A = 10.59 \text{ in.}^2$$
,  $S = 8.8 \text{ in.}^3$ ,  $I = 22.4 \text{ in.}^4$   
Compute Z and Mp  
 $I = 2Ad$ ,<sup>2</sup>  
 $d$ ,<sup>2</sup> =  $\frac{I}{ZA} = \frac{22.4}{2 \times 10.59} = 1.055$   
 $d$ , = 1.028  
 $Z = Ad$ , = 10.59 × 1.028 = 10.9  
Modular length of each sheet - 19.63 in.  
 $M_p = \frac{33.0 \times 10.9 \times 12}{19.63 \times 12} = 18.3^{1-k}/\text{Fr.}$ 

<u>5011 Pressure</u> w = w' × 1.57 w = 35.1 × 1.57 = 55.2 #/FT.<sup>3</sup>

Waler Location See Chapter III and Appendix F for formula and curve. 1st Waler Cantilevered beam 55.2 × L × = × = L = Mp = 18,300 1-# L3= 1,000 L = 10.0' Elev. -10.0 2nd Waler L=12.5', h=10.0',  $\frac{h}{L}=0.8$ ,  $\frac{a}{L}=0.582$ , a=7.28 $3hal = 3 \times 10 \times 7.28 \times 12.5 = 2,730$  $3al^{2} = 3 \times 7.28 \times 12.5^{2} = 3,410$ 

	$0, 7 \pm 0$
3ha2 = 3 × 10 × 7.282	= 1,590
$a^{-3} = 7.28^{-3}$	= 385
204 - 2×385×7.28	= 449
L 12.5	2,420

6,140 - 2,420 = 3,716

 $M_{P} = \frac{55.2}{12} \left( 3716 \right) = 17,100^{-4} \left( 18,300^{-41} \right)$ 

Elev. -22.5

$$\frac{3rd Waler}{L=11.0, h=22.5, \frac{h}{L}=2.04, \frac{a}{L}=0.556, a=6.12}$$

$$\frac{3haL=3 \times 22.5 \times 6.12 \times 11.0 = 4,550}{3aL^{2}=3 \times 6.12 \times 11.0^{2}} = \frac{2,220}{6,770}$$

$$\frac{3ha^{2}=3 \times 22.5 \times 6.12^{2}}{G=6.12^{3}} = 2.530$$

$$\frac{a^{3}}{L} = \frac{2 \times 229 \times 6.12}{11.0} = \frac{255}{3.014}$$

$$\frac{4 \, th}{L} \frac{Woler}{Woler}$$

$$L = 10.0', h = 33.5, \frac{h}{L} = 3.35, \frac{a}{L} = 0.541, a = 5.41$$

$$3 haL = 3 \times 33.5 \times 5.41 \times 10.0 = 5,440$$

$$3 aL^{2} = 3 \times 5.41 \times 10^{2} = \frac{1,623}{7,063}$$

$$3 ha^{2} = 3 \times 33.5 \times 5.41^{2} = 2,940$$

$$a^{3} = 5.41^{3} = 158$$

$$\frac{2a^{4}}{L} = \frac{2 \times 158 \times 5.41}{10.0} = \frac{1.71}{3,269}$$

$$Mp = \frac{55.2}{12} \left(7.063 - 3.269\right) = 17,400 \leqslant 18,300$$

$$Elev. - 43.5$$

. .

Unit Load at each waler  
Elev. -10.0  
From above  

$$55.2 \times \frac{10^2}{2} = 2,760 \frac{\#}{FT}$$
  
From below  
 $55.2 \times 10 \times \frac{12.5}{2} = 3,440$   
 $55.2 \times \frac{12.5^2}{6} = \frac{1,440}{7,640} \frac{\#}{FT}$ 

Elev. - 22.5 From above 3.440 1,440 xZ = 2,880 From below 55.2 × 22.5 × 11 = 6,830 55.2 × 11.0 × 1/3 = 1,115 14,265 #/FT. Elev. - 33.5 From above 6,830 1,115×2 2,230 From below 55.2 × 33.5 × 10 = 9,250 55.2 × 100 = 920 19,230 #/FT.

$$\frac{Elev. - 43.5}{From above}$$

$$\begin{array}{r} 9,250\\ 920 \times 2 = 1,840\\ From below\\ 5ee \ Chapter \ III \ and \ Appendix \ G.\\ Assuming \ failure \ mechanisn \ I.\\ Q = 452(55.2)(43.5)(8) + 0.865(55.2)64\\ + 1.1\left(\frac{18,300}{8}\right).\\ Q = 14,280 \ \#/Fr. \end{array}$$

	50mm	ory-W	aler L	oading
Waler Elev.	-10.0	-22.5	-33.5	-43.5
Load- K/Fr.	7.6	14.3	19.2	25.4

Compute Mp for beam with rectangular loading. E.W.=  $2\int_{0}^{\frac{4}{2}} w \phi x dx = w \phi \frac{L^{2}}{4}$ 1. W. = Mp. 40 E.W. = 1.W.  $w\phi \frac{l^2}{4} = 4\phi Mp.$  $MP = \frac{wL^2}{16}.$ 

