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# GRAPHICAL CORFELATION OF PAESSURE DROP AND TEMPERATURE DROP TO FRICTION AND VELOCITY FOR ADIABATIC FLOW OF COMPRESSIBLE FLUIDS 

BY

ANDRET A. GIACOBBE

A THESTS
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#### Abstract

Problems involving adiabatio flow of compressible fluids in horizontal conduits of constant orosesection usually require tedious trial and error solutions. Although several somewht simplified method have been developed, most of these methods are not ideally suited for problems involving flow between two sections of a conduit.

The graphical solutions presented in this paper, relating pressure and temperature ratios to friction and initial mach numbers, have been developed specifically for flow between sections of a conduit. Use of the graphs requires only the knowledge of initial conditions and determination of the friction factor.

Initial velocities from $30 \%$ to $70 \%$ of the initial acoustic velocity have been presented in this paper. Graphs for initial velocities of $5 \%$ to $30 \%$ of the initial scoustic velocity have previousiy been presented by straub. (7) Combination or doth papers provides a continous range of initial velocities from $5 \%$ to $70 \%$ of the initial acoustic velooity.


## INTRODUCTION

Flow equations for compressible fluids in conduits of constant crose-section have been developed for botn 1sothermal and adiabatic conditions. For isothermal conditions both algebraic and graphicsl metnods of calculation have been developed and presented in many texts. Equations for alabatic flon have been presented by Dodge and Phompson (3) and summarized by Perry. (6) These equations, containing specific heat ratios, pressure ration, velocity ratios, hach numbers, and friction, require the use of trial and error methods for most solutions. Because of the complexity

$$
\begin{aligned}
1 L / R_{H}= & -2 \cdot 3[(K+1) / K]\left[\log \left(V_{2} / V_{1}\right)\right] \\
& +(1 / K)\left[\left(1 / H_{1}\right)^{2}+(K-1) / 2\right]\left[1-\left(V_{1} / V_{2}\right)^{2}\right] \\
P_{2} / P_{1}= & \left(V_{1} / V_{2}\right)\left(1+[(K-1) / 2]\left(M_{1}\right)^{2}\left[1-\left(V_{2} / V_{1}\right)^{2}\right]\right) \\
T_{2} / T_{1}= & \left(P_{2} / P_{1}\right)\left(V_{2} / V_{1}\right)
\end{aligned}
$$

of these equations, several investigators nave presented varlous types of graphical solutions.

Dodge and Thompson, besides presenting the flow equations, illugtrated graphically the complex relationahips between velocity, friction, and pressure drop for gases With a specific heat ratio of $K=1.32$. Tnis graph, nowever, cannot be read with any degree of accuracy, and the presentation of only one specific heat ratio makes interpolation
for other specific heat ratios impossible. A similar graph, presented by Bincer ${ }^{(1)}$ for a specific heat ratio of $K=1.4$, involves the same accuracy and interpolation ilmitations.

A graphical solution employing conditions in a stagnant reservoir has been developed by Lapple. (5) Since velocity In a stagnant reservoir is negligible, theoretical isothermal flow trougn a frictionless nozzle was employed as a reference flow rate. For flow from a stagnant reservoir into a plpeline, the Lapple chart provides a rapla metnod of calculation. For flow between two sections of a conduit, however, a lengthly trial and error calculation is required to determine conditions in the theoretical reservolr before a solution can be obtained.

A nonograph developed iy Thompson, (8) relating velocity and friction to pressure arop for isotnermal flow can be adapted for adiabatic flow. (5) Although the use of the nomograph for isothermal ilon requires very little preliminary calculation, several ratner tedious calculations are necessary for adiabatic flow.

A method of calculation, based on the themodynamic properties of tine fluid, employing the Fanno lines" is also avallable. This method, although quite accurate, requires the use of a special set of rather lengthly tables.

Because of the difficulties involved when using the above methods of calculation, straub(7) presented an accurate graphical solution similar to those illustrated in Dodge and Thompson, and Binder. Additional parametric curves relating temperature ratios to friction and initial mach numbers were included to increase tne utility of the graphs. Hach numbers from 0.050 to 0.300 were employed, to cover as wae a range of conditions as possible witnout sacrificing accuracy.

Since many flow probleas involve inlet velooities greater than $=0.300$, it ass felt that construction of similar graphs with kacn number of 0.300 to 0.700 would provide a valuable contribution to the work of Straub. This paper has therefore been prepared as an extension of Straub's work. A comulnation of botn papers provides a continous range of Haon numbers from 0.050 to 0.700 .


#### Abstract

RESULTS

The graphs presented on the following pages represent the relationship of pressure drop and temperature drop to initial maon number and friction for perfect gases flowing adiabatically through conduita of constant orosssection. With these graphs and the graphe presented by Straub ${ }^{(7)}$ it is possible, knowing the inlet conditions and the friction factor, $f$, to determine the conditions of flow at any point along the conduit for initial Mach numbers up to 0.700 .

The curve on each graph marked, $P_{c} / P_{1}$ represents the maximum possible pressure drop attainable in the condult from the given inlet conditions. At this point the velocity at the conduit outlet equals the acoustic velocity, thereby producing maximum flow conditions.






## CONCLUSLONS

The solution to many problems concerning adiabatic flow of compressible fluids in norizontal conduits of oonstant crosemection can be greatly simplified by the use of the grapnical relationsinps presented in tnis thesis. These graphs, constructed for oricinal Mach numbers of 0.300 to 0.700 , represent an extension of the work of straub. (7) By combining both aets of graphe, it is possible to solve flow proolems ith inlet velocitiew for air from a minimum of $55 \mathrm{ft} . / \mathrm{sec}$. to a maximum of $790 \mathrm{ft} . / \mathrm{sec}$. at $75^{\circ} \mathrm{F}$.

Further work toward the construction of similar graphs for other specific heat ratios between 1.1 and 1.67 would simplify and increase the accuracy of interpolation Letween grapha.

Construction of grapna witn initial mach numbers above 0.700 would have only liaited application. maximum L/D ratios, for gases suca as air, would be less than 10, requiring conduits of snort lengtins or large diameters.

## DISCUSSION

The flow equations employed in this paper, sumarized by Perry, (6) are as followe:

$$
\begin{align*}
f L / R_{1}= & -2 \cdot 3[(K+1) / K]\left[\log \left(V_{2} / V_{1}\right)\right] \\
& +(1 / K)\left[\left(c_{1} / V_{1}\right)^{2}+(K-1) / 2\right]\left[1-\left(V_{1} / V_{2}\right)^{2}\right]  \tag{1}\\
P_{2} / P_{1}= & \left(V_{1} / V_{2}\right)\left[1+((K-1) / 2)\left(V_{1}\right)^{2}\left(1-\left(V_{2} / V_{1}\right)^{2}\right)\right]  \tag{2}\\
P_{2} / T_{1}= & \left(P_{2} / P_{1}\right)\left(V_{2} / V_{1}\right) \tag{3}
\end{align*}
$$

wach number ${ }_{1} \mathrm{~V}_{1} / \mathrm{C}_{1} \quad \mathrm{R}_{\mathrm{H}}=\mathrm{D} / 4$, for olrcular conduits.

By selecting constant values of specific heat ratioa, $K$, and mach numbers, M, and augtituting selected values for $V_{2} / V_{1}$ in equations 1 and 2 , points on the parametric maon number curves were determined. The resulting values of $P_{2} / P_{1}$ multiplied by $V_{2} / V_{1}$ gave $T_{2} / T_{1}$. The selected $V_{2} / V_{1}$ values together ith the determined values of $f / D, P_{2} / P_{1}$ and $T_{2} / T_{1}$ for specific values of $K$ and $H$, are included in the appendix as Tabulated Results.