Mp for loft and 13 ft. span at each elev. loft span

 $M_{P} = \frac{wL^{2}}{16} = \frac{w.10^{2}}{16} = 6.25 w$ 

13 ft. span

 $M_{p} = \frac{w L^{2}}{16} = \frac{w \cdot 13^{2}}{16} = 10.55w$ 

Summary Mr (external)

Span Elev.	-10.0	-22.5	-33.5	- 43.5
10' (EW)	47.5 1-*	89.5 -*	120 '-*	159'-*
13' (NS)	80.0'-*	151 -*	202 '-*	268'-*

Somma	y-14/011.	iems u	10 10100
Member	Mp	P	V
Elev10.0	:		
10'. EW	47.5	49.5	38.0
13'- NS	80.0	38.0	49.5
Elev22.5			
10'- EW	89.5	93.0	71.5
13'- N5	151.0	71.5	93.0
Elev33.5			
10'- EW	120	120	96.0
13'-N5	202	96.0	120.0
Elev43.5			
10' EW	159	165	127
13'-N5	268	127	165
	1	1	1

Summary- Moments and Forces

Compute members for each waler  
Waler- Elev. -10.0 - EW beam  

$$5_{trial} = \frac{47.5 \times 12}{1.14 \times 33.0} = 15.2 \text{ in}^{3}$$
  
Try 8 WE24  
A = 7.06, AF = 2 × 6.5 × 0.398 = 5.18  
Aw = 7.06 - 5.18 = 1.88  
 $\frac{AF}{Aw} = 2.75$   
Axial Load  
 $P_{g} = 7.06 \times 33.0 = 233^{K}$   
 $\frac{P}{R_{g}} = \frac{49.5}{233} = 0.212$   
From axial load diagram  
 $\frac{MR}{Mp} = 0.92$   
Shear  
 $2a = 5.0', a = 2.5, dw = 7.93 - 2(0.398) = 7.13$   
 $\frac{a}{dw} = \frac{2.5 \times 12}{7.13} = 4.2$   
From shear diagram  
 $\frac{MPS}{Mp} = 0.95$   
 $S_{reg'd} = \frac{15.2}{(0.92)(0.95)} = 17.4 \le 20.8$   
Use 8WF24

$$Valer - Elev. -10.0 - N5 beam$$

$$S_{Tent} = \frac{80 \times 12}{1.14 \times 33} = 25.6$$

$$Try \ 8WF31$$

$$A = 9.12, \ A_{F} = 2 \times 8.0 \times 0.433 = 4.94$$

$$A_{W} = 9.12 - 4.94 = 4.18$$

$$\frac{A_{F}}{A_{W}} = 1.18$$

$$A \times ial \ Load$$

$$\frac{P}{P_{y}} = \frac{38}{9.12 \times 33} = 0.126$$

$$\frac{M_{PC}}{M_{P}} = 0.98$$

$$Shear$$

$$2a = 6.5, \ a = 3.25, \ d_{W} = 8.0 - 2(.433) = 7.13$$

$$\frac{a}{d_{W}} = \frac{3.25 \times 12}{7.13} = 5.48$$

$$\frac{M_{PS}}{M_{P}} = 0.9$$

$$Sreg'd = \frac{25.6}{(.98)(.98)} = 26.7 \langle 27.4$$

Use 8WF31

Naler-Elev. -22.5 - EW beam  

$$5_{Tent.} = \frac{89.5 \times 12}{1.14 \times 33} = 28.6$$

$$Try \ 10WF 33A = 9.71 \ A_F = 2 \times 7.96 \times 0.433 = 6.99A_W = 9.71 - 6.99 = 2.72
$$\frac{A_F}{A_W} = 2.57$$
  
Axial Load  
$$\frac{P}{P_g} = \frac{93}{9.71 \times 33} = 0.290$$
  
$$\frac{M_{PC}}{M_P} = 0.83$$
  
$$\frac{M_{PC}}{M_P} = 0.83$$
  
$$\frac{A_F}{M_P} = 0.83$$
  
$$\frac{A_F}{M_P} = 0.83$$
  
$$\frac{A_F}{M_P} = 0.83$$
  
$$\frac{A_F}{M_P} = 0.93$$
  
$$\frac{M_{PS}}{M_P} = 0.93$$
  
$$\frac{S_{reg'd}}{(.83)(.93)} = 38.3 \ 35.0 \ 42.2$$$$

USE IOWE39
$$Waler - Elev. -22.5, N5 beam$$

$$5_{Tent.} = \frac{151 \times 12}{1.14 \times 33} = 48.2$$

$$Try \ 10WF \ 49$$

$$A = 14.4 \qquad A_F = 2(10.0)(.558) = 11.2$$

$$Aw = 14.4 - 11.2 = 3.2$$

$$\frac{A_F}{Aw} = \frac{11.2}{3.2} = 3.5$$

$$Axia/ \ Load$$

$$\frac{P}{Py} = \frac{71.5}{14.4 \times 33.0} = 0.15$$

$$\frac{M_{PC}}{M_{P}} = 0.95$$

$$Shear$$

$$\begin{aligned} a &= 3.25, \ d_w = 10.0 - 2 (0.558) = 8.88\\ \frac{a}{d_w} &= \frac{3.25 \times 12}{8.88} = 4.4\\ \frac{M_{PS}}{M_P} &= 0.95 \end{aligned}$$

$$5_{reg'd} = \frac{48.2}{(0.95)(0.95)} = 54.6 = 54.6$$

Use 10WF 49

Waler - Elev. -33.5, EW beam  

$$S_{Tent.} = \frac{120 \times 12}{1.14 \times 33} = 38.3$$
Try 12WF 40  

$$A = 11.77. \quad A_F = 2(8.0)(0.516) = 8.25$$

$$A_W = 11.77 - 8.25 - 3.52$$

$$\frac{A_F}{A_W} = 2.34$$

$$Axial \ Load$$

$$\frac{P}{P_g} = \frac{120}{11.77 \times 33} = 0.31$$

$$\frac{M_{PC}}{M_P} = 0.81$$

$$Shear$$

$$a = 2.5, \ d_W = 11.94 - 2(0.516) = 10.91$$

$$\frac{A}{d_W} = \frac{2.5 \times 12}{10.91} = 2.3$$

$$\frac{M_{PS}}{M_{P}} = 0.85$$

$$Sreq'd = \frac{38.3}{(0.81)(0.85)} = 55.8 \le 58.2$$

Use 12WF 45

Waler - Elev. - 33.5, NS beam  $S_{Tent.} = \frac{202 \times 12}{114 \times 33} = 64.5$ Try 12WF65 A = 19.11 AF = 2(12.0)(0.606) = 14.5 Aw = 19.11 - 14.5 = 4.6  $\frac{A_F}{A_W} = 3.15$ Axial Load  $\frac{P}{P_4} = \frac{96}{19.1 \times 33} = 0.152$  $\frac{M_{PC}}{M_{D}} = 0.95$ 5hear dw=12.12-2(0.606)=10.91  $\frac{a}{dw} = \frac{3.25 \times 12}{10.91} = 3.28$  $\frac{M_{PS}}{M_{P}} = 0.90$  $5_{reg'd} = \frac{64.5}{(0.95)(0.90)} = 75.5 \langle 88.0 \rangle$ 

Use IZWF65

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$$Waler - Elev. - 43.5, EW Span
Stant. =  $\frac{159 \times 12}{1.14 \times 33} = 50.8$   
Try 14WF53  
 $A = 15.59$   $A_F = 2(8.06)(0.658) = 10.6$   
 $A_{W} = 15.59 - 10.6 = 5.0$   
 $\frac{A_F}{A_W} = 2.12$   
 $Axial Load$   
 $\frac{P}{P_g} = \frac{1.65}{15.59 \times 33} = 0.321$   
 $\frac{M_{PC}}{M_P} = 0.8$   
 $Shear$   
 $d_W = 13.94 - 2(0.658) = 12.62$   
 $\frac{a}{d_W} = \frac{2.5 \times 12}{12.62} = 2.38$   
 $\frac{M_{PS}}{M_P} = 0.87$   
 $Sreg'd = \frac{50.8}{(0.8)(0.87)} = 73.0 < 77.8$$$

$$Waler - Elev. - 43.5, N5 Span$$

$$S_{Tent.} = \frac{268 \times 12}{1.14 \times 33} = 82.2$$

$$Try \ 14WF 68$$

$$A = 20.0 \qquad A_F = 2(10.04)(0.718) = 14.4$$

$$A_{W} = 20.0 - 14.4 = 5.6$$

$$\frac{A_F}{A_W} = 2.57$$

$$Axial \ Load$$

$$\frac{P}{R_g} = \frac{127}{20 \times 33} = 0.19$$

$$\frac{M_{PC}}{M_P} = 0.92$$

$$Shear$$

$$d_W = 14.06 - 2(0.718) = 12.62$$

$$\frac{a}{d_W} = \frac{12 \times 3.25}{12.62} = 0.31$$

$$\frac{M_{PS}}{M_P} = 0.87$$

$$5_{reg'd} = \frac{82.2}{(0.92)(0.87)} = 103 = 103$$

Use 14 W-68

Strut Design

Elev.	-10.0	-22.5	-33.5	-43.5
Load	76*	42 *	192*	254*
Member	8 VE24	8WE24	84531	BHE 31
r	1.61	1.61	2.01	2.01
A	7.06	7.06	9.12	9.12
P Allowable	97	97	77.5	77.5
Py Allowable	0.75	0.75	1.0	1.0
P Actual	0.33	0.61	0.83	0.85

7 From R.L. Ketter, <u>Stability of Beam Col</u>-<u>ums Above the Elastic Limit</u>, A.S.C.E Separate 692, May, 1955, p. 692-23, Fig. 17.