Since the work of Straub ${ }^{(7)}$ covered the range of Kach numbers from 0.050 to 0.300 , tris paper has been presented for hach numbers beginning at 0.300 . haximum hach numbers of 0.700 were considered sufficient to maintain maximum accuracy in the graphs. It was naturally neceasary to employ the same specific heat ratios as those of straub, $K=1.2,1.4$ and 1.67.

The curves of $T_{2} / T_{1}$ not only increase the utility of the graphs, but also increase the range of application. For instance, without the $T_{2} / T_{1}$ ourves it ie necessary to know inlet pressure and velocities for direct use of the graphs, In many practical problems, flow rates represent the condition to be determined. In this case knowledge of the upatrean and downstream temperatures and pressures give sufficient information to determine $P_{2} / P_{1}$ and $T_{2} / T_{1}$. These values are all that are required to determine the point on the graph necessary to give the initial velocity.

The limiting pressure ratio, represented by the curve $P_{e} / P_{1}$, indicates the point where the discharge velooity has reached the acoustic velocity. A further increase in pressure drop cannot be accomplished whout first changing inlet conditions. Limiting pressure ratio curves mere determined. from the following formula appearing in Dodge and Thompson. (3)

$$
P_{0} / P_{1}=M_{1}^{2} \sqrt{[(K-1) /(K+1)]\left[1+\left(2 /(K-1) M_{1}^{2}\right)\right]}
$$

Three assumptions were used in the development of the flow equations 1,2 , and 3 and were therefore employed in the construotion of the grapis presented in this problem. Tnese assumptions are as follows:

1. The friction factor remains constant trrouthout the conduit.
2. Veloaity distribution acrosa the conduit crosssection is uniform.
3. All gases are perfeot, requiring no correction for compressibility.

For certain design problems some correction will be necessary for these assumptions.

In problems involving large velocity changes it might be necessary to efirst solve the problem using the initial friction factor, f. Then from the downstream conditions, the final friction factor could be determined, and the solution repeated using the arithmetic average of the initial and final irietion factors.

Veloosty distribution soross conduits is disoussed in many texts. If necessary, correetions for actual frictional velocitien are casily appliea.

The compressibility correation for gases is negligible under normal pressure and temperature oonditions, but under extreme conaltions of pressure and temperature these correotions become significant,

## CONS AUCTION OF GRAPHS

The computation will be carried out to determine a point on the $M_{1}=.400$ curve for the graph of $x=1.4$. Arbitrarily selecting a velocity ratio, $V_{2} / V_{1}=2.0$ and employing the Mach number and specific hot ratio, M and $x$, as noted above, the following equations can then be solved for $1 \mathrm{~L} / \mathrm{D}$.

$$
\begin{aligned}
\mathrm{fL} / \mathrm{R}_{H}= & {[-2.3(\mathrm{~K}+1) / \mathrm{K}]\left[\log \left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)\right] } \\
& +(1 / \mathrm{K})\left[\left(1 / \mathrm{K}_{1}\right)^{2}+(\mathrm{K}-1) / 2\right]\left[1-\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{2}\right] \\
\mathrm{rL} / \mathrm{D}= & \mathrm{rL} / 4 \mathrm{R}_{\mathrm{H}} \\
\mathrm{rL} / \mathrm{R}_{\mathrm{H}}= & {[-2.3(1.4+1) / 1.4][\log 2.0] } \\
& +(1 / 1.4)\left[(1 / .40)^{2}+(1.4-1) / 2\right]\left[1-(1 / 2.0)^{2}\right] \\
\mathrm{fL} / \mathrm{R}_{\mathrm{H}}= & 2.2672 \\
\mathrm{fL} / \mathrm{D}= & 0.5668
\end{aligned}
$$

The pressure ratio is determined from the following equation.

$$
\begin{aligned}
& P_{2} / P_{1}=\left(V_{1} / V_{2}\right)\left[1+((1-1) / 2)\left(M_{1}\right)^{2}\left(1-\left(V_{2} / V_{1}\right)^{2}\right)\right] \\
& P_{2} / P_{1}=(1 / 2.0)\left[1+((1.4-1) / 2)(.400)^{2}\left(1-(2.0)^{2}\right)\right] \\
& P_{2} / P_{1}=0.4520
\end{aligned}
$$

## 11

The temperature ratio is then easily determined from the following expression.

$$
\begin{aligned}
& T_{2} / T_{1}=\left(P_{2} / P_{1}\right)\left(V_{2} / V_{1}\right) \\
& T_{2} / T_{1}=(.4520)(2.0) \\
& T_{2} / T_{1}=.90400
\end{aligned}
$$

## GAMPLE PROBLEM

To compare the several methods for nolving a problem of this type, a sample problem is presented.

## Data:

Alp enters a standard 4 inch steel line at $3000 \mathrm{ft} .3 / \mathrm{min}$. at a proseure of 14.0 psia and a temperature of $75^{\circ}$. Determine the aischarge conditions 20 feet from the inlet.
A. Kethod of Trial and Error.

$$
\begin{align*}
& P_{1}=(.0808)(14.0 / 14.7)(492 / 535)=0.070710 .14 .^{3} \\
& T_{1}=460+75=535^{\circ} \mathrm{R} \text {. } \\
& \mu_{1}=0.0178 \mathrm{ep.}(1 / 1488)=1.196 \times 10^{-5} 1 \mathrm{~b} \text {. mass/ft. sec. } \\
& \mathrm{D}=4.026 / 12=0.335 \mathrm{ft} \text {. } \\
& V_{1}=3000 / 60(144 / 12.73)=567 \mathrm{Pt} . / 800 \text {. } \\
& \text { Re }=\operatorname{DVP} / \mu=(.335)(567)(.0707) / 1.196 \times 10^{-5}=1.12 \times 10^{6} \\
& \text { From the Reynola's number curve in the Chemical } \\
& \text { Engineer's Handbook. }  \tag{6}\\
& f=0.0043 \\
& \text { For air the apecific neat ratio, } K=1.4
\end{align*}
$$

$$
\begin{aligned}
& \mathrm{C}_{1}=\sqrt{\mathrm{K}_{\mathrm{G}} \mathrm{RT}_{1} / \mathrm{m}}=\sqrt{1.4 \times 32.2 \times 1546 \times 535 / 29}=1133 \mathrm{ft} . / \mathrm{sec} . \\
& \mathrm{A}_{1}=\mathrm{V}_{1} / \mathrm{C}_{1}=567 / 1133=.500 \\
& \mathrm{IL} / \mathrm{D}=(.0043)(20) / .335=0.2565 \\
& \mathbf{I L} / \mathrm{R}_{\mathrm{H}}=4 \mathrm{IL} / \mathrm{D}=4(.2565)=1.026
\end{aligned}
$$

From the Chemical Engineer Handbook, ${ }^{(6)}$

$$
\begin{aligned}
& \mathrm{fL} / \mathrm{R}_{\mathrm{H}}= {[-2.3(\mathrm{~K}+1) / \mathrm{K}]\left[\log \left(\mathrm{v}_{2} / \mathrm{v}_{1}\right)\right] } \\
&\left((1 / \mathrm{K})\left[\left(\mathrm{c}_{1} / \mathrm{v}_{1}\right)^{2}+(\mathrm{K}-1) / 2\right]\left[1-\left(\mathrm{v}_{1} / \mathrm{v}_{2}\right)^{2}\right]\right. \\
& \mathrm{P}_{2} / \mathrm{P}_{1}=\left(\mathrm{v}_{1} / \mathrm{v}_{2}\right)\left[1+((\mathrm{K}-1) / 2)\left(\mathrm{H}_{1}\right)^{2}\left(1-\left(\mathrm{v}_{2} / \mathrm{v}_{1}\right)^{2}\right)\right]
\end{aligned}
$$

Assuming $V_{2} / V_{1}=1.50$

$$
\begin{aligned}
& \mathrm{fL} / \mathrm{R}_{\mathrm{H}}= {[-2.3(1.4+1) / 1.4][\log 1.50] } \\
&+(1 / 2.4)\left[(1 / 500)^{2}+(1.4-1) / 2\right]\left[1-(1 / 1.50)^{2}\right]
\end{aligned}
$$

$f L / R_{H}=0.957$, which does not check the correct value of $\mathrm{fL} / \mathrm{H}_{\mathrm{H}}=1.026$.
$\begin{aligned} & \text { Adjusting } V_{2} / V_{2}= 1.60 \text { and repeating the above } \\ & \text { calculations gives } f L / R_{H}=1.026 .\end{aligned}$
Therefore $p_{2} / P_{1}$

$$
\begin{aligned}
& =(1 / 1.60)\left[1+((1.4-1) / 2)(.500)^{2}\left(1-(1.60)^{2}\right)\right]=.576 \\
P_{2} & =P_{1}\left(P_{2} / P_{1}\right)=(14.0)(.576)=8.06 \text { psia. } \\
T_{2} & =P_{1}\left(P_{2} / P_{1}\right)\left(V_{2} / V_{1}\right)=(535)(.576)(1.60)=493^{\circ} \mathrm{R} .
\end{aligned}
$$

## B. Method Using Fino Tables.

When using the Fino Tables (4) a length of Plotitious duct must be added to the actual duct so that the exit velocity equals the acoustic velocity.


The Iriction factor used in the fanno Tables is equal to 41
therefore $f^{\prime}=4 f=4(.0043)=.0172$.
From the Fanno Tables with $K=2.4$ and $M_{1}=.500$
1:L/D=P: $\left(X^{*}-X_{1}\right) / D=1.069 \quad T_{1} / T^{*}=1.143 \quad P_{1} / P^{*}=2.133$

From the original adata $L=X_{2}-X_{1}=20$
thereforef'L/D $=f^{\prime \prime}\left(X_{2}-X_{1}\right) / D=1.026$

$$
\begin{gathered}
f^{\prime}\left(x^{*}-x_{2}\right) / D=\left[f^{\prime}\left(x^{*}-x_{1}\right) / D\right]-\left[f^{\prime}\left(X_{2}-X_{1}\right) / D\right] \\
=1.069-1.026=0.043
\end{gathered}
$$

From the Panno Tables with $K=1.4$ and $f(L / D=0.043$

$$
T_{2} / T^{*}=1.052 \quad P_{2} / P^{*}=1.224
$$

therefor $T_{2}=T_{1}\left(T_{2} / T^{*}\right)\left(T T_{1} / T_{1}\right)=535(1.052)(1 / 1.143)=492^{\circ} \mathrm{R}$. and $P_{2} P_{1}\left(P_{2} / P^{*}\right)\left(P / P_{1}^{*}\right)=14.0(1.224)(1 / 2.138)=8.02$ pa1a.

## C. Lapple Method.

When using the Lapple charts (2) it muet be assumed that the air enters the pipe through a frictionless nozzle from a reservoir of atagnant alr. Therefore conditions in the theoretical reservoir must be determined.


$$
\begin{aligned}
& G_{1}=v_{1} P_{1}=(567)(.0707)=40.1 \mathrm{Ib} . / \mathrm{ft}_{*}^{2} \text { sec. } \\
& P_{1}=14.0(144)=2015 \mathrm{Ib} . / \mathrm{ft} .^{2} \\
& N=\pi L / R_{H}=1.026
\end{aligned}
$$

Assuming $p_{1} / p_{0}=0.900$, from the Lapple chart of $\mathrm{K}=1.4$ with $\mathrm{N}=0$

$$
\sigma / G_{\mathrm{cni}}=0.688 \text { and } T_{1} / T_{0}=0.972
$$

$$
\text { then } T_{0}=T_{1} / .972=535 / .972=5500 \mathrm{R}
$$

$$
\sigma_{\mathrm{cn} 1}=G / .688=40.1 / .688=58.2 \mathrm{Ib} . / \mathrm{ft} .^{2} \mathrm{sec}
$$

$$
a_{\mathrm{cni}}=p_{o} \sqrt{\mathrm{~g}_{\mathrm{c}}^{\mathrm{m} / \mathrm{oRT}_{\mathrm{o}}}}
$$

$$
p_{0}=58.2 / \sqrt{(32.2)(29) /(2.718)(1546)(550)}
$$

$P_{0}=3890 \mathrm{lb} . / 4 t{ }^{2}$
$P_{1} / P_{0}=2015 / 2890=0.698$, which does not check the assumed value of $p_{1} / p_{0}=.900$

Adjusting $p_{1} / p_{0}=.841$ gives $G / C_{\text {oni }}=0.840$ and $T_{1} / T_{0}$ * 0.952 from the Lapple chart.

Repeating the previous calculations gives $T_{0}=562^{\circ} \mathrm{R}$, $a_{\text {eni }}=47.7$ and $p_{0}=2395$
therefore $p_{1} / p_{0}=2015 / 2395=0.841$. This checks the assumed value.
Then for $\mathrm{W}=1.026, \mathrm{X}=1.4$ and $\mathrm{Q} / \mathrm{G}_{\mathrm{cni}}=0.840$ the Lapple chart gives $p_{2} / p_{0}=0.495$ and $T_{2} / T_{0}=.872$

$$
\begin{aligned}
& P_{2}=P_{1}\left(p_{2} / p_{0}\right)\left(p_{0} / p_{1}\right)=14.0(.495)(1 / .841)=8.23 \mathrm{pala} \\
& T_{2}=T_{1}\left(T_{2} / T_{0}\right)\left(T_{0} / T_{1}\right)=535(.872)(1 / .952)=4900 \mathrm{R} .
\end{aligned}
$$

D. Method Using Thompson's Nomography. (8) (5)

The use of the nomograph first requires the solution for $Y$ and $Z$ in the following equations:

$$
\begin{aligned}
& Y=(K+1) M_{1}^{2} /\left[2+(K-1) M_{1}^{2}\right] \\
& Y=(1.4+1)(0.5)^{2} /\left[2+(1.4-1)(0.5)^{2}\right] \\
& Y=0.286 \\
& 1-Z=M_{1}^{2} K /\left[2+(K-1) M_{1}^{2}\right] \\
& N=f(L / D=1.026 \\
& 1-Z=(1.026)(0.5)^{2}(1.4) /\left[2+(1.4-1)(0.5)^{2}\right] \\
& 1-Z=0.171 \quad Z=0.829
\end{aligned}
$$