Waler Corner Reinforcing

Required web thickness =  $\frac{25}{d^2}$ Diagonal stiffener thickness =  $\frac{\sqrt{2}}{b} \left(\frac{5}{d} - \frac{wd}{z}\right)$ 

and the second state of th	and a state of the second s	and the second state of th	
-10.0	-22.5	-33.5	-43.5
8WF31	10WF49	12WF65	141468
27.4	54.6	88.0	103.0
8.0	10.0	12.12	14.06
0.788	0.340	0.390	0.418
0.857	1.09	1.2	1.05
5/8"	3/4"	13/16"	5/8"
8.0	10.0	12.0	10.04
0.402	0.53	0.578	0.616
7/16"	9/16"	5/8"	5/8
	-10.0 8WF31 27.4 8.0 0.288 0.857 5/8" 8.0 0.402 7/16"	-10.0 -22.5 8WF31 10WF49 27.4 54.6 8.0 10.0 0.288 0.340 0.857 1.09 5/8" 3/4" 8.0 10.0 0.402 0.53 7/16" 9/16"	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$



8 L.S. Beedle, B. Thurlmann, R.L. Ketter, <u>Plastic</u> <u>Design in Structural Steel</u>, Lecture Notes, Summer Course, Sept. 1955 (Bethleham, Pa., 1957).

Typical Details





### APPENDIX A MOMENT AND AXIAL LOAD

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 $\frac{P}{P_{y}} = \frac{2y_{o}}{dw(1 + \frac{A_{F}}{A_{o}})} \quad Equation \quad A-1$ 

In a similar way the ratio of the available moment with an axial load, Mpc, to the moment with an axial load can be found. If no axial load were present Mp= T- ( = AF dF + = Aw dw). From Figure A.1 the available moment with an axial force 15: Mpc= Ty ( = AF dF + + Awdw - W yo2). There fore:  $\frac{M_{PC}}{M} = I - \frac{4y_o^2}{d_w^2 (1 + \frac{2A_F d_F}{A_W d_W})} Equation A.2$ Solving for yo in Equation A.1 and sub-stituting in Equation A.2 we obtain:  $\frac{M_{PC}}{M_{P}} = I - \left[\frac{P}{P_{y}}\left(1 + \frac{A_{F}}{A_{W}}\right)\right]^{2} \frac{1}{\left(1 + 2\frac{A_{F}}{A_{W}}d_{W}\right)} Equation A.3$ Equation A.3 shows reduction of available plastic moment due to an axial load when the neutral axis is in the flange and is plotted in Figure For the case of the neutral axis in the web a similar expression can be developed:  $\frac{M_{PC}}{M_{P}} = \frac{2 \frac{\partial}{\partial w}}{1 + (1 + \frac{\partial}{\partial w}) \frac{A_{F}}{A_{W}} \left( \frac{A_{F}}{A_{W}} - \frac{P}{P_{g}} \left( 1 + \frac{A_{F}}{A_{W}} \right) - 1 \right)}.$ Equation A.4

#### APPENDIX B SHEAR

The fundamental relationship between moment and shear is M=Vd where "d" is the distance along the beam to a point of contraflexture and "V" is the shear at that point. In the case of a uniform load an equivelant "V" and "d" is used. The moment, then, to be resisted is Vd.



Figure B.1 Stress Distribution-Section Partially Plastic

Considering Figure B.I, the available resisting moment is  $MP = \overline{\nabla y} \frac{x \cdot y \cdot y}{2}$ . Substituting Vd for MPV we get: 3

 $y_{o}^{2} = -\frac{3}{\sigma_{y}} \left( Vd - Mp \right) Equation B.1$ or  $y_{o}^{2} = -\frac{3}{\sigma_{y}} \left( Mpv - Mp \right)$ 

It is necessary now to find a relationship between  $y_0, \sigma_y$  and V. In an elemen-tary section,  $\sigma_x = \sigma_y$ . Therefore, in the elastic range  $\sigma_x = \sigma_y \frac{y}{y_0}$ . Differentiating with respect to x,

 $\frac{\partial \sigma_{x}}{\partial r} = \sigma_{y} \left[ - \frac{y_{o}' y}{y_{o}^{2}} \right].$  $Since \frac{\delta T}{\delta y} = -\frac{\delta \sigma x}{\delta \tau},$  $\frac{\delta Z}{\delta y} = \frac{\nabla y}{y_0} \frac{y_0}{y_0} \cdot \frac{$ 

Integrating between y and yo gives  $\mathcal{T} = \frac{\sigma_y}{2} \cdot \frac{g_0}{\left(\frac{y}{y_0}\right)^2} - \frac{g_0}{1}$  Equation B.2

Differentiating Equation B.1  $2y_0y_0' = -\frac{3}{\sigma_y w} V \quad (d \text{ and } Mp \text{ are } f(x))$   $y_0 = \frac{3}{2w y_0} \cdot \frac{V}{\sigma_y}$ Substituting in Equation B.2,  $\mathcal{T} = \frac{3}{4} \cdot \frac{V}{w y_0} \left[ 1 - \left(\frac{y}{y_0}\right)^2 \right]$ .

Assuming that  $T_{max} = \frac{\sigma_y}{\sigma_y}^{9,10}$  and exists in the center of the section  $\sqrt{3}$  where  $\frac{y}{y} = 0$ ,  $y_0 = \frac{3\sqrt{3}}{4} \frac{\sqrt{3}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{$ Substituting this in Equation B.1  $0.563 \frac{V^2}{w \sigma_y} = M_p \left( 1 - \frac{Vd}{M_p} \right) Equation B.3$ The plastic moment (Mp) in Equation B.3 is the available moment if there were no shear force present and is: Mp= AFdF + Awdw J Jg Substituting this value in Equation B.3 and using Mev for Vd gives:  $0.563 \left(\frac{M_{PV}}{M_{P}}\right)^{2} \left(\frac{d_{W}}{2d}\right)^{2} \left(1+2\frac{A_{F}d_{F}}{A_{W}d_{W}}\right)^{2} + \frac{M_{PV}}{M_{P}} - 1 = 0$ 

9 G. Murphy, <u>Advanced Mechanics</u> of <u>Materials</u> (New York: McGraw-HILL Book Company, Inc., 1946), p. 76. 10 F. B. Seely, <u>Resistance of Materials</u> (New York: John Wiley & Sons, Inc., 1947), p. 273.