Connecting $Y$ and $Z$ with a straight line on the homograph gives two values of $x, \quad X=.470$ and $x=.620$
The value of $X$ olosest to $Z$ is the correct value to be used, therefore $x=.620$
The $P_{2} / P_{1}$ and $T_{2} / T_{1}$ ration can then be calculated
from the following equations:

$$
\begin{aligned}
P_{2} / P_{1} & =x\left(1+\left[(x-1) m_{1}^{2} / 2\right]\left[1-\left(1 / x^{2}\right)\right]\right) \\
& =.620\left(1+\left[(1.4-1)(0.5)^{2} / 2\right]\left[1-\left(1 / .620^{2}\right)\right]\right) \\
P_{2} / P_{1} & =.570
\end{aligned}
$$

$$
\begin{aligned}
& T_{2} / T_{1}=\left(P_{2} / P_{1}\right) / X=.570 / .620=.920 \\
& P_{2}=P_{1}\left(P_{2} / P_{1}\right)=14.0(.570)=7.98 \mathrm{ps} 1 \mathrm{a} \\
& T_{2}=T_{1}\left(P_{2} / P_{1}\right)=535(.920)=492^{\circ} \mathrm{R}
\end{aligned}
$$

## E. Hethod Using the Graphs Developed in This Thesis.

$$
\begin{aligned}
& \text { Using the } K=1.4 \text { ohart, with } M=.500 \text { and } \mathrm{PL} / \mathrm{D}=.2565 \\
& P_{2} / P_{1}=.573 \text { and } T_{2} / T_{1}=.920 \\
& P_{2}=P_{1}\left(P_{2} / P_{1}\right)=14.0(.573)=8.02 \mathrm{p} 1 \mathrm{a} \\
& T_{2}=T_{1}\left(T_{2} / T_{1}\right)=535(.920)=4920 \mathrm{R} .
\end{aligned}
$$

## Summary of Resulte

Gethod
Trial and Error
$\mathrm{P}_{2}$
Panno Tables 8.02 ..... 4928.06 psia$493^{\circ} \mathrm{R}$

- Lapple Charta 8.23 ..... 490
Thompson's Nomograph ..... 7.98 ..... 492
Author's Graph 8.02 ..... 492
$\underline{T}$


## TABLE O NOMENCLATURE

$$
\begin{aligned}
& \rho=\text { density of fluid, lbs. mase/ft. }{ }^{3} \\
& \text { T = absolute temperature, }{ }^{\circ} \mathrm{R} \text {. } \\
& \mu=\text { viscosity of fluid, tba. mass/ft. sec. } \\
& \text { D = diameter or pipe, ft. } \\
& \text { v = velocity of fluid, it./sec. } \\
& p=\text { pressure of fluid, lbs./in. }{ }^{2} \text { abs. } \\
& \text { Re }=\text { Reynolds number, no units. } \\
& f=\text { friction factor, no units. } \\
& f^{\prime}=4 t=\text { friction factor, no units. } \\
& K=C_{p} / C_{v}=\text { specific heat ratio for fluid, no units. } \\
& C=\text { acoustic velocity in fluid, ft./sec. } \\
& g_{c}=\text { conversion factor, (aus, lass.) ft./(1bs, force) sec. }{ }^{2} \\
& \mathrm{R}=\text { gas constant, (lbs. force)ft./(lb, mole) }{ }^{\circ} \mathrm{R} \text {. } \\
& m=\text { molecular welgnt of fluid, lbs. mass/lb. mole. } \\
& \mathrm{L}=\mathrm{leng} \mathrm{th}, \mathrm{ft} . \\
& \mathrm{R}_{\mathrm{H}}=\text { hydraulic radius, } \mathrm{ft} .=\mathrm{D} / 4 \text { for circular ducts. } \\
& X \text { distance from zero reference point, ft. (fino method) } \\
& X^{*}=\text { distance from outlet of fictitious acct to zero } \\
& \text { reference point, ft. (kano method) } \\
& G=\text { mass velocity of fluid, lbs, mass } / \mathrm{ft}^{2} \text { sem. }
\end{aligned}
$$

```
M= Mach number, ratio of velooity to acoustic velocity
    in fluid, no units.
p = pressure of fluld, lbs./ft.' abs. (Lapple method)
sub l = conditions at pipe inlet.
sub 2 = conditions at plpe outlet.
sub O = conditions in stagnant reservoir.(Lapple method)
```


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## APPENDIX

TABULATED RESULGS
$K=1.1 \quad \%=0.300$

| $v_{2} / v_{1}$ | $\mathrm{fL} / \mathrm{D}$ | $P_{2} / P_{1}$ | $\mathrm{T}_{2} / \mathrm{T}_{1}$ |
| :---: | :---: | :---: | :---: |
| 1.05 | 0.2068 | 0.9519 | 0.99954 |
| 1.1 | 0.3949 | 0.9082 | 0.99905 |
| 1.2 | 0.6882 | 0.8317 | 0.99802 |
| 1.3 | 0.9105 | 0.7668 | 0.99689 |
| 1.4 | 1.0818 | 0.7112 | 0.99568 |
| 1.5 | 1.2158 | 0.6629 | 0.99437 |
| 1.6 | 1.3215 | 0.6206 | 0.99298 |
| 1.7 | 1.4057 | 0.5832 | 0.99149 |
| 1.8 | 2.4733 | 0.5500 | 0.98992 |
| 1.9 | 1.5276 | 0.5201 | 0.98825 |
| 2.0 | 1.5716 | 0.4933 | 0.98650 |
| 2.1 | 1.6072 | 0.4689 | 0.98465 |
| 2.2 | 1.6362 | 0.4467 | 0.95272 |
| 2.3 | 1.6596 | 0.4264 | 0.98069 |
| 2.4 | 1.6784 | 0.4077 | 0.97858 |
| 2.5 | 1.6934 | 0.3905 | 0.97637 |
| 2.6 | 1.7054 | 0.3746 | 0.97408 |
| 2.7 | 1.7145 | 0.3599 | 0.97169 |
| 2.8 | 1.7215 | 0.3462 | 0.96922 |

$$
K=1.1
$$

$v_{2} / v_{1}$

| 2.9 | 1.7268 |
| :--- | :--- |
| 3.0 | 1.7304 |
| 3.1 | 1.7325 |
| 3.2 | 1.7336 |
| 3.3 | 1.7339 |

$11=0.300$
$P_{2} / P_{1} \quad T_{2} / T_{1}$

| 0.3333 | 0.96665 |
| :--- | :--- |
| 0.3213 | 0.96400 |
| 0.3103 | 0.96125 |
| 0.2995 | 0.95842 |
| 0.2595 | 0.95549 |
| $0.2934=P_{c} / P_{1}$ |  |

## TABULATED RESULPS

$$
K=1.1 \quad i n=0.325
$$

| $v_{2} / v_{1}$ | PL/D | $P_{2} / P_{1}$ | $\mathrm{s}_{2} / \mathrm{s}_{1}$ |
| :---: | :---: | :---: | :---: |
| 1.05 | 0.1729 | 0.9519 | 0.99946 |
| 1.1 | 0.3300 | 0.9081 | 0.99889 |
| 1.2 | 0.5740 | 0.8314 | 0.99768 |
| 1.3 | 0.7580 | 0.7664 | 0.99636 |
| 1.4 | 0.8989 | 0.7107 | 0.99493 |
| 1.5 | 1.0083 | 0.6623 | 0.99340 |
| 1.6 | 1.0939 | 0.6199 | 0.99176 |
| 1.7 | 1.1614 | 0.5824 | 0.99002 |
| 1.8 | 1.2150 | 0.5490 | 0.98817 |
| 1.9 | 1.2576 | 0.5191 | 0.98622 |
| 2.0 | 1.2915 | 0.4921 | 0.98416 |
| 2.1 | 1.3184 | 0.4676 | 0.98199 |
| 2.2 | 1.3399 | 0.4453 | 0.97972 |
| 2.3 | 1.3567 | 0.4249 | 0.97734 |
| 2.4 | 2.3697 | 0.4062 | 0.97486 |
| 2.5 | 1.3797 | 0.3839 | 0.97227 |
| 2.6 | 1.3371 | 0.3729 | 0.96958 |
| 2.7 | $1.39<3$ | 0.3581 | 0.96678 |
| 2.8 | 1.3957 | 0.3442 | 0.96388 |


|  | $K=1.1$ | $N=0.325$ |  |
| :--- | :--- | :--- | :--- |
| $\mathbf{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{PL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
|  |  | 0.3313 | 0.96083 |
| 2.9 | 1.3977 | 0.3193 | 0.95775 |
| 3.0 | 1.3934 | 0.3079 | 0.95453 |
| 3.1 | 1.3980 | $0.3180=\mathrm{P}_{\mathrm{c}} / \mathrm{P}_{1}$ |  |