## APPENDIX C COLUMN BUCKLING



and  $\Delta y_0 = \Delta \phi_0 \frac{L^2}{\pi r_2}$ .  $\Delta M = P \Delta y_0 = P \Delta \phi_0 \frac{L^2}{TT^2}$  $\frac{\Delta M}{\Delta \phi_0} = \frac{PL^2}{\pi^2} \quad Equation \quad C.1$ now:  $\phi_y = \frac{My}{FT}$  $\frac{\phi_{y}}{M_{y}} = \frac{1}{ET} \quad Equation \ C.2$ Multiplying Equation C.I by Equation C.Z.  $\frac{\Delta M}{\Delta \phi_{a}} \cdot \frac{\phi_{y}}{M_{y}} = \frac{PL^{2}}{\pi^{2} FT} Equation C.3$ also  $P_y = \sigma_y A$ ,  $\frac{\sigma_y A}{P_y} = 1$ . Multiplying the right hand side of Equation C.3 by the above:  $\frac{\Delta M}{\Delta \phi_0} \cdot \frac{\phi_y}{M_y} = \frac{P_1^2}{\pi^2 E_1^2} \cdot \frac{\nabla y A}{P_4} \,.$  $\frac{\Delta M}{\Delta \phi_0} \cdot \frac{\phi_y}{M_y} = \frac{P}{P_y} \frac{\nabla y}{\pi^2 F} \left(\frac{L}{T}\right)^2$  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\phi_{0}}{D}\right)} = \frac{P}{P_{y}} \frac{\sigma_{y}}{\pi^{2}E} \left(\frac{L}{T}\right)^{2} Equation C.4$ 

Equation C.4 represents the change in moment with respect to curvature for the external moments. The use of My and by in the left hand side of the equation are for convenience when considering the internal equations.

The derivation of internal moment equations for wide-flange shapes follows. The objective in this analysis is to relate the internal moment resisting capacity of a section to the curvature. By equating the first derivative of this equation to Equation C.4 the ratio  $\frac{4}{50}$  is obtained. Substituting this ratio in the original internal moment equation,  $\frac{M}{My}$  is obtained. We then will be able to plot  $\frac{M}{My}$  versus  $\frac{E}{Fy}$  with various values of  $\frac{L}{F}$ .



Figure C.2 shows the deformation and stress distribution of a wide-flange beam under moment with the plastic condition in the flange only.  $M = Z\left(\frac{\phi}{\phi_y}\sigma_y\right) - \frac{W_F}{Z}\sigma_y\left(\frac{\phi}{\phi_y} - 1\right)\frac{d(\phi - \phi_y)}{Z\phi}$  $\left[d-\frac{2}{3}d\left(\frac{\phi-\phi_y}{z\phi}\right)\right].$ Dividing the above equation by My= Zoy and reducing,  $\frac{M}{M_{y}} = \frac{\phi}{\phi_{y}} - \frac{W_{F} d^{2} \left(\frac{\phi}{\phi_{y}}\right)^{2}}{6Z} \left(\frac{\phi}{\phi_{y}}\right)^{2} - 3 \left(\frac{\phi}{\phi_{y}}\right)^{-3} \left(\frac{\phi}{\phi_{y}}\right)^{-3}$ Equation C.5  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\Phi}{d}\right)} = 1 - \frac{WFd^2}{6Z} \left[ 1 - 4\left(\frac{\Phi}{\Phi y}\right)^{-3} + 9\left(\frac{\Phi}{\Phi y}\right)^{-47} \right].$ 

Equation C.G



Figure C.3 Stress and Strain Distribution Plastic Portion in Flange and Web

Figure C.3 shows the deformation and stress distribution of a wide-flange beam under moment with plastic condition in the web.

 $M = \overline{vy} A_F \left( d - T_F \right) + \overline{vy} T_W \left( \frac{d_W}{z} - \frac{d_y}{\varphi} \cdot \frac{d}{z} \right) \left( \frac{d_W}{z} - \frac{d_W}{\varphi} \cdot \frac{d}{z} \right) \left( \frac{d_W}{z} - \frac{d_W}{\varphi} \cdot \frac{d}{z} \right) \left( \frac{d_W}{\varphi} - \frac{d_W}{\varphi} \cdot \frac{d_W}{\varphi} \right) \left( \frac{d_W}{\varphi} - \frac{d_W}{\varphi} \right) \left( \frac{d_W}{\varphi}$  $\left( d_{W} - \frac{\phi_{y}}{\phi} \cdot \frac{\partial}{\partial} \right) + \frac{\sigma_{y}}{2} T_{W} \cdot \frac{\phi_{y}}{\phi} \cdot \frac{\partial}{\partial} \cdot \frac{\partial}{\partial} \cdot \frac{\partial}{\partial} \cdot \frac{\partial}{\partial} \cdot \frac{\partial}{\partial}$ 

Dividing the above equation by M=Zoy and reducing,

 $\frac{M}{M_{u}} = \frac{AF}{Z} \left( d - T_{F} \right) + \frac{T_{w}}{Z} \left( \frac{dwd}{Z} \left( \frac{\phi}{\phi_{y}} \right)^{-} - \frac{d^{2}}{1Z} \left( \frac{\phi}{\phi_{y}} \right)^{-} \right).$ Equation C.T





Figure C.4 Stress and Strain Distribution Entire Section in Elastic Range

Figure C.4 shows the deformation and stress distribution of a wide-flange beam under moment with the entire section in the elastic range.

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 $M = \sigma Z = \frac{\phi}{\phi_y} \sigma_y Z; M_y = \sigma_y Z$  $\frac{M}{M_y} = \frac{\phi}{\phi_y} \quad Equation \ C.9$  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\Phi}{\Phi_{y}}\right)} = 1 \quad Equation \ C.10$ 

We now have three pairs of equations relating moment and curvature. We must now add the effect of an axial load.

$$P_{y} = \overline{\sigma_{y}} A, \quad P = \overline{\sigma_{A}}$$

$$\frac{P}{P_{y}} = \frac{\overline{\sigma_{y}}}{\overline{\sigma_{y}}}, \quad \overline{\sigma} = \frac{P\overline{\sigma_{y}}}{P_{y}}$$

Multiplying both sides of the above equations by Z,

Equations C.5, C.7, and C.9 are modified by subtracting  $\frac{1}{p_y}$  from the right hand side. A summary of the moment-curvature equations follows.

SUMMARY OF M-& EQUATIONS Internal Equations  $0 < \frac{\phi}{\phi_{4}} < 1$  $\frac{M}{M_{4}} = \frac{\phi}{\phi_{4}} - \frac{P}{P_{4}}$  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\phi}{\phi_{y}}\right)} = 1$  $1 < \frac{\phi}{\phi_y} < \frac{\partial}{\partial w}$  $\frac{M}{My} = \frac{\phi}{\phi_y} - \frac{WFd^2}{6T} \left[ \frac{\phi}{\phi_y} + 2\left(\frac{\phi}{\phi_y}\right)^{-2} \left(\frac{\phi}{\phi_y}\right)^{-3} - \frac{F}{F_y} \right].$  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\phi}{m_{y}}\right)} = 1 - \frac{W_{F}d^{2}}{6Z} \left[1 - 4\left(\frac{\phi}{\varphi_{y}}\right)^{-3} + 9\left(\frac{\phi}{\varphi_{y}}\right)^{-4}\right].$  $\frac{\partial}{\partial w} \left\langle \frac{\phi}{\phi y} \right\rangle^{\infty}$  $\frac{M}{My} = \frac{AF}{Z} \left( d - TF \right) + \frac{Tw}{Z} \left[ \frac{dwd}{Z} \left( \frac{\phi}{\phi_4} \right)^{-1} - \frac{d^2}{IZ} \left( \frac{\phi}{\phi_4} \right)^{-1} \right] - \frac{P}{P_4}.$  $\frac{d\binom{M}{M_y}}{d\binom{\Phi}{D_y}} = \frac{T_w}{Z} \left[ -\frac{d_w d}{Z} \left( \frac{\Phi}{d_y} \right)^{-2} + \frac{d^2}{G} \left( \frac{\Phi}{d_y} \right)^{-3} \right].$ External Equation  $\frac{d\left(\frac{M}{My}\right)}{d\left(\frac{\Phi}{Dy}\right)} = \frac{P}{Py} \frac{\overline{Uy}}{\overline{T}^{2}E} \left(\frac{L}{\overline{T}}\right)^{2}.$ 

APPENDIX D SAMPLE PROBLEM-EXTERNAL MOMENTS



Figure D.I. Sample Problem

The various failure mechanisms are as follows:







Equate External Work to Internal Work and determine lowest value of P in terms of Mp.

Mechanism I  
E.W. = 
$$ZP \cdot \phi \cdot L$$
  
I.W. =  $M_P \phi + 2M_P \cdot 2\phi + M_P \cdot \phi = 6M_P \phi$ .  
 $ZP \phi L = 6M_P \phi$   
 $P = 3\frac{M_P}{L}$ .

Mechanism II

$$P\phi L + ZP\phi L = M_P\phi + 2M_PZ\phi + M_PZ\phi + M_P\phi.$$

$$P = \frac{8}{3} \frac{M_P}{L}$$

Mechanism III  $P\phi L = 4M_P\phi$  $P = \frac{4M_P}{L}$  The lowest value of P is Mechanism II. A plasticity check is now made by drawing the moment diagram to insure that no moments exceed the plastic moment





Moment Diagram

No moments in the above diagram exceed the plastic moment of the member.