TABULATED RESULTE

$$
K=1.1 \quad K=0.350
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1440 | 0.9518 | 0.99939 |
| 1.1 | 0.2746 | 0.9079 | 0.99871 |
| 1.2 | 0.4765 | 0.8311 | 0.99730 |
| 1.3 | 0.6276 | 0.7660 | 0.99577 |
| 1.4 | 0.7425 | 0.7101 | 0.99412 |
| 1.5 | 0.8310 | 0.6616 | 0.99234 |
| 1.6 | 0.8993 | 0.6190 | 0.99044 |
| 1.7 | 0.9527 | 0.5814 | 0.98842 |
| 1.8 | 0.9943 | 0.5479 | 0.98628 |
| 1.9 | 1.0268 | 0.5179 | 0.98401 |
| 2.0 | 1.0521 | 0.4908 | 0.98162 |
| 2.1 | 1.0716 | 0.4662 | 0.97911 |
| 2.2 | 1.0866 | 0.4439 | 0.97648 |
| 2.3 | 1.1060 | 0.4234 | 0.97372 |
| 2.4 | 1.1115 | 0.4045 | 0.97085 |
| 2.5 | 1.1151 | 0.3871 | 0.96784 |
| 2.6 | 1.1168 | 0.3710 | 0.96472 |
| 2.7 | 1.1172 | 0.3561 | 0.96147 |
| 2.8 |  | 0.3422 | 0.95811 |
|  |  |  |  |

## TABULATED RESULPS

$$
K=1.1 \quad \hat{k}=0.375
$$

| $V_{2} / V_{1}$ | LL/D | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1243 | 0.9517 | 0.99928 |
| 1.1 | 0.2370 | 0.9077 | 0.99852 |
| 1.2 | 0.4103 | 0.8308 | 0.99691 |
| 1.3 | 0.5393 | 0.7655 | 0.99515 |
| 1.4 | 0.6366 | 0.7095 | 0.99325 |
| 1.5 | 0.7107 | 0.6608 | 0.99121 |
| 1.6 | 0.7675 | 0.6181 | 0.98903 |
| 1.7 | 0.811 | 0.5804 | 0.98671 |
| 1.8 | 0.8703 | 0.5468 | 0.98425 |
| 1.9 | 0.9043 | 0.489 | 0.98165 |
| 2.0 | 0.9149 | 0.4224 | 0.4423 |

## TABULATED RESULTS

$$
K=1.1 \quad M=0.400
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1066 | 0.9515 | 0.99912 |
| 1.1 | 0.2031 | 0.9076 | 0.99836 |
| 1.2 | 0.3506 | 0.8304 | 0.99648 |
| 1.3 | 0.4594 | 0.7650 | 0.99448 |
| 1.4 | 0.5407 | 0.7088 | 0.99232 |
| 1.5 | 0.6020 | 0.6600 | 0.99000 |
| 1.6 | 0.6482 | 0.6172 | 0.98752 |
| 1.7 | 0.6831 | 0.5793 | 0.98488 |
| 1.8 | 0.7094 | 0.5456 | 0.98208 |
| 1.9 | 0.7288 | 0.5153 | 0.97912 |
| 2.0 | 0.7430 | 0.4880 | 0.97600 |
| 2.1 | 0.7530 | 0.4632 | 0.97272 |
| 2.2 | 0.7597 | 0.4406 | 0.96928 |
| 2.3 | 0.7637 | 0.4199 | 0.96568 |
| 2.4 | 0.7654 | 0.4008 | 0.96192 |
| 2.5 | 0.7654 | 0.3832 | 0.95800 |
|  |  | $0.3919=P_{c} / P_{1}$ |  |

## TABULATED RESULTS

$$
K=1.1 \quad N=0.425
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{fL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0919 | 0.9515 | 0.99907 |
| 1.1 | 0.1749 | 0.9074 | 0.99810 |
| 1.2 | 0.3010 | 0.8300 | 0.99603 |
| 1.3 | 0.3932 | 0.7644 | 0.99377 |
| 1.4 | 0.4613 | 0.7081 | 0.99133 |
| 1.5 | 0.5119 | 0.6591 | 0.98871 |
| 1.6 | 0.5494 | 0.6162 | 0.98591 |
| 1.7 | 0.5771 | 0.5782 | 0.98293 |
| 1.8 | 0.5973 | 0.5443 | 0.97977 |
| 1.9 | 0.6116 | 0.5139 | 0.97643 |
| 2.0 | 0.6214 | 0.4865 | 0.97291 |
| 2.1 | 0.6275 | 0.4615 | 0.96920 |
| 2.2 | 0.6310 | 0.4388 | 0.96532 |
| 2.3 | 0.6321 | 0.4179 | 0.96126 |
| 2.4 | 0.6314 | 0.3988 | 0.95701 |

## TABULATED RESULTS

$$
x=2.1 \quad 3=0.450
$$

| $v_{2} / v_{1}$ | LL/D | $\mathrm{P}_{2} / \mathrm{P}_{2}$ | $S_{2} / T_{1}$ |
| :---: | :---: | :---: | :---: |
| 1.05 | 0.0795 | 0.9514 | 0.99896 |
| 2.1 | 0.1513 | 0.9072 | 0.99787 |
| 1.2 | 0.2594 | 0.8296 | 0.99554 |
| 1.3 | 0.3377 | 0.7639 | 0.99301 |
| 1.4 | 0.3947 | 0.7073 | 0.99028 |
| 1.5 | 0.4364 | 0.6582 | 0.98734 |
| 1.6 | 0.4666 | 0.6151 | 0.98420 |
| 1.7 | 0.4882 | 0.5770 | 0.98086 |
| 1.8 | 0.5033 | 0.5430 | 0.97732 |
| 1.9 | 0.5133 | 0.5124 | 0.97357 |
| 2.0 | 0.5195 | 0.4848 | 0.96962 |
| 2.1 | 0.5225 | 0.4597 | 0.96547 |
| 2.2 | 0.5232 | 0.4369 | 0.96112 |
|  |  | 0.4414 |  |

## TABULATED RESULTS

$$
\mathrm{K}=1.1 \quad \text { w}=0.500
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0602 | 0.9512 | 0.99872 |
| 1.1 | 0.1143 | 0.9067 | 0.99738 |
| 1.2 | 0.1943 | 0.8288 | 0.99450 |
| 1.3 | 0.2506 | 0.7626 | 0.99137 |
| 1.4 | 0.2903 | 0.7057 | 0.98800 |
| 1.5 | 0.3179 | 0.6562 | 0.98437 |
| 1.6 | 0.3366 | 0.6128 | 0.98050 |
| 1.7 | 0.3559 | 0.5743 | 0.97637 |
| 1.8 | 0.3592 | 0.5400 | 0.97200 |
| 1.9 | 0.3596 | 0.4813 | 0.96737 |
| 2.0 |  |  |  |