Assume last hinge to form is "A."

$$\begin{aligned} \theta_{A} &= \theta_{A}' + \frac{\Delta}{L} + \frac{L}{2EI} \left[ M_{AB} - \frac{M_{BA}}{2} \right]^{2} \\ \theta_{A} &= 0, \quad rotation \quad has \quad not \quad begun. \\ \theta_{A}' &= 0, \quad no \quad loads \quad between \quad "A" \quad and \quad "B. \\ M_{AB} &= -M_{P} \\ M_{BA} &= -\frac{M_{P}}{2} \\ \theta_{Z} &= 0 + \frac{\Delta}{L} + \frac{L}{2EI} \left[ -M_{P} + \frac{M_{P}}{4} \right]^{2}. \end{aligned}$$

$$\Delta = \frac{3}{8} \frac{1}{EI} M_{P}.$$

Assume last hinge at "B." Even though continuity is assumed at "B" there will be rotation at that point. Therefore, it will be necessary to write equations for

beam BA and beam BC and solve simul-  
taneously.  
Beam BA  

$$\Theta_{BA} = \frac{\Delta}{L} + \frac{L}{ZEI} \left( M_{BA} - \frac{M_{AB}}{Z} \right)$$
  
 $\Theta_{BA} = \frac{\Delta}{L} + \frac{L}{ZEI} \left( -\frac{M_P}{Z} + \frac{M_P}{Z} \right)$ .  
 $\Theta_{BA} = \frac{\Delta}{L}$ .  
Beam BC  
 $\Theta_{BC} = -\frac{\Delta}{L} + \frac{L}{ZEI} \left( \frac{M_P}{Z} - \frac{ZM_P}{Z} \right)$ .  
 $\Theta_{BC} = -\frac{\Delta}{L} - \frac{L}{4EI} M_P$   
 $\Theta_{BA} = + \Theta_{BC}$   
 $\frac{\Delta}{L} = -\frac{\Delta}{L} - \frac{L}{4EI} M_P$   
 $\Delta = -\frac{L^2}{KEI} M_P$   
Assume the last hinge at "C."  
 $O = \frac{\Delta}{L} + \frac{L}{ZEI} \left( M_{CB} - \frac{M_{BC}}{Z} \right)$ .  
 $O = \frac{\Delta}{L} + \frac{L}{ZEI} \left( ZM_P - \frac{M_P}{4} \right)$ .  
 $\Delta = -\frac{7}{8} \frac{L^2}{EI} M_P$ , the largest deflection.

#### APPENDIX F DISTRIBUTED LOADS

The purpose of this section is to derive general equations for the plastic moment and the location of the plastic hinge for a beam with triangular and rectangular loading.



Figure F.1 Failure Condition with Distributed Load

External Work E.W. = wh  $\int \phi \frac{(1-\alpha)}{\alpha} \chi d\chi + w \int_{0}^{\alpha} \frac{(1-\alpha)}{\alpha} \chi^{2} d\chi$  $+wh \left| \phi(L-a) dx + w \right| \phi(L-x) x dx.$ 

 $E.W. = \frac{w\phi}{a} \left[ -3hal - La^2 - 2a^3 + 3hl^2 + 3l^3 \right].$ Internal Work • I.W. =  $M_P \left( \phi + \frac{\phi L}{a} + \phi \frac{(L-a)}{a} \right)^2$  $I.W. = 2Mp \phi \frac{L}{D}$ External Work = Internal Work  $2M_p\phi \frac{L}{a} = \frac{w\phi}{a} \left[ -3hal - la^2 - 2a^3 + 3hl^2 + 3l^3 \right]$  $M_{P} = \frac{1}{2} \left[ -\frac{3}{2}ha^{2} - a^{3} - \frac{2a^{4}}{2} + \frac{3}{2}hal + \frac{3}{2}al^{2} \right]$ Equation F.1 Differentiating with respect to "a" to determine minimum value of Mp.  $\frac{dM_{P}}{da} = \frac{w}{12} \left[ -6ha - 3a^{2} - 8a^{3} + 3hL + 3L^{2} \right] = 0$  $-6ha - 3a^2 - \frac{8a^3}{7} + 3hL + 3L^2 = 0$ Equation F.2

# APPENDIX G SHEET PILE PENETRATION



mh= Coeffecient of Horizantal Subgrade Reaction. "

Passive Force = 1/3 mh \$ L(L-h)2.  $\Xi H=0 \quad Q + \frac{1}{3} mh \, dL \left(L-h\right)^2 = qL + \frac{wL^2}{2} \cdot Equation \ G.1$  $\frac{\mathcal{E}M^{=0}}{\frac{1}{3}}mh\phi L(L-h)^{2}\left(\frac{3L+h}{4}\right) = \frac{9L^{2}}{2} + \frac{wL^{3}}{3} - \frac{Mp}{Equation} G.2$ 11 K. Terzaghi, <u>Theoretica</u> Soil <u>Mechanics</u> (New York: John Wiley & Sons, Inc., 1947)

External Work  $E.W. = q \left( \chi \phi d\chi + \left( w \chi d\chi \phi \chi - \frac{1}{2} \right) \left( m_h \chi \phi^2 (h + \chi) d\chi \right) \right)$  $E.W = \frac{q\phi L^{2}}{2} + \frac{w\phi L^{3}}{3} - \frac{mh\phi^{2} \left[\frac{h^{2}(L-h)}{2} + \frac{2h(L-h)^{3}}{3} + \frac{(L-h)^{4}}{4}\right]^{2}}{2}$ Internal Work 1.W. = Mp \$ F.W. = I.W. $M_{P}\phi = 9 \frac{\phi L^{2}}{2} + \frac{w \phi L^{3}}{3} - \frac{m h \phi^{2}}{2} \left[ \frac{h^{2}(L-h)^{2}}{2} + \frac{2h(L-h)^{3}}{3} \right]$  $+ \frac{(2-h)}{4}$ Rearranging terms,  $q_{\frac{L}{2}}^{2} + \frac{wl}{3} - Mp = \frac{mh\phi}{h\phi} \left[ \frac{h^{2}(L-h)^{2}}{h^{2}(L-h)^{2}} + \frac{2h(L-h)^{3}}{2} \right]$ +  $(L-h)^{+}$ . Equation G.3 Combining Equations G.2 and G.3,  $\frac{1}{3}mh\phi(L-h)^{2}\left(\frac{3L+h}{4}\right) = \frac{mh\phi}{2}\left(\frac{h^{2}(L-h)^{2}}{4} + \frac{2h}{3}(L-h)^{3}\right)$ +(1-1) +7. Reducing,  $\frac{L}{h} = 1.55$ h2+2hL+31-61-2h=0

Solve for Q: Rearranging Equation G.I.  $Q = qL + \frac{wL^2}{3} - \frac{1}{3} mh \phi L (L-h)^2$ . Equation G.1 Rearranging Equation G.2,  $\frac{1}{3}mh\phi(L-h)^{2} = \frac{4}{3L+h}\left[\frac{9L^{2}}{2} + \frac{\omega}{3}L^{3} - M_{P}\right].$ Solving simultaneously:  $Q = qL + \frac{wL^2}{2} - \frac{4}{3L+h} \left[ \frac{qL^2}{2} + \frac{wL^3}{3} - Mp \right].$ For h=0.645L Q=0.4529L+0.865 wh +1.1 Mp



Figure G.2 Sheet Pile Penetration-Mechanism 2

Unknowns: a, b, L, Q and ¢ Equations:

$$\begin{split} \mathcal{E}H=0 & \frac{d(M_P)}{db} = 0\\ \mathcal{E}M=0 & \frac{d(M_P)}{db} = 0\\ \mathcal{I}W. = \mathcal{E}.W. & \frac{d(M_P)}{da} = 0\\ \mathcal{P}assive \ Force = mh\phi \Big( (1+a-h-x) \times dx \,. \\ \int_{0}^{L-h} \mathcal{P}F. = \frac{mh\phi}{6} \int_{0}^{L^{3}} - 2hL^{2} + h^{2}L + 3aL^{2} - 6hLa + 3ah^{2} \\ - hL^{2} + 2h^{2}L - h^{3} \Big]. \end{split}$$

EH=0 Q + mho [13-2h1 + h1 + 3al-6hla + 3ah2  $-hL^2+2h^2L-h^3]=qL+wL^2$ . Equation G.3

Moment of Passive Force about Q. Measure =  $mh\phi/(L+a-h-x)(h+x)xdx$ Measure =  $l^2mh\phi/(L+zh)^3 + 4l^3a - 18l^2a$ 

MPASSIVE = 12 mhp/L 4-2hL3 + 4L3a-18Lah +24h2/a-10h3a+2h3/-h47.

EM=0 12 mhd/L+-2hL3+4L3a-18Lah +24h2La -10h3a+2h3L-h47=96+W13-Mp. Equation G.4

Fxternal Work  $E.W. = \phi q \int_{0}^{b} \frac{(L+a-b)}{b} \chi d\chi + \phi w \frac{(L+a-b)}{b} \int_{0}^{z} \frac{\chi^{2}}{2} d\chi$ +  $(q+wb)\phi((l+a-x)dx + \phi w(x-b)(l+a-x)dx)$  $-\frac{mh\phi}{2}\Big/(L+a-h-\chi)(L+a-h-\chi)\phi\chi\,d\chi.$ 

Internal Work  $I.W. = M_{p}\phi \frac{(L+a-b)}{b} + M_{p} \frac{(L+a)}{b}\phi$ 

Equating Internal Work to external work and solving for MP.

 $M_{P} = \frac{b}{2L+2a-b} \begin{cases} \frac{bqL}{2} - \frac{abq}{2} - \frac{abq}{2} - \frac{qb^{2}}{3} - \frac{abq}{3} - \frac$ 

- $-2\frac{wb^{2}a}{3} + \frac{wb^{3}}{L} + \frac{9L^{2}}{2} + alg + 2awbl \\ + \frac{2}{3}l^{3}w wl^{2}a \frac{mh\phi}{24} \int [l^{4} + 4l^{3}a 4l^{3}h 12l^{2}ah \\ 6a^{2}l^{2} + 3l^{2}h^{2} + 12h^{2}al + 12a^{2}hl 4h^{3}l 4h^{3}a$
- $-6a^{2}h^{2} + h^{4}]. Equation G.5$

Equation G.5 should be differentiated with respect to "a" and "b" and equated to zero. These two equations together with Equations G.3, G.4 and G.5 should then be solved simultaneously for Mp, Q and L.

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