## TABULATTD RESULTE

$$
K=1.1 \quad M=0.550
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $P_{2} / P_{1}$ | $T_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0459 | 0.9509 | 0.99845 |
| 1.1 | 0.0869 | 0.9062 | 0.99682 |
| 1.2 | 0.1461 | 0.8278 | 0.99334 |
| 1.3 | 0.1862 | 0.7612 | 0.98954 |
| 1.4 | 0.2130 | 0.7039 | 0.98548 |
| 1.5 | 0.2302 | 0.6541 | 0.98109 |
| 1.6 | 0.2405 | 0.6103 | 0.97640 |
| 1.7 | 0.2455 | 0.5714 | 0.97141 |
| 1.8 |  | 0.5367 | 0.96612 |

## TABULATED RESULTS

K $=1.1$

$$
B=0.600
$$

$v_{2} / v_{1}$
1L/D
$P_{2} / F_{1}$
$T_{2} / T_{1}$
1.05
1.1
1.2
1.3
1.4
1.5
1.6
1.7
$0.9506 \quad 0.99815$
$0.9057 \quad 0.99622$
0.1094
$0.8267 \quad 0.99208$
0.1372
0.7597
0.98758
0.1542
0.7019
0.98272
0.1636
0.6517
0.97750
0.1673
0.6075
0.97192
0.1671
0.5682
0.96598
$0.5907=P_{0} / P_{1}$

## TABULATED RESULTS

|  | $\mathrm{K}=1.1$ | $=0.650$ |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{~V}_{2} / \mathrm{V}_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| 1.05 | 0.0265 | 0.9503 | 0.99783 |
| 1.1 | 0.0499 | 0.9051 | 0.99556 |
| 1.2 | 0.0809 | 0.8256 | 0.99070 |
| 1.3 | 0.0991 | 0.7580 | 0.98542 |
| 1.4 | 0.1085 | 0.6998 | 0.97972 |
| 1.5 | 0.1117 | 0.6491 | 0.97359 |
| 1.6 | 0.1104 | 0.6044 | 0.96704 |
|  |  | $0.6410=\mathrm{P}_{\mathrm{c}} / \mathrm{P}_{1}$ |  |

## TABULATED RESULTS

$$
K=1.1 \quad K=0.700
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{TL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0198 | 0.9500 | 0.99749 |
| 1.1 | 0.0370 | 0.9044 | 0.99485 |
| 1.2 | 0.0582 | 0.8244 | 0.98922 |
| 1.3 | 0.0688 | 0.7562 | 0.98309 |
| 1.4 | 0.0722 | 0.6975 | 0.97648 |
| 1.5 | 0.0705 | 0.6462 | 0.96937 |
|  |  | $0.6914=\mathrm{P}_{\mathrm{o}} / \mathrm{P}_{1}$ |  |

## TABULATED RESULTS

$$
K=1.4 \quad N=0.300
$$

| $V_{2} / V_{1}$ | IL/D | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1623 | 0.9506 | 0.99815 |
| 1.1 | 0.3098 | 0.9057 | 0.99622 |
| 1.2 | 0.5391 | 0.8267 | 0.99208 |
| 1.3 | 0.7122 | 0.7597 | 0.98758 |
| 1.4 | 0.8451 | 0.7019 | 0.98272 |
| 1.5 | 0.9484 | 0.6517 | 0.97750 |
| 1.6 | 1.0294 | 0.6075 | 0.97192 |
| 1.7 | 1.1446 | 0.5682 | 0.96592 |
| 1.8 | 1.2178 | 0.5332 | 0.95968 |
| 1.9 | 1.2437 | 1.2646 | 0.5016 |

$$
K=1.4 \quad \text { 煺 }=0.300
$$

| $V_{2} / V_{1}$ | IL/D | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 2.9 | 1.3234 | 0.2988 | 0.86662 |
| 3.0 | 1.3246 | 0.2853 | 0.85600 |
| 3.1 | 1.3235 | 0.2726 | 0.84502 |
|  |  | $0.2763=P_{0} / P_{1}$ |  |

## TABULATED RESULTS

$$
K=1.4 \quad M=0.325
$$

| $V_{2} / V_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1357 | 0.9503 | 0.99783 |
| 1.1 | 0.2589 | 0.9051 | 0.99556 |
| 1.2 | 0.4494 | 0.8256 | 0.99070 |
| 1.3 | 0.5924 | 0.7580 | 0.98542 |
| 1.4 | 0.7013 | 0.6998 | 0.97972 |
| 1.5 | 0.7854 | 0.6491 | 0.97359 |
| 1.6 | 0.8506 | 0.6044 | 0.96704 |
| 1.7 | 0.9016 | 0.5647 | 0.96007 |
| 1.8 | 0.9417 | 0.5293 | 0.95268 |
| 1.9 | 0.9730 | 0.4973 | 0.94486 |
| 2.0 | 1.0168 | 0.4683 | 0.93662 |
| 2.1 | 1.0318 | 0.4419 | 0.92796 |
| 2.2 | 1.0431 | 0.4177 | 0.91888 |
| 2.3 | 1.0514 | 0.3954 | 0.90934 |
| 2.4 | 1.0574 | 0.3748 | 0.89944 |
| 2.5 | 1.0615 | 0.3556 | 0.88909 |
| 2.6 | 1.0638 | 0.3378 | 0.87832 |
| 2.7 | 1.0648 | 0.3055 | 0.86712 |
| 2.8 | 1.0647 | 0.2908 | 0.85550 |
| 2.9 |  | $0.2998=P_{0} / P_{1}$ |  |
|  |  |  | 0.84346 |

## TABULATED RESULTE

$$
K=1.4 \quad u=0.350
$$

| $V_{2} / V_{1}$ | $P_{1} / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1145 | 0.9500 | 0.99749 |
| 1.1 | 0.2184 | 0.9044 | 0.99485 |
| 1.2 | 0.3783 | 0.8244 | 0.98922 |
| 1.3 | 0.4973 | 0.7562 | 0.98309 |
| 1.4 | 0.5873 | 0.6975 | 0.97648 |
| 1.5 | 0.6560 | 0.6462 | 0.96937 |
| 1.6 | 0.7087 | 0.6011 | 0.96178 |
| 1.7 | 0.7493 | 0.5610 | 0.95369 |
| 1.8 | 0.7806 | 0.5251 | 0.94512 |
| 1.9 | 0.8047 | 0.4927 | 0.93605 |
| 2.0 | 0.8230 | 0.4633 | 0.92650 |
| 2.1 | 0.8367 | 0.4364 | 0.91645 |
| 2.2 | 0.8470 | 0.4118 | 0.90592 |
| 2.3 | 0.8542 | 0.3891 | 0.89489 |
| 2.4 | 0.8590 | 0.3681 | 0.88338 |
| 2.5 | 0.8618 | 0.3485 | 0.87137 |
| 2.6 | 0.8630 | 0.3303 | 0.85888 |
| 2.7 | 0.8628 | 0.3133 | 0.84589 |

## TABULATED RESULTS

$$
K=1.4 \quad M=0.375
$$

| $V_{2} / V_{2}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0975 | 0.9496 | 0.99712 |
| 1.1 | 0.1858 | 0.9037 | 0.99409 |
| 1.2 | 0.3209 | 0.8230 | 0.98762 |
| 1.3 | 0.4206 | 0.7543 | 0.98059 |
| 1.4 | 0.4953 | 0.6950 | 0.97300 |
| 1.5 | 0.5516 | 0.6432 | 0.96484 |
| 1.6 | 0.5942 | 0.5976 | 0.95612 |
| 1.7 | 0.6265 | 0.5570 | 0.94684 |
| 1.8 | 0.6508 | 0.5206 | 0.93700 |
| 1.9 | 0.6689 | 0.4877 | 0.92659 |
| 2.0 | 0.6821 | 0.4578 | 0.91562 |
| 2.1 | 0.6915 | 0.4305 | 0.90409 |
| 2.2 | 0.6980 | 0.4055 | 0.89200 |
| 2.3 | 0.7019 | 0.3823 | 0.57934 |
| 2.4 | 0.7038 | 0.3609 | 0.86612 |
| 2.5 | 0.7040 | 0.3409 | 0.85234 |

## TABULATED RESULTS

$$
K=1.4 \quad 3=0.400
$$

| $V_{2} / V_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0636 | 0.9493 | 0.99672 |
| 1.1 | 0.1591 | 0.9030 | 0.99328 |
| 1.2 | 0.2739 | 0.8216 | 0.98592 |
| 1.3 | 0.3578 | 0.7522 | 0.97792 |
| 1.4 | 0.4200 | 0.6923 | 0.96928 |
| 1.5 | 0.4662 | 0.6400 | 0.96000 |
| 1.6 | 0.5005 | 0.5988 | 0.95008 |
| 1.7 | 0.5259 | 0.5527 | 0.93952 |
| 1.8 | 0.5445 | 0.5157 | 0.92832 |
| 1.9 | 0.5577 | 0.4824 | 0.91648 |
| 2.0 | 0.5668 | 0.4520 | 0.90400 |
| 2.1 | 0.5726 | 0.4242 | 0.89088 |
| 2.2 | 0.5759 | 0.3987 | 0.87712 |
| 2.3 | 0.5772 | 0.3751 | 0.86272 |
| 2.4 | 0.5767 | 0.3532 | 0.84768 |

## TABULATED RESULTS

$$
K=1.4 \quad M=0.450
$$

| $V_{2} / V_{1}$ | $P L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0623 | 0.9484 | 0.99585 |
| 1.1 | 0.1185 | 0.9014 | 0.99149 |
| 1.2 | 0.2023 | 0.8185 | 0.98218 |
| 1.3 | 0.2622 | 0.7477 | 0.97205 |
| 1.4 | 0.3052 | 0.6865 | 0.96112 |
| 1.5 | 0.3360 | 0.6329 | 0.94937 |
| 1.6 | 0.3577 | 0.5855 | 0.93682 |
| 1.7 | 0.3727 | 0.5432 | 0.92345 |
| 1.8 | 0.3825 | 0.5052 | 0.90928 |
| 1.9 | 0.3883 | 0.4707 | 0.89429 |
| 2.0 | 0.3911 | 0.4393 | 0.87850 |
| 2.1 | 0.3915 | 0.4104 | 0.86189 |
|  |  | $0.4190=P_{c} / P_{1}$ |  |

## TABULATED RESULTE

$$
K=1.4 \quad M=0.500
$$

| $V_{2} / V_{1}$ | $1 \mathrm{~L} / \mathrm{D}$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0471 | 0.9475 | 0.99487 |
| 1.1 | 0.0894 | 0.8995 | 0.98950 |
| 1.2 | 0.1511 | 0.8150 | 0.97800 |
| 1.3 | 0.1938 | 0.7427 | 0.96550 |
| 1.4 | 0.2232 | 0.6800 | 0.95200 |
| 1.5 | 0.2429 | 0.6250 | 0.93750 |
| 1.6 | 0.2556 | 0.5763 | 0.92200 |
| 1.7 | 0.2667 | 0.5326 | 0.90550 |
| 1.8 | 0.2672 | 0.4933 | 0.88800 |
| 1.9 |  | 0.4576 | 0.86950 |

## TABULATED RESULTS

$$
K=1.4 \quad M=0.600
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0273 | 0.9454 | 0.99262 |
| 1.1 | 0.0515 | 0.8953 | 0.98488 |
| 1.2 | 0.0844 | 0.8069 | 0.96832 |
| 1.3 | 0.1047 | 0.7310 | 0.95032 |
| 1.4 | 0.1163 | 0.6649 | 0.93088 |
| 1.5 | 0.1217 | 0.6067 | 0.91000 |
| 1.6 | 0.1227 | 0.5548 | 0.88768 |
|  |  | $0.5671=P_{0} / P_{1}$ |  |

## Tabulated assults

$$
x=2.4 \quad u=0.700
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{PL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0131 | 0.9428 | 0.98995 |
| 1.1 | 0.0243 | 0.8904 | 0.97942 |
| 1.2 | 0.0365 | 0.7974 | 0.95688 |
| 1.3 | 0.0407 | 0.7172 | 0.93238 |
| 1.4 | 0.0395 | 0.6471 | 0.90592 |
|  |  | $0.6696=P_{\mathrm{c}} / \mathrm{P}_{1}$ |  |

## TABULATED RESULTS

$$
x=1.67 \quad u=0.300
$$

| $V_{2} / V_{1}$ | $\mathrm{IL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1359 | 0.9494 | 0.99691 |
| 1.1 | 0.2594 | 0.9033 | 0.99367 |
| 1.2 | 0.4508 | 0.3223 | 0.98673 |
| 1.3 | 0.5948 | 0.7532 | 0.97920 |
| 1.4 | 0.7048 | 0.6936 | 0.97106 |
| 1.5 | 0.7900 | 0.6415 | 0.96231 |
| 1.6 | 0.8564 | 0.5956 | 0.95297 |
| 1.7 | 0.9085 | 0.5547 | 0.94302 |
| 1.8 | 0.9498 | 0.5180 | 0.93246 |
| 1.9 | 0.9823 | 0.4849 | 0.92131 |
| 2.0 | 1.0081 | 0.4548 | 0.90955 |
| 2.1 | 1.0283 | 0.4272 | 0.39719 |
| 2.2 | 1.0443 | 0.4019 | 0.83422 |
| 2.3 | 1.0567 | 0.3785 | 0.37066 |
| 2.4 | 1.0731 | 0.3569 | 0.85649 |
| 2.5 | 1.0782 | 0.3367 | 0.84171 |
| 2.6 | 1.0814 | 0.3001 | 0.82634 |
| 2.7 | 1.0833 | 0.2835 | 0.81036 |
| 2.8 |  |  | 0.79377 |

$$
K=1.67 \quad u=0.300
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | PL/D | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 2.9 | 1.0842 | 0.2678 | 0.77659 |
| 3.0 | 1.0840 | 0.2529 | 0.75880 |
|  |  | $0.2635=P_{\mathrm{c}} / P_{1}$ |  |

## TABULATED RESULTS

$$
x=1.67 \quad M=0.325
$$

| $V_{2} / V_{1}$ | $R L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.1136 | 0.9489 | 0.99637 |
| 1.1 | 0.2167 | 0.9023 | 0.99257 |
| 1.2 | 0.3756 | 0.8204 | 0.98443 |
| 1.3 | 0.4943 | 0.7505 | 0.97559 |
| 1.4 | 0.5842 | 0.6900 | 0.96603 |
| 1.5 | 0.6532 | 0.6372 | 0.95577 |
| 1.6 | 0.7064 | 0.5905 | 0.94480 |
| 1.7 | 0.7476 | 0.5489 | 0.93312 |
| 1.8 | 0.7796 | 0.5115 | 0.92074 |
| 1.9 | 0.8044 | 0.4777 | 0.90765 |
| 2.0 | 0.8235 | 0.4469 | 0.89385 |
| 2.1 | 0.8380 | 0.4187 | 0.87934 |
| 2.2 | 0.8491 | 0.3928 | 0.86413 |
| 2.3 | 0.8571 | 0.3688 | 0.84820 |
| 2.4 | 0.8627 | 0.3465 | 0.83157 |
| 2.5 | 0.8664 | 0.3257 | 0.81423 |
| 2.6 | 0.8685 | 0.3062 | 0.79619 |
| 2.7 | 0.8691 | 0.2879 | 0.77743 |
| 2.8 | 0.8686 | 0.2707 | 0.75797 |
|  |  | $0.2862=P_{0} / P_{1}$ |  |

## TABULATED RESULTS

$$
X=1.67 \quad K=0.350
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0959 | 0.9484 | 0.99579 |
| 1.1 | 0.1828 | 0.9013 | 0.99138 |
| 1.2 | 0.3159 | 0.8183 | 0.98194 |
| 1.3 | 0.4146 | 0.7474 | 0.97168 |
| 1.4 | 0.4886 | 0.6861 | 0.96060 |
| 1.5 | 0.5448 | 0.6325 | 0.94870 |
| 1.6 | 0.5874 | 0.5850 | 0.93598 |
| 1.7 | 0.6199 | 0.5426 | 0.92243 |
| 1.8 | 0.6447 | 0.5045 | 0.90807 |
| 1.9 | 0.6633 | 0.4699 | 0.89289 |
| 2.0 | 0.6771 | 0.4384 | 0.87688 |
| 2.1 | 0.6871 | 0.4095 | 0.86005 |
| 2.2 | 0.6942 | 0.3829 | 0.84241 |
| 2.3 | 0.6988 | 0.3582 | 0.82394 |
| 2.4 | 0.7014 | 0.3353 | 0.80465 |
| 2.5 | 0.7024 | 0.3138 | 0.75454 |
| 2.6 | 0.7021 | 0.2937 | 0.76362 |

## TABULATED RESULTS

$$
K=1.67 \quad K=0.375
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0816 | 0.9478 | 0.99517 |
| 1.1 | 0.1554 | 0.9001 | 0.99011 |
| 1.2 | 0.2678 | 0.8161 | 0.97927 |
| 1.3 | 0.3503 | 0.7442 | 0.96749 |
| 1.4 | 0.4115 | 0.6820 | 0.95477 |
| 1.5 | 0.4573 | 0.6274 | 0.94111 |
| 1.6 | 0.4914 | 0.5791 | 0.92651 |
| 1.7 | 0.5169 | 0.5359 | 0.91096 |
| 1.5 | 0.5358 | 0.4969 | 0.39448 |
| 1.9 | 0.5494 | 0.4616 | 0.87704 |
| 2.0 | 0.5590 | 0.4293 | 0.85867 |
| 2.1 | 0.5653 | 0.3997 | 0.83935 |
| 2.2 | 0.5693 | 0.3723 | 0.81910 |
| 2.3 | 0.5711 | 0.3469 | 0.79790 |
| 2.4 | 0.5712 | 0.3232 | 0.77576 |

## TABULATED RESULTE

$$
x=1.67 \quad K=0.400
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0699 | 0.9472 | 0.99451 |
| 1.1 | 0.1330 | 0.8989 | 0.98874 |
| 1.2 | 0.2284 | 0.8137 | 0.97642 |
| 1.3 | 0.2976 | 0.7408 | 0.96302 |
| 1.4 | 0.3483 | 0.6775 | 0.94854 |
| 1.5 | 0.3857 | 0.6220 | 0.93300 |
| 1.6 | 0.4129 | 0.5727 | 0.91638 |
| 1.7 | 0.4326 | 0.5286 | 0.89870 |
| 1.8 | 0.4466 | 0.4889 | 0.87994 |
| 1.9 | 0.4562 | 0.4527 | 0.86010 |
| 2.0 | 0.4623 | 0.4196 | 0.83920 |
| 2.1 | 0.4657 | 0.3892 | 0.81722 |
| 2.2 | 0.4670 | 0.3610 | 0.79418 |
| 2.3 | 0.4666 | 0.3348 | 0.77006 |

## TABULATED RESULTS

$x=1.67$
$M=0.450$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{TL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0521 | 0.9458 | 0.99305 |
| 1.1 | 0.0990 | 0.8961 | 0.98575 |
| 1.2 | 0.1684 | 0.8085 | 0.97015 |
| 1.3 | 0.2275 | 0.7332 | 0.95319 |
| 1.4 | 0.2522 | 0.6678 | 0.93488 |
| 1.5 | 0.2765 | 0.6101 | 0.91520 |
| 1.6 | 0.2932 | 0.5589 | 0.89417 |
| 1.7 | 0.3042 | 0.5128 | 0.87179 |
| 1.8 | 0.3109 | 0.4711 | 0.84804 |
| 1.9 | 0.3142 | 0.4331 | 0.82294 |
| 2.0 | 0.3150 | 0.3982 | 0.79649 |
|  |  | $0.4025=P_{c} / \mathrm{P}_{1}$ |  |

## TABULATED RESULTS

$$
x=1.67 \quad M=0.500
$$

| $V_{2} / V_{1}$ | $I L / D$ | $P_{2} / P_{1}$ | $T_{2} / T_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0394 | 0.9442 | 0.99141 |
| 1.1 | 0.0746 | 0.8931 | 0.98241 |
| 1.2 | 0.1255 | 0.8026 | 0.96315 |
| 1.3 | 0.1601 | 0.7248 | 0.94221 |
| 1.4 | 0.1834 | 0.6569 | 0.91960 |
| 1.5 | 0.1985 | 0.5969 | 0.89531 |
| 1.6 | 0.2076 | 0.5433 | 0.86935 |
| 1.7 | 0.2124 | 0.4951 | 0.84171 |
| 1.8 | 0.2138 | 0.4513 | 0.81240 |
| 1.9 | 0.2127 | 0.4113 | 0.78141 |
|  |  | $0.4453=P_{0} / P_{1}$ |  |

## TABULATED RESULTS

$$
K=1.67 \quad x=0.600
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{rL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
| 1.05 | 0.0228 | 0.9406 | 0.98764 |
| 1.1 | 0.0428 | 0.8861 | 0.97467 |
| 1.2 | 0.0695 | 0.7891 | 0.94694 |
| 1.3 | 0.0854 | 0.7052 | 0.91679 |
| 1.4 | 0.0938 | 0.6316 | 0.88422 |
| 1.5 | 0.0969 | 0.5662 | 0.84925 |
| 1.6 | 0.0961 | 0.5074 | 0.81186 |
|  |  | $0.5497=\mathrm{P}_{\mathrm{c}} / \mathrm{P}_{1}$ |  |

## TABULATED RESULTS

$$
x=1.67 \quad u=0.700
$$

| $\mathrm{V}_{2} / \mathrm{V}_{1}$ | $\mathrm{PL} / \mathrm{D}$ | $\mathrm{P}_{2} / \mathrm{P}_{1}$ | $\mathrm{~T}_{2} / \mathrm{T}_{1}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 1.05 | 0.0128 | 0.9364 | 0.98317 |
| 1.1 | 0.0236 | 0.8778 | 0.96553 |
| 1.2 | 0.0358 | 0.7731 | 0.92777 |
| 1.3 | 0.0403 | 0.6821 | 0.88674 |
| 1.4 | 0.0397 | 0.6017 | 0.84242 |
|  |  | $0.6537=\mathrm{P}_{0} / \mathrm{P}_{1}$ |  |

